1. A central particle of charge \(-3q\) is surrounded by a hexagonal array of other charged particles \((q>0)\). The length of a side is \(s\), and charges are placed at each corner.

(a) [6 points] Find the component of the force along the \(x\)-axis \((F_x)\) on the central particle.

The force along the horizontal direction and along the direction from bottom left to upper right cancels. Thus, only the force from the upper left charge and bottom right charges matter. The upper left charge is greatest in magnitude, and the force is attractive since the middle charge is negative. Thus:

\[
F_x = -K \frac{(4q)(3q)}{s^2} \cos 60^\circ + K \frac{(2q)(3q)}{s^2} \cos 60^\circ \\
F_x = -K \frac{6q^2}{s^2} \frac{1}{2} = -3K \frac{q^2}{s^2}
\]

(b) [6 points] Find the component of the force along the \(y\)-axis \((F_y)\) on the central particle.

\[
F_x = +K \frac{(4q)(3q)}{s^2} \sin 60^\circ - K \frac{(2q)(3q)}{s^2} \sin 60^\circ \\
F_x = +K \frac{6q^2\sqrt{3}}{s^2} \frac{1}{2} = +3\sqrt{3}K \frac{q^2}{s^2}
\]
2. Consider electric charge distributed along a one-dimensional path in the form shown as two sections of $\frac{1}{4}$ of a circle each. The circle is centered at the origin with a radius of $R$, and the linear charge density is $+\lambda$ in the left quadrant and $-\lambda$ in the right.

   (a) [6 points] Find the component of the electric field along the $x$-axis ($E_x$) at the origin (0,0).

   The field contribution from the left arc equals that of the right arc. So we can calculate twice the $x$-component of the field from the right arc:

   \[
   E_x = 2 \int K \frac{dq}{R^2} \cos \theta = 2 \int_0^{\pi/2} \frac{\lambda R d\theta}{R^2} \cos \theta
   \]

   \[
   E_x = 2K \frac{\lambda}{R} \left[ \sin \theta \right]_0^{\pi/2} = \frac{2K\lambda}{R}
   \]

   (b) [6 points] Find the component of the electric field along the $y$-axis ($E_y$) at the origin (0,0).

   The field cancels in the $y$ direction.
3. Consider a cube with side length \( s = 2 \) m and one corner at the origin \((0,0,0)\) as shown.

(a) [6 points] What is the total charge enclosed by the cube if the electric field is
\[
\mathbf{E} = (3x^2 \hat{i} + 5\hat{j} - 2z \hat{k}) \text{ N/C.}
\]

\[
\Phi = \frac{q_{enc}}{\varepsilon_0} = \sum_{i=1}^{6} \Phi_i = \Phi(z = s) + \Phi(x = s) \quad \text{others equal 0 because field in those directions is constant}
\]

\[
= 3s^2s^2 - 2ss^2 = s^3 (3s - 2)
\]

\[
q_{enc} = \varepsilon_0 s^3 (3s - 2) = 32\varepsilon_0 = 2.8 \times 10^{-10} \text{ C}
\]

(b) [6 points] What is the electric charge density \((\text{C/m}^3)\) at the center of the right face at \( x = 2 \) m if the electric field is the same as in part (a)?

\[
\rho = \varepsilon_0 \nabla \cdot \mathbf{E} = \varepsilon_0 \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right) = \varepsilon_0 (6x + 0 - 2)_{x=s} = \varepsilon_0 (6s - 2) = 8.85 \times 10^{-11} \text{ C/m}^3
\]

By the way, we could integrate this to get the answer for part (a)

\[
q_{enc} = \int \rho dV = \varepsilon_0 \int_0^s dy \int_0^s dz \int_0^s dx (6x - 2) = \varepsilon_0 s^3 (3s^2 - 2s)
\]

\[
q_{enc} = \varepsilon_0 s^3 (3s - 2)
\]
4. [8 points] A flat nonconducting surface infinite in extent carries a uniform charge density of \( \sigma = 3 \times 10^{-9} \text{ C/m}^2 \). A small circular hole of radius \( R = 1.5 \text{ m} \) has been cut in the middle of the sheet as shown. Calculate the electric field at a point \( z = 5 \text{ m} \) away from the center of the hole along an axis perpendicular to the surface. (In other words, consider \( z \gg R \), but don’t set \( R / z \) exactly equal to zero. You may find the superposition principle useful.)

Using the superposition principle, add the field due to an infinite sheet of charge (without a hole) plus a small disk of negative charge (same magnitude but opposite sign charge density).

\[
E = E_{\text{sheet}} + E_{\text{hole}}
\]

\[
E_{\text{sheet}} = \frac{\sigma}{2\varepsilon_0} \hat{z}
\]

\[
E_{\text{hole}} = K \frac{q}{z^2} \hat{z} \quad \text{when viewed far away} \ (z \gg R) \quad \text{and where} \ q = \left( \pi R^2 \right)(-\sigma)
\]

\[
E_{\text{hole}} = -\frac{1}{4\pi\varepsilon_0} \frac{\pi R^2 \sigma}{z^2} \hat{z} = -\frac{\sigma}{2\varepsilon_0} \left( \frac{R^2}{2z^2} \right)
\]

\[
E = \frac{\sigma}{2\varepsilon_0} \left( 1 - \frac{R^2}{2z^2} \right) \hat{z} = 0.955 \frac{\sigma}{2\varepsilon_0} \hat{z} = 162 \hat{z} \text{ N/C}
\]
5. [8 points] An electron is launched away from the surface of an infinite nonconducting sheet of charge with a velocity of $v = 2 \times 10^6 \, \text{m/s}$ on a trajectory perpendicular to the surface. The charge density of the sheet is $\sigma = +5 \, \text{nC/m}^2$. Is the electron able to reach a distance infinitely far away from the charged sheet, and if not, how far does it travel before turning around? The charge of the electron is $q = -e = -1.6 \times 10^{-19} \, \text{C}$, and the electron mass is $m_e = 9.11 \times 10^{-31} \, \text{kg}$.

Because the electron is negatively charged and the sheet positive, the electron is attracted to the sheet by a force $F = qE$, where $E = \frac{\sigma}{2\varepsilon_0} \hat{x}$ taking $x$ in the direction of the particle (left).

Thus, it will stop and turn around.

The difference in electric potential from the surface of the sheet to position $x$ is:

$$\Delta V = -\frac{\sigma}{2\varepsilon_0} x$$

$$\Rightarrow \Delta U = q \Delta V = (-e) \left( -\frac{\sigma}{2\varepsilon_0} \right) x$$

At the turn-around

$$\Delta U = K_{\text{initial}}$$

$$\Rightarrow \frac{e\sigma}{2\varepsilon_0} x = \frac{1}{2} m v^2$$

$$x = \frac{e\sigma m v^2}{e\sigma} = 0.04 \, \text{m}$$
6. A conducting sphere of radius $R_1$ contains a charge $Q$. It is surrounded by a concentric spherical conducting shell of radius $R_2 > R_1$ and charge $-Q$.

(a) [6 points] What is the difference in electric potential between the shell and the sphere?

$$E = K \frac{Q}{r^2} \hat{r} \quad R_i < r < R_2 \text{ by Gauss' Law and symmetry}$$

$$\Delta V = -\int_{r_i}^{r_2} E \cdot ds = -|E| |ds| = |E| dr$$

$$\Delta V = -\int_{R_i}^{R_2} K \frac{Q}{r^2} dr = -K \frac{Q}{r} \bigg|_{R_i}^{R_2} = KQ \left( \frac{1}{R_i} - \frac{1}{R_2} \right)$$

(b) [6 points] What is the capacitance of the arrangement of conductors?

$$Q = C \Delta V$$

$$Q = \frac{1}{K} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \Delta V$$

$$C = \frac{R_1 R_2}{K (R_2 - R_1)}$$
7. [6 points] Initially two electrons are fixed in place with a separation of 2.15 µm. How much work must be done to bring a third electron in from infinity to complete an equilateral triangle?

Note that for a point charge, the electric potential a distance $r$ away is $V = k \frac{q}{r}$.

The work to place a negative charged particle a distance $r$ away from 2 points charges also of negative charge is the inverse of the change in the potential energy:

$$W = -\Delta U = -q\Delta V = 2k \frac{e^2}{r} = 2.14 \times 10^{-22} \text{ J}$$

8. [6 points] Sketch the electric field lines for a negatively charged particle above the surface of a flat perfectly conducting surface (both above and below the surface).
9. A 1.5 \( \mu \)F capacitor is charged to a potential difference of 12 V, and the charging battery is disconnected.

(a) [6 points] What is the energy stored in the capacitor?

\[
q = C_1 \Delta V = \left(1.5 \times 10^{-6} \text{ F}\right)(12)
\]

\[
U = \frac{q^2}{2C_1} = \frac{1}{2} C_1 (\Delta V)^2 = 1.08 \times 10^{-4} \text{ J}
\]

(b) [6 points] If the charged capacitor is then connected in parallel with a second (initially uncharged) capacitor, and if the potential difference across the first capacitor subsequently drops to 9 V, what is the capacitance of this second capacitor?

*Charge cannot be destroyed, so it remains on the respective plates even though they are connected to additional plates. But the capacitance must change (capacitors in parallel), so the potential difference must change to maintain constant \( q \):*

\[
C_{\text{eq}} = C_1 + C_2 \quad \text{(parallel)}
\]

\[
\Delta V' = \frac{q}{C_{\text{eq}}} = \frac{q}{C_1 + C_2} = \frac{C_1}{C_1 + C_2} \Delta V
\]

rearranging:

\[
C_2 = C_1 \left(\frac{\Delta V}{\Delta V'} - 1\right) = 0.5 \mu \text{F}
\]
10. [6 points] The electric potential along the x-axis (in V) is plotted versus the value of x, (in cm). Evaluate the x-component of the electrical force (in Newtons, including sign) on a proton located on the x-axis at x = 10 cm.

The electric field is given by the negative rate of change of the electric potential per unit length:

\[ E = -\frac{\Delta V}{\Delta x} = -\frac{500 \text{ V}}{0.2 \text{ m}} = -2500 \text{ V/m} \quad \text{(from graph)} \]

\[ F = qE = (+1.6 \times 10^{-19} \text{ C})(-2500 \text{ V/m}) \]

so \( F = -4.0 \times 10^{-16} \hat{x} \quad \text{N} \)

Force is directed in negative x direction.

11. [6 points] Two wires are made out of the same material (copper). One has a circular cross section with radius \( r = 1 \text{ mm} \) and a length of 10 cm, the other has a square cross section with width \( s = 1 \text{ mm} \) and a length of 5 cm. Which wire has the larger resistance?

\[ R = \rho \frac{L}{A} \]

circular: \( R_1 = \rho \frac{L_1}{\pi r^2} = \frac{\rho 0.1}{\pi 10^{-6}} = \frac{\rho}{\pi} 10^5 \)

square: \( R_2 = \rho \frac{L_2}{s^2} = \frac{\rho 0.05}{\pi 0^{-6}} = \frac{\rho}{2} 10^5 > \frac{\rho}{\pi} 10^5 \)

Square wire has larger resistance.