A self-force primer

- The *Self-Force* on a “charge” arises from the interaction of the “charge” with its own “retarded field.” This includes the dissipative effects of radiation reaction and may also have some conservative effects as well.

- The acceleration from the self-force is proportional to $q^2/m$. For gravity this acceleration is proportional to $m$.

- Example: The orbital frequency of a circular Newtonian binary is

\[
\Omega^2 = \frac{M}{R^3(1+m/M)^2} \approx \frac{M}{R^3(1 - 2m/M)}
\]

where $R$ is the radius of the orbit of $m$, *not* the separation between the masses. The $-2m/M$ term is a consequence of the Newtonian gravitational self-force.

- For gravity, analysis of the self-force uses perturbation theory, $g_{ab} = g^o_{ab} + h_{ab}$. 

Some Details

• A gauge choice in perturbation theory is not just a choice of coordinates, but rather is a choice of how to associate an event on the background manifold with an event on the perturbed manifold. The change in $h_{ab}$ from a gauge transformation (an infinitesimal change in coordinates) $\xi^a$, is $\Delta h_{ab} - \nabla_a \xi^b - \nabla_b \xi^a$.

• Freedom in the choice of gauge adds confusion to a self-force calculation. There is no gauge-invariant definition of a self-force.

• The field from a charge is singular at the location of the charge — which is precisely where we need to know the field to calculate the self-force.

• Regularizing the field is the act of removing the singular part from the actual field leaving behind the regular part of the field, which is then used to calculate the self-force.

• Common ways of distinguishing between the singular and regular parts are

$$
\begin{align*}
  h^\text{act}_{ab} & = h^S_{ab} + h^R_{ab} \\
  h^\text{ret}_{ab} & = h^\text{direct}_{ab} + h^\text{tail}_{ab} \\
  h^\text{full}_{ab} & = h^\tilde{S}_{ab} + h^\tilde{R}_{ab}
\end{align*}
$$
Capra Issues

- Evolution of the Carter constant.
- Metric perturbations of the Kerr geometry.
- Gauge invariant descriptions of self-force effects.
- Formal analysis of second order metric perturbations with a point (small, black hole) source.
- A “practical” scheme for implementing the second order perturbation theory for a point source.

Capra Goals

- Better waveforms for LISA and LIGO.
- Comparison between perturbation analysis and the post-Newtonian approximation.
- Better understanding of the relativistic two body problem.