Chapters 34, 36: Electromagnetic Induction
Topics

⇒ Electromagnetic Induction
  - Magnetic flux
  - Induced emf
    - Faraday’s Law
    - Lenz’s Law
    - Motional emf
  - Magnetic energy
  - Inductance
  - RL circuits
  - Generators and transformers
Reading Quiz 1

Magnetic flux through a wire loop depends on:

- 1) thickness of the wire
- 2) resistivity of the wire
- 3) geometrical layout of the wire
- 4) material that the wire is made of
- 5) none of the above

\[ \Phi_B = \int_A \mathbf{B} \cdot d\mathbf{A} \]  
Flux depends only on geometrical properties
Reading Quiz 2

An induced emf produced in a motionless circuit is due to

1) a static (steady) magnetic field
2) a changing magnetic field
3) a strong magnetic field
4) the Earth’s magnetic field
5) a zero magnetic field

Faraday’s law
Reading Quiz 3

Motional emf relates to an induced emf in a conductor which is:

◆ 1) long
◆ 2) sad
◆ 3) stationary
◆ 4) insulated
◆ 5) moving

Potential difference proportional to velocity
Reading Quiz 4

Faraday’s law says that

- a) an emf is induced in a loop when it moves through an electric field
- b) the induced emf produces a current whose magnetic field opposes the original change
- c) the induced emf is proportional to the rate of change of magnetic flux
Reading Quiz 5

A generator is a device that:

- a) transforms mechanical into electrical energy
- b) transforms electrical into mechanical energy
- c) transforms low voltage to high voltage
Electromagnetic Induction

➔ Faraday discovered that a changing magnetic flux leads to a voltage in a wire loop
   • Induced voltage (emf) causes a current to flow !!

➔ Symmetry: electricity magnetism
   • electric current ———————— magnetic field
   • magnetic field ———————— electric current

➔ We can express this symmetry directly in terms of fields
   • Changing E field ————> B field (“displacement current”)
   • Changing B field ————> E field (Faraday’s law)

➔ These & other relations expressed in Maxwell’s 4 equations
   • (Other 2 are Gauss’ law for E fields and B fields)
   • Summarizes all of electromagnetism
   • See Chapter 32
Experimental Observation of Induction

This effect can be quantified by Faraday’s Law
Magnetic Flux

- Define magnetic flux $\Phi_B$
  
  $$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta$$
  
  - $\theta$ is angle between $\mathbf{B}$ and the normal to the plane
  - Flux units are T-m² = “webers”

- When $\mathbf{B}$ field is not constant or area is not flat
  
  - Integrate over area
  
  $$\Phi_B = \int_A \mathbf{B} \cdot d\mathbf{A}$$
\[ \Phi_B = \mathbf{B} \cdot \mathbf{A} = BA \cos \theta \]

\[ \begin{align*}
\Phi_B &= 0 & \Phi_B &= \frac{1}{\sqrt{2}} BA & \Phi_B &= BA
\end{align*} \]
Faraday’s Law of Induction

\[ E = -N \frac{d\Phi_B}{dt} \]

- The faster the change, the larger the induced emf
- Flux change caused by changing B, area, or orientation
- The induced emf is a *voltage*
Faraday’s Law & Flux Change

Rotating coil

$\Phi_B = BA \cos \omega t$

$E = -N \frac{d\Phi_B}{dt} = N \omega BA \sin \omega t$

- $\Phi_B$ is maximum when coil faces up
- $E$ is maximum when coil faces sideways

Stretched coil

- $B$ constant, $\theta$ constant
- Area shrinks
- $\Rightarrow$ Flux decreases

$\Phi = \int B \sin \omega t dt$
Faraday’s Law of Induction

\[ E = -N \frac{d\Phi_B}{dt} \]

- Minus sign from Lenz’s Law:
- Induced current produces a magnetic field which \textit{opposes} the original change in flux
Comment on Lenz’s Law

➡ Why does the induced current oppose the change in flux?

➡ Consider the alternative

◆ If the induced current reinforced the change, then the change would get bigger, which would then induce a larger current, and then the change would get even bigger, and so on . . .

◆ This leads to a clear violation of conservation of energy!!
Direction of Induced Current

Bar magnet moves through coil
- Current induced in coil

Reverse pole
- Induced current changes sign

Coil moves past fixed bar magnet
- Current induced in coil as in (A)

Bar magnet stationary inside coil
- No current induced in coil
ConcepTest: Lenz’s Law

If a North pole moves towards the loop from above the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Must counter flux change in downward direction with upward B field
ConcepTest: Induced Currents

A wire loop is being pulled through a uniform magnetic field. What is the direction of the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

No change in flux, no induced current
ConcepTest: Induced Currents

In each of the 3 cases above, what is the direction of the induced current?

(Magnetic field is into the page and has no boundaries)

(a) clockwise
(b) counter-clockwise
(c) no induced current
ConcepTest: Lenz’s Law

➔ If a coil is shrinking in a B field pointing into the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux is decreasing, so need to create downward B field
Induced currents

A circular loop in the plane of the paper lies in a 3.0 T magnetic field pointing into the paper. The loop’s diameter changes from 100 cm to 60 cm in 0.5 s

- What is the magnitude of the average induced emf?
- What is the direction of the induced current?
- If the coil resistance is 0.05 Ω, what is the average induced current?

\[ |V| = \frac{d\Phi_B}{dt} = 3.0 \times \left| \frac{\pi \left( 0.3^2 - 0.5^2 \right)}{0.5} \right| = 3.016 \text{ Volts} \]

- Direction = clockwise (Lenz’s law)

- Current = 3.016 / 0.05 = 60.3 A
ConcepTest: Induced Currents

A wire loop is pulled away from a current-carrying wire. What is the direction of the induced current in the loop?

(a) clockwise
(b) counter-clockwise
(c) no induced current

Downward flux through loop decreases, so need to create downward field.
ConcepTest: Induced Currents

A wire loop is moved in the direction of the current. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Flux does not change when moved along wire
ConcepTest: Lenz’s Law

➔ If the B field pointing out of the page suddenly drops to zero, in what direction is the induced current?
  ◆ (a) clockwise
  ◆ (b) counter-clockwise
  ◆ (c) no induced current

➔ If a coil is rotated as shown, in a B field pointing to the left, in what direction is the induced current?
  ◆ (a) clockwise
  ◆ (b) counter-clockwise
  ◆ (c) no induced current

Upward flux through loop decreases, so need to create upward field

Flux into loop is increasing, so need to create field out of loop
ConcepTest: Induced Currents

Wire #1 (length L) forms a one-turn loop, and a bar magnet is dropped through. Wire #2 (length 2L) forms a two-turn loop, and the same magnet is dropped through. Compare the magnitude of the induced currents in these two cases.

(a) $I_1 = 2I_2$
(b) $I_2 = 2I_1$
(c) $I_1 = I_2 \neq 0$
(d) $I_1 = I_2 = 0$
(e) Depends on the strength of the magnetic field

Voltage doubles, but R also doubles, leaving current the same
**Motional EMF**

Consider a conducting rod moving on metal rails in a uniform magnetic field:

\[
\mathcal{E} = \frac{d}{dt}(BA) = \frac{d}{dt}(BLx) = BLv
\]

Current will flow counter-clockwise in this “circuit”. Why?
Force and Motional EMF

- Pull conducting rod out of B field
- Current is clockwise. Why?
  \[ i = \frac{\mathcal{E}}{R} = \frac{BLv}{R} \]
- Current within B field causes force
  \[ F = iLB = \frac{B^2L^2v}{R} \]
  - Force opposes pull (RHR)
  - Also follows from Lenz’s law
- We must pull with this force to maintain constant velocity
Power and Motional EMF

- Force required to pull loop: \( F = iLB = \frac{B^2L^2v}{R} \)

- Power required to pull loop: \( P = Fv = \frac{B^2L^2v^2}{R} \)

- Energy dissipation through resistance

\[
P = i^2R = \left(\frac{BLv}{R}\right)^2R = \frac{B^2L^2v^2}{R}
\]

- Same as pulling power! So power is dissipated as heat
  - Kinetic energy is constant, so energy has to go somewhere
  - Rod heats up as you pull it
Example

- Pull a 30cm x 30cm conducting loop of aluminum through a 2T B field at 30cm/sec. Assume it is 1cm thick.
  - Circumference = 120cm = 1.2m, cross sectional area = $10^{-4}$ m$^2$
  - $R = \rho L/A = 2.75 \times 10^{-8} \times 1.2 / 10^{-4} = 3.3 \times 10^{-4}\Omega$

- EMF
  \[ \varepsilon = BLv = 2 \times 0.3 \times 0.3 = 0.18 \text{ V} \]

- Current
  \[ i = \varepsilon / R = 0.18 / 3.3 \times 10^{-4} = 545 \text{ A} \]

- Force
  \[ F = iLB = 545 \times 0.3 \times 2 = 327 \text{ N} \quad \text{74 lbs!} \]

- Power
  \[ P = i^2 R = 98 \text{ W} \quad \text{About 0.330 C per sec (from specific heat, density)} \]
Electric Generators

→ Rotate a loop of wire in a uniform magnetic field:

- changing $\theta \Rightarrow$ changing flux $\Rightarrow$ induced emf
- $\Phi_B = BA \cos \theta = BA \cos(\omega t)$

Rotation: $\theta = \omega t$
Electric Generators

Flux is changing in a sinusoidal manner

- Leads to an alternating emf (AC generator)

\[ |\mathbf{E}| = N \frac{d\Phi_B}{dt} = NBA \frac{d\cos(\omega t)}{dt} = NBA\omega \sin(\omega t) \]

- This is how electricity is generated
- Water or steam (mechanical power) turns the blades of a turbine which rotates a loop
- Mechanical power converted to electrical power
A generator has a coil of wire rotating in a magnetic field. If the B field stays constant and the area of the coil remains constant, but the rotation rate increases, how is the maximum output voltage of the generator affected?

- (a) Increases
- (b) Decreases
- (c) Stays the same
- (d) Varies sinusoidally

\[ \mathcal{E} = NBA\omega \sin(\omega t) \]
Induction in Stationary Circuit

- Switch closed (or opened)
  - Current induced in coil B (directions as shown)

- Steady state current in coil A
  - No current induced in coil B
Inductance

- Inductance in a coil of wire defined by $L = \frac{N\Phi_B}{i}$
- Can also be written $Li = N\Phi_B$
- From Faraday’s law $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$

◆ This is a more useful way to understand inductance

- Inductors play an important role in circuits when current is changing!
Self - Inductance

Consider a single isolated coil:

- Current (red) starts to flow clockwise due to the battery.
- But the buildup of current leads to changing flux in loop.
- Induced emf (green) opposes the change.

This is a self-induced emf (also called “back” emf).

\[ \mathcal{E} = -N \frac{d\Phi}{dt} = -L \frac{di}{dt} \]

L is the self-inductance

units = “Henry (H)”
Inductance of Solenoid

→ Total flux (length \( l \))

\[
B = \mu_0 i n
\]

\[
N \Phi_B = (nl)(BA) = \mu_0 n^2 Al i
\]

\[
\mathcal{E} = -N \frac{d\Phi_B}{dt} = -\mu_0 n^2 Al \frac{di}{dt} = -L \frac{di}{dt}
\]

\[
L = \mu_0 n^2 Al
\]

To make large inductance:
- Lots of windings
- Big area
- Long
LR Circuits

- Inductance and resistor in series with battery of EMF $V$

- Start with no initial current in circuit
  - Close switch at $t = 0$
  - Current is initially 0 (initial increase causes voltage drop across inductor)

- Find $i(t)$
  - Resistor: $\Delta V = Ri$
  - Inductor: $\Delta V = L \frac{di}{dt}$

$$V - Ri - L \frac{di}{dt} = 0$$
Analysis of LR Circuit

⇒ Differential equation is \( \frac{di}{dt} + i \left( \frac{R}{L} \right) = \frac{V}{R} \)

⇒ General solution: \( i = \frac{V}{R} + Ke^{-tR/L} \)

◆ (Check and see!)
◆ \( K = -\frac{V}{R} \) (necessary to make \( i = 0 \) at \( t = 0 \))

\[
i = \frac{V}{R} \left( 1 - e^{-tR/L} \right)
\]

Rise from 0 with time constant \( \tau = \frac{L}{R} \)

Final current (maximum)
Current vs Time in RL Circuit
(Initially Zero Current in Inductor)

\[ i(t) = i_{\text{max}} \left(1 - e^{-tR/L}\right) \]
L-R Circuits (2)

Switch off battery: Find $i(t)$ if current starts at $i_0$

$$0 = L\frac{di}{dt} + Ri$$

$$i = i_0 e^{-\frac{tR}{L}}$$

Exponential fall to 0 with time constant $\tau = \frac{L}{R}$

Initial current (maximum)
Current vs Time in RL Circuit
(For Initial Current $i_{\text{max}}$ in Inductor)

\[ i(t) = i_0 e^{-t/(R/L)} \]
Exponential Behavior

$\tau = L/R$ is the “characteristic time” of any RL circuit

- Only $t / \tau$ is meaningful

$\Rightarrow t = \tau$

- Current falls to $1/e = 37\%$ of maximum value
- Current rises to $63\%$ of maximum value

$\Rightarrow t = 2\tau$

- Current falls to $1/e^2 = 13.5\%$ of maximum value
- Current rises to $86.5\%$ of maximum value

$\Rightarrow t = 3\tau$

- Current falls to $1/e^3 = 5\%$ of maximum value
- Current rises to $95\%$ of maximum value

$\Rightarrow t = 5\tau$

- Current falls to $1/e^5 = 0.7\%$ of maximum value
- Current rises to $99.3\%$ of maximum value
ConcepTest: Generators and Motors

A current begins to flow in a wire loop placed in a magnetic field as shown. What does the loop do?

- (a) moves to the right
- (b) moves up
- (c) rotates around horizontal axis
- (d) rotates around vertical axis
- (e) moves out of the page

This is how a motor works!!
Electric Motors

- Current is supplied from an external source of emf (battery or power supply)
- Forces act to rotate the wire loop
- A motor is essentially a generator operated in reverse!
Motor

- Forces act to rotate the loop towards the vertical.

- When loop is vertical, current switches sign and the forces reverse, in order to keep the loop in rotation.

- This is why alternating current is necessary for a motor to operate.
Motors

Electrical $\Rightarrow$ mechanical energy

Generators

Mechanical $\Rightarrow$ electrical energy
Energy Stored in Magnetic Field

- Just like electric fields, magnetic fields store energy.

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 \quad \text{Electric field} \]

\[ u_B = \frac{B^2}{2\mu_0} \quad \text{Magnetic field} \]

- Let’s see how this works.
Energy of an Inductor

➔ How much energy is stored in an inductor when a current is flowing through it?

➔ Start with loop rule

\[ \mathcal{E} = iR + L \frac{di}{dt} \]

➔ Multiply by \( i \) to get power equation

\[ \mathcal{E} i = i^2 R + L i \frac{di}{dt} \quad \text{and} \quad P_L = L i \frac{di}{dt} = \frac{d}{dt} \left( \frac{1}{2} Li^2 \right) \]

\[ P_{\text{in}} = P_R \left( \text{heat} \right) + P_L \left( \text{store} \right) \]

➔ \( P_L = \) rate at which energy is being stored in inductor

◆ Energy stored in inductor

\[ U_L = \frac{1}{2} Li^2 \]

◆ Similar to capacitor:

\[ U_C = \frac{q^2}{2C} \]
Energy in Magnetic Field (2)

→ Apply to solenoid (constant B field)

\[ U_L = \frac{1}{2} Li^2 = \frac{1}{2} \left( \mu_0 n^2 lA \right) i^2 \]

→ Use formula for B field: \( B = \mu_0 ni \)

\[ U_L = \frac{B^2}{2 \mu_0} lA \]

→ Calculate energy density: \( u_B = \frac{U_L}{V} \quad V = Al \)

\[ u_B = \frac{B^2}{2 \mu_0} \quad \text{B field} \]

\[ u_E = \frac{1}{2} \varepsilon_0 E^2 \quad \text{E field} \]

→ This is generally true even if B is not constant
Energy Calculation Examples

Calculate \( u_B \) for earth field, \( B = 5 \times 10^{-5} \) T

\[
u_B = \frac{B^2}{2\mu_0} = \frac{(5 \times 10^{-5})^2}{2 \times 4\pi \times 10^{-7}} \approx 10^{-3} \text{ J/m}^3
\]

Calculate \( u_B \) for neutron star, \( B = 10^8 \) T

\[
u_B = \frac{B^2}{2\mu_0} = \frac{(10^8)^2}{2 \times 4\pi \times 10^{-7}} \approx 4 \times 10^{21} \text{ J/m}^3
\]

Calculate \( u_B \) for magnetar, \( B = 10^{11} \) T

\[
u_B = \frac{B^2}{2\mu_0} = \frac{(10^{11})^2}{2 \times 4\pi \times 10^{-7}} \approx 4 \times 10^{27} \text{ J/m}^3
\]

\[
\rho = \frac{u_B}{c^2} \approx 4 \times 10^{10} \text{ kg/m}^3
\]

Use \( E = mc^2 \)
Web Sites

⇒ Original magnetar discovery
  ◆ http://science.nasa.gov/newhome/headlines/ast20may98_1.htm
  ◆ http://www.firstscience.com/site/articles/solarflares.asp

⇒ More recent magnetar discovery (Feb. 2005)

⇒ Online articles on magnetars
  ◆ http://solomon.as.utexas.edu/~duncan/magnetar.html
  ◆ http://www.space.com/scienceastronomy/magnetar_formation_050201.html

⇒ Articles on neutron stars (second one has videos_
  ◆ http://www.astro.umd.edu/~miller/nstar.html
  ◆ http://antwrp.gsfc.nasa.gov/htmltest/rjn_bht.html
Gigajoule Magnet at CERN

CMS experiment at CERN

- p-p collisions at world’s highest energy in 2007
- Hope to discover new particles, find the origin of mass and new fundamental forces

Compact Muon Solenoid
Compact Muon Solenoid
CMS Experiment Magnet

Large central solenoid magnet to study particle production

- $B = 4T$, $R = 3.15\ m$, $L = 12.5\ m$
- $U_B = 2.6 \times 10^9\ J = 2.6\ \text{gigajoules!!}$

$$U_B = \frac{B^2}{2\mu_0} lA = \frac{4^2}{2 \times 4\pi \times 10^{-7}} \left(\pi \times 3.15^2\right)(12.5)$$

CMS Articles and Pictures

» Home page
  ◆ http://cmsinfo.cern.ch/Welcome.html/

» Pictures of detector
  ◆ http://cmsinfo.cern.ch/Welcome.html/CMSdetectorInfo/CMSdetectorInfo.html
  ◆ http://cmsinfo.cern.ch/Welcome.html/CMSmedia/CMSmedia.html

» Interesting article on solenoid, with pictures

» Other documents & pictures about CMS