Problem

In the figure shown below, a 24 V battery is connected across capacitors of capacitance $C_1 = 6 \mu F$, $C_2 = 4 \mu F$, $C_3 = 2 \mu F$, $C_4 = 3 \mu F$, $C_5 = 2 \mu F$ and $C_6 = 1 \mu F$. (a) What is the effective capacitance of the entire circuit? (b) What is the voltage across $C_4$? (c) What is the charge on $C_4$? (d) What is the voltage across $C_6$?

Solution

In class we found that the equivalent capacitor had a value of $C_{eq} = 3.73 \mu F$. We can start getting information about the charges in the circuit by computing the total charge that the circuit is drawing from the battery. This is simply given by $Q_{total} = C_{eq}V = 24 \times 3.73 \mu F = 89.6 \mu C$.

Now, all of this charge provided by the battery (the positive one say) gets distributed among capacitors $C_4$, $C_5$ and $C_2$ since those are the only possible places where the (positive) charge from the battery can go to. If we call $Q_4$, $Q_5$ and $Q_2$ the charges on capacitors $C_4$, $C_5$ and $C_2$ respectively, we have;

$$89.6 \mu C = Q_4 + Q_5 + Q_2 \quad (1)$$

We have here one equation and three unknowns. If we want to solve for all of these charges we need two more equations that relate these three variables. We can obtain two more equations as follows: capacitors $C_2$ and $C_4$ are in parallel, thus their voltages are the same,
i.e.:

\[
\frac{Q_2}{C_2} = \frac{Q_4}{C_4} \quad \rightarrow \quad Q_4 = \frac{3}{2}Q_2 \tag{2}
\]

One more equation to go. The voltage across \( C_4 \) is the same as the one across the combination of \( C_5 \) and \( C_3 \) which are in series. Since the voltage across these capacitors is simply the sum of the voltages across each, we have

\[
\frac{Q_4}{C_4} = \frac{Q_5}{C_5} + \frac{Q_5}{C_3} \tag{3}
\]

notice that we have used the fact that the charge on capacitor \( C_3 \) is the same as the one on capacitor \( C_5 \) (they are in series). Now we have three equations (1), (2) and (3) for the three unknowns \( Q_4, Q_5 \) and \( Q_2 \), so now the problem has become a simple algebraic one.

**Problem**

The capacitors in the figure are initially uncharged. The capacitances are \( C_1 = 4.0\mu F \), \( C_2 = 8.0\mu F \) and \( C_3 = 12\mu F \), and the battery’s potential difference is \( V = 3V \). When the switch is closed, how many electrons travel through (a) point \( a \), (b) point \( b \), (c) point \( c \), and (d) point \( d \)? In the figure, do the electrons travel up or down through (e) point \( b \) and (f) point \( c \)?

**Solution**

Capacitors 1 and 2 are in parallel while this combination is in series with capacitor 3, therefore

\[
C_{eq} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = 6\mu F \tag{4}
\]

Going down from the negative side of the battery we only encounter \( C_3 \) thus all of the negative charge provided by the battery gets deposited on it. Hence

\[
q_3 = Q_{eq} = 3V \times 6\mu F = 18\mu C \tag{5}
\]

Capacitors 1 and 2 combined also hold the net charge \( Q_{net} = 18\mu F \), then

\[
q_1 + q_2 = 18\mu F \tag{6}
\]
In addition to this they also are in parallel, thus their voltages are the same, i.e.

\[ \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow q_1 = \frac{1}{2} q_2 \quad (7) \]

Putting (6) and (7) together yields \( q_1 = 6 \mu F \) and \( q_2 = 12 \mu F \). The charge of a single electron is \( 1.6 \times 10^{-19} \) C, therefore

(a) Since all of the positive charge provided by the battery passes through point \( a \), the number of electrons that passed through there is \( N_a = \frac{18 \mu C}{1.6 \times 10^{-19}} = 11.3 \times 10^{13} \) electrons.

(b) Through point \( b \) passes only the charge that goes to \( C_1 \), then \( N_b = \frac{6 \mu C}{1.6 \times 10^{-19}} = 3.75 \times 10^{13} \) electrons.

(c) Through \( c \) we have \( N_c = 11.3 \times 10^{13} - 3.75 \times 10^{13} = 7.55 \times 10^{13} \) electrons.

(d) Same as in \( a \)

(e) Up

(f) Up