Impulse and Momentum

Experiment AQ

Objective
In this experiment you will explore the relationship between the impulse applied to an object during a collision and the resulting change in the motion of the object.

Theory
The time interval over which a collisional force acts on an object is often quite short — a fraction of second in these experiments. Such short-duration forces are called impulsive forces and their effect on the motion of the object can be quantified by the impulse-momentum theorem.

Many quantities discussed in this experiment (force, velocity, acceleration, momentum, and impulse) are vector quantities. However, as we will only consider the restricted case of motion along a straight line, the equations and variables associated with these quantities will be reduced to their one-dimensional or one-component versions. The one-dimensional treatment is equivalent to the treatment of any one of the three component equations associated with any vector equation. Vector equations translate directly to component or one-dimensional equations by translating the vector quantities appearing in them to signed components using a sign convention.

For example, consider Newton’s second law, $\mathbf{F} = ma$ relating the net force $\mathbf{F}$ acting on an object of mass $m$ and the object’s acceleration $a$. Applied to an object constrained to move along a horizontal line, its one-dimensional (component) version becomes

$$F = ma \quad (1)$$

Using a sign convention in which the positive direction is to the right, the signed component quantities $F$ and $a$ would be positive if the corresponding vector force and acceleration point to the right, and negative if the vectors point to the left. Normally, such a component equation would have subscripts, e.g., $F_x = ma_x$ for a line along the $x$-direction. However, because all components for one-dimensional motion are along one line, the subscripting is unnecessary and will not be used. Furthermore, we can and will dispense with the word component and consider it implied in speaking of vector quantities. On the Symbol Definitions page those vector quantities which are reduced to their one-dimensional versions are listed as 1-D.

If the force $F$ is constant, the acceleration will also be constant. For this case, over any time interval $\Delta t$ during which the force acts, $a = \Delta v / \Delta t$ where $\Delta v = v_f - v_i$ and $v_i$ and $v_f$ are the velocities at the start and end of the interval, respectively.Rewriting Eq. 1 with this substitution for $a$ and then moving the $\Delta t$ to the left gives

$$F \Delta t = m \Delta v \quad (2)$$

The product of an object’s mass and its velocity ($mv$) occurs often in physics. It is called
momentum and is given the symbol \( p \). Like velocity, momentum is a vector quantity. However, we will only need its 1-D version. In terms of momentum, Eq. 2 becomes

\[ F \Delta t = \Delta p \]  

where \( \Delta p = p_f - p_i \), is the change in the object’s momentum over the time interval \( \Delta t \).

When a batter hits a baseball, the force on the baseball changes significantly during the collision. The force that the bat exerts on the ball starts off low as the bat just begins to make contact with the ball, reaches a peak near the middle of the collision, and decreases to zero when the ball loses contact with the bat. The whole process may only last a fraction of a second, and the force on the ball might be similar to the solid curve labeled \( F \) vs. \( t \) in Fig. 1. Equation 3, valid for a constant force, will have to be modified to treat such a time-varying force.

Even though an impulsive force may act for quite a short time interval, it will prove useful to break up the time interval into even smaller intervals. If these subintervals are made short enough, the force can be considered constant over each of them and Eq. 3 will apply. Consider one such subinterval from time \( t_a \) to \( t_b \) as in Fig. 1. In applying Eq. 3, we use the average force for \( F \) — the mid value for that subinterval (\( F_{ab} \) in the figure).

The change in the object’s momentum during this subinterval is (by definition) \( p_b - p_a \), i.e. the momentum of the object at \( t_b \) minus its value at \( t_a \). According to Eq. 3 for this subinterval

\[ F_{ab} \Delta t = p_b - p_a \]  

The left side of this equation, \( F_{ab} \Delta t \), is just the area under the \( F \) vs. \( t \) graph for this subinterval (the shaded strip in the figure). The base of the rectangular strip (in time units) is \( \Delta t = t_b - t_a \) and its height (in force units) is \( F_{ab} \).

To take into account the entire impulsive force, we consider a large number of such short subintervals and narrow strips completely covering the \( F \) vs. \( t \) graph with no overlapping and no gaps between them. Let \( p_i \) represent the object’s initial momentum at \( t_i \) just before the force starts and let \( p_f \) be its final momentum at \( t_f \) just as the force ends. Let the subintervals be defined by increasing intermediate times between \( t_i \) and \( t_f \) at \( t_1, t_2, ..., t_N \). Over the first subinterval from \( t_i \) to \( t_1 \) the momentum change is \( \Delta p = p_1 - p_i \); over the second subinterval it is \( \Delta p = p_2 - p_1 \). Continuing this way to the last subinterval, the final momentum change is \( \Delta p = p_f - p_N \). According to Eq. 3, associated with each of these momentum changes is a particular strip area of the \( F \) vs. \( t \) graph, with a height given by the mid value of the force for that subinterval and a width equal to the time interval involved.

Next, we could write out all the equations of the form of Eq. 3 for each subinterval. Summing all the right sides of the equations (\( \Delta p \)’s) gives the simple result \( p_f - p_i \) because all the intermediate \( p \)’s cancel, occurring with a negative sign in one equation and a positive sign.
in the next. Summing all the left sides of the equations (\(F\Delta t\) strip areas), gives simply the total area under the \(F\) vs. \(t\) graph. The area under the force versus time graph is called the impulse and will be given the symbol \(J\). Consequently, impulse has units of force times time and thus SI units of N·s (Newton-seconds).

\[
\Delta p = p_f - p_i
\]

is simply the change in the momentum between the start and end of the impulse, and thus Newton’s second law is now seen to have the elegant consequence

\[
J = \Delta p
\]  

(This equation is the 1-D version of the impulse-momentum theorem — same equation with \(J\) and \(p\) as full three-dimensional vectors).

The impulse-momentum theorem is really quite non-specific. The \(J\) could be from a large force acting over a short time or a smaller force acting over a longer time. Only the area of the \(F\) vs. \(t\) curve is involved. The object subject to the force may have a large or a small mass and may have a large or small initial velocity. Only the momentum change is involved.

One of the most basic conservation laws of physics — the law of conservation of momentum — results from the impulse-momentum theorem coupled with Newton’s third law regarding action-reaction forces. According to Newton’s third law, when two objects \(a\) and \(b\) exert forces on one another, the force \(F_{ab}\) on \(a\) due to \(b\) is equal in magnitude and opposite in direction to the force \(F_{ba}\) on \(b\) due to \(a\). That is, \(F_{ab} = -F_{ba}\). Because this must be true at all times, the impulse delivered to object \(a\) by \(F_{ab}\) must be equal and opposite the impulse delivered to object \(b\) by \(F_{ba}\). Then, by the impulse-momentum theorem, object \(a\) and \(b\) must experience equal and opposite momentum changes and the net momentum (sum of the two) does not change.

Data Acquisition

The apparatus shown in Fig. 2 will be used to explore the impulse-momentum theorem. The cart is launched through the photogate, collides with a bumper, and rebounds back through the photogate. The bumper is mounted on a force probe attached to the track. The cart velocity just before and just after the collision is determined from data collected as the picket fence mounted to the cart passes through the photogate on the way into
and out of the collision. The force acting on the cart — the $F$ vs. $t$ graph — is obtained from data supplied by the force probe.

The impulse-momentum theorem applies to the cart; it is the cart’s initial and final momentum and the net force acting on the cart that are under consideration. If the track is level, the downward force due to gravity (the cart’s weight) is cancelled by the upward normal force due to the track. And friction is small enough to be neglected during the collision. Thus, at all times during the collision, the net force on the cart is the force due to the bumper. According to Newton’s third law, the cart exerts an equal but opposite force on the bumper. Continuing and again using Newton’s third law, the bumper exerts a force on the probe, which exerts an equal but opposite force on the bumper. It is the force on the force probe due to the bumper which is measured by our apparatus, while it is the force on the cart due to the bumper that is needed in applying the impulse-momentum theorem to the cart. Thus, we would like these two forces to be equal in magnitude at all times.

Newton’s third law does not guarantee the two forces always have equal magnitudes. In fact, it can be used to show that they differ by the net force on the bumper. However, any net force on the bumper must, by Newton’s second law, be associated with acceleration of the bumper or its parts. Because the bumpers hardly move and have small mass, the net force on the bumper is always small and therefore the probe will accurately, if not perfectly, reflect the force on the cart. It is interesting to note that the measured impulse starting from the time the cart hits the (stationary) bumper and ending when the bumper stops moving is predicted to be identical to the impulse delivered to the cart even if the net force on the bumper is not small at all times. This is because over such an interval, the bumper would have no momentum change and, consequently, must deliver equal and opposite impulses to the cart and probe.

The data acquisition program for this experiment is complex and worth reviewing and understanding before using it in the experiment. The principles are reasonably straightforward if not simple. Recall from Experiment AP Acceleration due to Gravity how you used the spreadsheet to analyze a list of times measured as each stripe of a picket fence blocked the photogate. Particularly important to this experiment, recall how you were able to make a straight-line graph of $v$ vs. $t$.

In this experiment, there will be two $v$ vs. $t$ graphs. Both will be obtained as in Experiment AP from the blockage times of the cart-mounted picket fence. However, the graphs and straight line fits will be displayed automatically by the data acquisition program; you will not have to use the spreadsheet. One graph will be based on the data obtained as the cart moves into the collision (in green at the top of the computer window) and the other will be based on the data obtained as the cart moves out of the collision (in yellow, below). This data and the analysis of each $v$ vs. $t$ graph will be used to determine the velocities just before and just after the collision.

Starting immediately after the first passage of the picket fence through the photogate, the signal from the force probe will be read by the computer at a rate of 5000 times per second, i.e., with a subinterval between readings $\Delta t = 0.0002$ s. Force values so obtained will then be displayed on the computer screen as an $F$ vs. $t$ graph which should show an impulsive force like that of Fig. 1. On this graph there will be two adjustable cursors (vertical lines you can move with the mouse). You will set one cursor (color coded in green) just
ahead of the force to indicate where the collision has started and you will set the other cursor (yellow) just after to indicate where the collision is complete. The impulse \( J \) will be determined by the computer by calculating the area under the curve in the \( F \) vs. \( t \) graph (between the cursors).

Both \( v \) vs. \( t \) graphs and the \( F' \) vs. \( t \) graph display the time \( t \) using a common “clock time” (in seconds) which runs continuously starting at first blockage of the photogate.

You will see that the incoming (green) \( v \) vs. \( t \) graph is indeed a straight line and that it is therefore reasonable to extend the line forward (to the time specified by the green cursor on the \( F' \) vs. \( t \) graph) to determine the cart velocity just before the collision starts. Similarly, the outgoing (yellow) \( v \) vs. \( t \) graph can be extended backward (to the time specified by the yellow cursor) to determine the cart velocity just after the collision. The magnitude of these velocities will be displayed in green and yellow indicators just to the right of the corresponding \( v \) vs. \( t \) graphs. (You will easily determine the signs of these 1-D velocities from a convention specified in the procedure section.) From these velocities and the measured cart mass you will then be able to calculate the momentum change.

**Procedure**

The force probe has a threaded hole and each bumper is equipped with a screw. The bumpers should screw in rather easily and then stop abruptly after about four turns. Do not overtighten. If you start off with the screw a bit crooked, it will jam after a turn or so. If this happens gently unscrew and start again.

We will choose the positive direction to be the direction away from the force sensor. Thus, the initial velocity \( v_i \) of the cart colliding with the bumper will have a negative value and the final velocity \( v_f \) after the collision will have a positive value. Consequently, \( \Delta p = mv_f - mv_i \) will be positive.

**Collision Measurement**

The following is the setup and basic procedure for a collision measurement. Perform these steps as given for the conditions specified. In later work, you will be asked to change various factors, such as the initial launch speed, the cart mass, or the bumper type. Refer back to this section as needed.

1. Plug the photogate into the DÅSI box’s DIGITAL IN 3. Plug the force probe into the special connector just under the flower on the DÅSI box. Make sure the track is backed against the room wall so that it will not recoil when the cart hits the bumper.

2. Activate the Impulse-Momentum data acquisition program.

3. Screw on the less-stiff of the two spring bumpers. Click on the Zero Probe button, and then click on the big rectangular ZERO button in the new window. (You will have to zero each time you change the bumper.) Push gently on the bumper with your hand, you should see the Force Reading go positive. (The program makes and displays readings slowly in this part of the program, but much faster when measuring collisions.) Note that the bumper is now pushing your hand in a direction away from the force probe. Thus, the sign of the force is now correct in the sense that it will be positive when the force on the cart is away from the force sensor. Click on the Return button.
4. Measure and record the total mass of the cart including the picket fence. Also measure and record the mass of the metal bar (for later use).

5. There are 13 stripes along the top of the picket fence. This is the default value for the program as indicated by the control at the top of the $v$ vs. $t$ graphs. Lay the picket fence along the tape measure attached to the track and note that the distance from the start of one stripe to the next is very close to 1 cm. While the picket fence is on the track scale, check that twelve such intervals span nearly exactly 12 cm. Thus, the default value used in the analysis and indicated in the control at the top of the $v$ vs. $t$ graphs is 0.01 m. Report to the instructor if you believe your picket fence has a spacing other than 0.01 m.

6. Place the photogate about 25 cm ahead of the spring bumper. Adjust its height so that it detects blockages on the top row of the picket fence. Adjust its position so that the picket fence passes close to one of the photogate arms, not in the center between the arms. The red LED on the photogate should blink on and off once for each stripe as you slowly push the cart through the photogate.

7. Click on the Start button and launch the cart (with no added bars) toward the bumper. Make sure the cart is completely free of your hand (coasting) as it goes through the photogate. If the run is successful there should be one green and one yellow straight-line $v$ vs. $t$ graph and a bump on the $F$ vs. $t$ graph. To see the bump you may need to click on the autoscale button in the palette below the graph display. The autoscale button appears as a doublesided arrow ↔ with an x above it. If there is a problem, perhaps the photogate position was not set properly. You might have to block and unblock the photogate with your fingers to run the data acquisition to completion. Then, check the photogate positioning. If you still have trouble, ask the instructor for help.

8. Having a level track is important. Even a small component of the gravitational force along the track will affect the cart’s momentum change but will not be measured by the force probe. However, having a perfectly level track is not critical. Arising from both friction and track tilt, the accelerations for motion in each of the two directions are displayed above the corresponding $v$ vs. $t$ graphs. With a level track, the two decelerations should be equal and the computer would report both values as negative. Any systematic difference would be due to track tilt, which would make the deceleration larger (more negative) for the uphill motion and smaller (less negative) for the downhill motion. Note that the computer may display the acceleration or other numerical values in engineering notation. With this notation, 5.12E-2 is $5.12 \times 10^{-2} = 0.0512$. Adjust the track’s leveling feet if either acceleration is positive or if they are not within 0.06 m/s$^2$ of each other. Ask the instructor for help if this takes more than a few minutes.

9. After a successful run, set the green and yellow cursors on the $F$ vs. $t$ graph just before and after the impulse bump (not vice versa). Click on the Expand button to see how that works.
10. Click on the **Calculate Impulse and Velocities** button and the program will calculate and display the impulse \( J \) and the velocities before and after the collision (at the times set by the cursors in the \( F \) vs. \( t \) graph).

11. For comparison with later experiments, try to get a collision having an initial velocity near 0.35 m/s; the closer the better but between 0.32 and 0.38 m/s should be good enough. When you are trying to “hit” any particular initial velocity, just set the green cursor once, it doesn’t need to be accurately placed (until you are ready to accept and analyze the run), nor do you need to use the **Expand** button of otherwise adjust the graph axes. Click **Start**, launch the cart, and then hit the **Calculate Impulse and Velocities** button. Check the initial velocity and, if necessary, increase or decrease your launch speed to get the desired \( v_i \).

12. Record the bumper type, the cart mass (including any steel bars), the before and after collision velocities (with the proper sign; remember the initial velocity is negative), and the impulse \( J \). Also calculate and record the momentum change \( \Delta p = m(v_f - v_i) \).

13. **The rest of the lab procedure is described in each Comprehensive Question.** Go on to the Comprehension Questions. Record the results on the data sheet, as in the previous step, for each additional measurement described there.
Symbol Definitions

$F$ 1-D net force.
$m$ The cart mass including any steel bars.
$a$ 1-D acceleration.
$v$ 1-D velocity.
$v_i, v_f$ 1-D initial and final velocities.
$t_i, t_f$ Starting and ending times for the impulsive force.
$\Delta t$ Time interval, e.g., from $t_i$ to $t_f$, $\Delta t = t_f - t_i$
$\Delta v$ 1-D change in velocity, e.g., from $v_i$ to $v_f$, $\Delta v = v_f - v_i$
$J$ 1-D impulse of a force, the area under the $F$ vs. $t$ graph.
$p$ 1-D momentum, product of object’s mass and velocity.
$\Delta p$ 1-D change in momentum, e.g., from $p_i$ to $p_f$, $\Delta p = p_f - p_i$
$D$ Distance between the stripes on the picket fence.
Title Sheet

Name: _____________________  Date: _____________________
Partner: ___________________  SSN: _____________________
Course: ____________________  Section: ___________________
Instructor: _________________

Comments on the experiment or writeup:
**Data Sheet**

Cart mass (incl. picket fence): \( m_{\text{cart}} = \ \) 

Steel bar mass: \( m_{\text{bar}} = \ \) 

Picket fence stripe separation: \( D = \ \) 

<table>
<thead>
<tr>
<th>Bumper type</th>
<th>( m )</th>
<th>( v_i )</th>
<th>( v_f )</th>
<th>( J )</th>
<th>( \Delta p )</th>
<th>( J/\Delta p )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Comprehension Questions

1. This question refers to the first collision experiment — with the less-stiff spring bumper, no added bars, and $v_i \approx 0.35 \text{ m/s}$. How well did the impulse and momentum change match? Express the comparison as the ratio $J/\Delta p$ and enter this value (to the hundredths place) in the final data sheet column. Differences from unity should be in the few percent range. This difference should be calculated for each run to check whether it is systematically above or below 1. Make a rough, hand drawing of the $F$ vs. $t$ graph. Label the starting and ending times and use them to determine the duration of the collision. (Use the cursors and note the times in the cursor display below the graph.) Also label the maximum force attained. Use your drawing to determine a rough estimate of $J$ and check that the computer provided $J$ value has the right magnitude. (Show your work.)

2. This question explores how the impulsive force depends on the spring stiffness. Suppose the cart is launched with exactly the same velocity but the bumper spring is made stiffer. Would you expect an increase, decrease, or no change in the duration of the force? The maximum force? The impulse? Explain. (Here and in any other questions, you will not be penalized for incorrect predictions or the explanations associated with them.)

Now change to the bumper with the stiffer spring and rezero the probe. Make a collision measurement with an initial velocity of about 0.35 m/s as described in the procedure section Steps 7 and 9-12. Make a rough drawing of the $F$ vs. $t$ graph showing the values of the maximum force and the starting and ending times. Do the experimental results agree with your predictions? If not, explain the results.
3. This question explores how the impulse depends on the cart mass. If the cart were launched with exactly the same velocity but the cart mass were doubled, would you expect the impulse to increase, decrease, or remain the same? By how much? Explain.

Now, using the stiff spring bumper on the cart from the last run, add the steel bar to the cart and make a collision measurement with an initial velocity near 0.35 m/s. Calculate the ratio of the impulse with the added bar to the impulse without it from the C.Q. 2. Was your prediction correct? If not, explain the experimental result, pointing out where your prediction was incorrect.

4. This question explores how the impulse depends on the initial velocity. If the mass stays the same but the initial velocity is doubled, would you expect the impulse to increase, decrease, or remain the same? By how much? Explain.

Remove the added bar. It will not be used again. Make a collision measurement with an initial velocity between 0.60 and 0.80 m/s. Find the ratio of this impulse with the impulse of C.Q. 2 under the same conditions except for the lower velocity. Compare this ratio with the ratio of the initial velocities for the two collisions. Why should these ratios be the same?
5. Change to the magnetic bumper. Note the small post which must be matched up with a small hole on the force probe. Rezero the force probe. Now make a collision measurement (no added bar) at $v_i$ around 0.35 m/s. Note and explain why the duration of this force is much longer than it was for the spring bumpers. How must the cursors be set to measure the whole impulse?

How are the final velocities related to the initial velocities for this collision and for the collisions with the springs? Is much energy lost in any of these collisions?

Did the impulse-momentum theorem check out as well for this case as for the previous cases? It should. If it didn’t, check how you set the cursors.

6. Change to the black, rubber bumper. Rezero the force probe. First observe (without taking data) how the cart (no added bar) behaves as it bounces off this bumper. Note in particular the large decrease in the magnitude of the velocity after the collision. With all the previous bumpers you should have noticed that the magnitude of the velocities before and after the collision were nearly equal. Were they exactly equal, the collision would be called elastic. In elastic collisions the kinetic energy $\frac{1}{2}mv^2$ is the same before and after the collision. With this bumper, you are seeing an inelastic collision — the cart has less kinetic energy after the collision. For this bumper, do you expect the impulse-momentum theorem to apply? Why or why not? If not, would you expect the impulse $J$ to be less than or greater than the momentum change? Why?

For a collision with the same mass and initial velocity, would you expect the impulse to be less than, equal to, or greater than the impulse obtained with a nearly elastic collision using, say, a spring bumper? Explain.

Now, make a collision measurement with $v_i \approx 0.35$ m/s and check both your predictions above. If any of your predictions were incorrect, correct them and their explanations.
Experiment AQ

7. Change to the clay bumper. Shape the clay into a long cone at least 2 cm long, so that it will deform as the cart collides with and sticks to it. Observe (without taking data) how collisions with this bumper behave. What is the final velocity? What fraction of the cart’s kinetic energy is lost in this collision?

For collisions with the same mass and initial velocities, would you expect the impulse for the clay bumper to be larger, smaller or the same when compared to a spring bumper? By how much? Why?

Reshape the clay, rezero the force probe and make a collision measurement with a \( v_i \approx 0.35 \text{ m/s} \) (no added bar). (You will have to manually push the cart back through the photogate after it sticks to the clay in order to get the program to run to completion. Thus, the final velocity from the computer is meaningless.) Enter the correct \( v_f \) in the data sheet. Check your prediction and, if they are incorrect, give a correct explanation of the observations.

8. Comment on the observed discrepancies between \( J \) and \( \Delta p \). Do you notice any systematic differences? Keep in mind the force probe may give force readings that are systematically high or low; in the worst case it is probably off by no more than around three percent. Could there be any other systematic effects? Comment on any conditions under which the impulse-momentum theorem can be expected to fail — or, whether it always valid.
Name: __________

Prelab Questions

Answer the following question and turn this in to the instructor at the beginning of the lab.

1. Show that Eq. 5 is dimensionally correct. That is, show that the dimensions or the SI units of impulse and momentum are the same. You need to start from the definitions of impulse and momentum.

2. Assume the force in Fig. 1 (reproduced below) is the net eastward force acting on a ball. Assume this force starts rising from zero at $t = 0.012\ s$, has fallen back to zero at $t = 0.062\ s$, and reaches a maximum force of 35 N at the peak. Determine with an error no bigger than 25% (high or low) the magnitude and direction of the impulse delivered to the ball by this force. Hint: Do not use $J = F\Delta t$. Look at the figure. Find the area. If the ball comes into this force with an initial horizontal velocity of 2 m/s to the west and leaves with a 3 m/s velocity to the east, what is the mass of the ball?

![Graph showing force (F) vs. time (t)]
Experiment AQ

Impulse and Momentum