1. Cosmic string is a possible component of the universe that has constant mass per unit length, and so \( M = \lambda \ell \); a domain wall has \( M = \sigma A \). What is the average mass density within a sphere of radius \( R \) that is crossed by a cosmic string? What is the mass density within a sphere of radius \( R \) crossed by a domain wall? If \( R \) expands with cosmological scale factor as \( a(t) \), how do these densities scale with \( a \)? Thus, what is the effective value of \( w \) for cosmic string and for domain walls?

2. Compute \( \zeta(3) \) numerically by any technique you choose to as many digits as you can. Show that \( \zeta(4) = \pi^4/90 \). Show that for fermions \( n_F = \frac{2}{3} n_{B} \) and \( \rho_F = \frac{7}{8} \rho_B \). [Hint: look at the difference in the occupation number, \( 1/(e^{\beta E} - 1) - 1/(e^{\beta E} + 1) \).]

3. What is the present-day neutrino number density (in one species)? What value of a neutrino mass (in eV) would provide \( \Omega_\nu = 1 \)?

4. In an open universe without cosmological constant, if \( \Omega \approx 0.3 \) today, what is \( \Omega \) when the universe passes through the epoch of nucleosynthesis, \( T \approx 10^9 \) K? At the electroweak scale, \( T \approx 100 \) GeV?

5. Take \( n_b/n_\gamma = \eta = 10^{-9} \).
   (a) Estimate the epoch (temperature) when an ionized universe becomes transparent (optical depth across the Hubble or horizon distance becomes less than 1).
   (b) Estimate the epoch when the hydrogen ionization fraction becomes small.
   (c) Estimate the epoch when the universe becomes matter dominated.