1. A particle following an orbit characterized by a conserved energy $E > 0$ and a conserved angular momentum $\ell$ scatters in a potential $U(r)$ that goes to zero as $r \to \infty$. Write the energy as $E = \frac{1}{2} \mu v^2$ (this defines $v$) and write $\ell = \mu \nu b$ (defines $b$).

(a) Starting from an expression derived in class or an expression in the text, write the deflection angle from closest approach $r_0$ to $r \to \infty$ as an integral over the variable $x = b/r$ (this $x$ is not the Cartesian position $x$).

(b) Evaluate your integral for the hard sphere potential, $U = 0$ for $r > a$ and $U = \infty$ for $r < a$. Compute the relation between the scattering angle $\theta$ (the difference between incoming and outgoing directions) and the impact parameter $b$. Compute the differential scattering cross section $d\sigma/d\Omega$.

(c) Evaluate your integral for the Coulomb potential $U = -K/r$. Compute the relation between the scattering angle $\theta$ (the difference between incoming and outgoing directions) and the impact parameter $b$. Compute the differential scattering cross section $d\sigma/d\Omega$.

2. Two galaxies collide at a relative speed $v_0 = 1000 \text{ km s}^{-1}$. A star of mass $m = 1 M_\odot$ in galaxy A has an encounter with a black hole of mass $M = 10^6 M_\odot$ at the center of galaxy B. Before the collision $m$ is bound to its galaxy with an escape velocity $v_{\text{esc}} = 300 \text{ km s}^{-1}$. What are the conditions on $v_0$ and $b$ such that after the collision $m$ is kicked out of its galaxy? If galaxy A contains $10^{11}$ stars distributed more or less uniformly within a diameter of $30 h^{-1}$ kpc, how many of them are kicked out in the encounter?

(Bonus) Let $U(r)$ be the truncated Coulomb potential
\[
U(r) = \begin{cases} 
-\frac{K}{r} + \frac{K}{c} & (r < c), \\
0 & (r > c).
\end{cases}
\]

For an orbit characterized by $v$ and $b$, find the closest approach $r_0$. Find the relation between $\theta$ and $b$ ($r_0 > c$ is different from $r_0 < c$). Compute the differential scattering cross section $d\sigma/d\Omega$ and total cross section $\sigma$. 