Due at the start of class on Wednesday, November 30. Half credit will be available for homework submitted after the deadline but turned no later than the start of class on Friday, December 2.

Answer all questions. Please write neatly and include your name on the front page of your answers. You must also clearly identify all your collaborators on this assignment. To gain maximum credit you should explain your reasoning and show all working.

1. An ideal full-wave rectifier converts an input current \( I_{in}(t) = I_0 \sin \omega t \) to an output current \( I_{full}(t) = |I_{in}(t)| \). An ideal half-wave rectifier converts the same input current to an output current \( I_{half}(t) = I_{in}(t) \Theta(I_{in}(t)) \), where \( \Theta(x) \) is the Heaviside function.
   (a) Find the Fourier sine and cosine series for the output of the full-wave rectifier.
   (b) By looking at the Fourier series for \( I_{full} \) at suitably chosen times \( t \), use your answer to (a) to evaluate
   \[ \sum_{k=1}^{\infty} \frac{1}{(2k)^2 - 1} \]
   (ii) \[ \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k)^2 - 1} \]
   (c) Find the Fourier sine and cosine series for the output of the half-wave rectifier.
   Hint: To reduce your work, examine the quantity \( I_{half}(t) - \frac{1}{2} I_{full}(t) \).

2. Verify that
   \[ \delta(\phi - \phi_0) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{in(\phi - \phi_0)} \]
   is a faithful representation of the Dirac delta function for polar angles in 2D or azimuthal angles in 3D, in that it obeys
   \[ \int_{0}^{2\pi} f(\phi) \delta(\phi - \phi_0) \, d\phi = f(\phi_0) \]

3. The Fourier series for both of the following functions \( f(x) \) was derived in class. Use each of these Fourier series to evaluate the corresponding sum \( S \).
   (a) \( f(x) = x^2 \) for \( |x| < \pi \); \( S = \sum_{k=1}^{\infty} (-1)^{k-1} k^{-2} \).
   (b) \( f(x) = 0 \) for \( -\pi < x < 0 \), \( f(x) = 1 \) for \( 0 < x < \pi \); \( S = \sum_{k=1}^{\infty} (-1)^{k-1} (2k + 1)^{-1} \).

   The Fourier coefficients of any square-normalizable function \( f(x) \) decrease in magnitude with increasing \( n \) (or \( n! \) for exponential series). If \( f(x) \) is discontinuous, the coefficients vary as \( 1/n \) for \( n \to \infty \), while if \( f(x) \) and its first \( k - 1 \) derivatives are continuous but the \( k \)’th derivative is discontinuous, then its coefficients decay as \( 1/n^{k+1} \).

4. Calculate the Fourier transform of the function \( f(x) = h(1 - |x|/w)\Theta(w - |x|) \), where \( h \) and \( w \) are constants and \( \Theta(x) \) is the Heaviside function. Hint: Use integration by parts.