No other materials allowed. If you can't do one part of a problem, solve subsequent parts in terms of unknown answer—define clearly. **Do 5 of first 6 problems, clearly indicating which you want graded!** You may attempt extra credit problems as well. All regular parts are worth 10 pts., extra credit 5 each, for maximum of 60 points. Good luck!
1. Expand $x/(e^x - 1)$ to order $x^2$ for $x \ll 1$.

2. The equation of state for a van der Waals gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT,$$

where $a$, $b$ and $R$ are constants. Consider two experiments on such a gas confined to a cylinder where you may control $p$, $V$ and/or $T$.

(a) Hold $T$ constant and find $dV/dp$.

(b) Hold $p$ constant and find $dV/dT$.

3. Change variables $x = u + v$, $y = u - v$, to rewrite the differential equation

$$\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} = 1$$

in terms of $u$ and $v$ (no need to solve the equation).
4. Evaluate the integral

\[
\int_{y=0}^{\pi} dy \int_{x=y}^{\pi} dx \frac{\sin x}{x}.
\]

5. If \( \nabla \cdot \vec{A} = 0 \) and \( \nabla \cdot \vec{B} = 0 \), show that

\[
\nabla \times (\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B}.
\]

[Hint: \( \epsilon_{ijk} \epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \)]
6. Look for a minimum of the function \( 1/x + 4/y + 9/z \) for \( x, y, z > 0 \) and \( x + y + z = 12 \) by the method of Lagrange multipliers.

7. (Extra credit.) Consider the vector \( \vec{V} = 4y\hat{i} + x\hat{j} + 2z\hat{k} \) and the scalar field \( \psi(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2} \).

(a) show \( \nabla \times \vec{V} = -3\hat{k} \)

(b) evaluate \( \int \vec{V} \cdot d\vec{r} \) from the origin \((0,0,0)\) to \((1,1,1)\) along the line \( x = t, y = t^2, z = t^3 \).

(c) evaluate \( \nabla \psi \) and \( \nabla \times \nabla \psi \).

8. (Extra credit.) Calculate the radii of convergence of the following series:

(a) \[ \sum_{n=1}^{\infty} \frac{(nx)^n}{n!} \]  

(b) \[ \sum_{n=1}^{\infty} \frac{x^n}{n^2 + 1} \]