1. A potential $\Phi(\rho, \phi, z)$ satisfies $\nabla^2 \Phi = 0$ in the volume $V = \{z \geq 0\}$ with boundary condition $\partial \Phi/\partial n|_S = F_S(\rho, \phi)$ on the surface $S = \{z = 0\}$.

(a) Write the Neumann Green’s function $G_N(\mathbf{x}, \mathbf{x}')$ within $V$ in cylindrical coordinates $\rho$, $\phi$, $z$ (and $\rho'$, $\phi'$, $z'$). Evaluate $G$ and its normal derivative $\partial G/\partial n'$ for $\mathbf{x}'$ on $S$.

(b) For zero charge density and with boundary condition $F_S = E_0$ (constant) within the circle $\rho < a$ and zero outside, find the potential on the $z$-axis. Compare the limit $z \to 0$ of your solution with the given boundary condition.

(c) Find the first two nonvanishing terms in the potential for $r = \sqrt{\rho^2 + z^2} \gg a$. Compare with (b) where the two overlap. What is the charge inferred from the large-$r$ potential?

2. A potential $\Phi(r, \theta, \phi)$ satisfies $\nabla^2 \Phi = -\rho/\varepsilon_0$ in the volume $V = \{r \geq a\}$ with boundary condition $\Phi|_S = V_S(\theta, \phi)$ specified on the surface $S = \{r = a\}$.

(a) Write the Dirichlet Green’s function $G_D(\mathbf{x}, \mathbf{x}')$ within $V$ and its normal derivative $\partial G/\partial n'$ for $\mathbf{x}'$ on $S$ in spherical coordinates $r$, $\theta$, $\phi$ (and $r'$, $\theta'$, $\phi'$).

(b) If the charge density vanishes, find $\Phi$ on the $z$-axis for $V_S = V_0 \sin^2 \theta$.

(c) Find $\Phi$ far from the sphere, $r \gg a$. Compare with (b) where the two overlap. What is the charge inferred?