Consider a system, composed of $N$ classical spins, which is in thermal equilibrium with a heat reservoir at temperature $T$, and which is subjected to a uniform magnetic field of strength $H$. The spins are distinguishable and independent of one another. Each spin has precisely three possible orientations, directed at angles $\theta = 0, \pm 2\pi/3$ to the direction of the magnetic field. The magnetic energy of each spin is $\epsilon(\theta) = -\mu H \cos \theta$, where $\mu$ is the magnetic moment, a (known) constant.

(a) Write down the partition function for this system. Neglect all non-magnetic degrees of freedom, i.e., just take into account the magnetic energy for each spin.

(b) Calculate the Helmholtz free energy $F$ of the system.

(c) The total magnetic energy of the system can be written $E_{\text{mag}} = -MH$, where $M$ is the magnetization. Calculate $M$ by using the total differential $dF = -SdT - MdH$.

(d) Derive the limiting forms for $M$ in the limits $k_B T \ll \mu H$ and $k_B T \gg \mu H$. In each case, include the leading nonvanishing dependence on both $T$ and $H$.

(e) Sketch $M$ as a function of $H$ at fixed $T$. You should pay special attention to getting the low-field and high-field limits qualitatively correct.