1. **Canonical ensemble with continuous microstates.** (Based on Reif Problem 6.11.)

Consider a pair of atoms which are constrained to move only along the $x$ direction. Let $x_j$ and $p_j$ be the position and momentum of atom $j$ ($j = 1, 2$). Assume that the total energy of the pair can be written

$$E = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + U(x_1 - x_2)$$  \hspace{1cm} (1)

where $m$ is the mass of each atom and $U(r)$ is the Lennard-Jones potential, which describes the atomic interaction in the group VIII elements He, Ne, Ar Kr, and Xe:

$$U(r) = U_0 \left[ \left( \frac{a}{r} \right)^{12} - 2 \left( \frac{a}{r} \right)^{6} \right].$$  \hspace{1cm} (2)

Here $U_0$ and $a$ characterize the strength and range of the potential, respectively.

Provided that the atoms are maintained in thermal contact with a heat reservoir, the expectation (mean) value of any quantity $y(x_1, p_1, x_2, p_2)$ can be written

$$\langle y \rangle = \int dx_1 dp_1 dx_2 dp_2 y e^{-\beta E} \int dx_1 dp_1 dx_2 dp_2 e^{-\beta E}.$$  \hspace{1cm} (3)

Using the preceding information,

(a) calculate the mean separation between the atoms, $\bar{x}(T) = \langle |x_1 - x_2| \rangle$;

(b) calculate the linear coefficient of thermal expansion, $\alpha = \frac{1}{\bar{x}} \frac{\partial \bar{x}}{\partial T}$.

The following hints may simplify the problem:

(i) Given the form of Eq. (1), it will prove advantageous to change integration variables in Eq. (3) from $x_1$ and $x_2$ to $x_1 + x_2$ and $x_1 - x_2$.

(ii) You should be able to take advantage of the separability of Eq. (1) to greatly simplify Eq. (3) for the case $y = |x_1 - x_2|$.

(iii) Expand the potential $U(r)$ about its minimum. You will need to figure out how many terms in the expansion you need to keep to obtain the leading term in $\alpha$.

(iv) When performing integrals of the form $\int_0^\infty dz \exp(-a_2 x^2 + a_3 x^3 + \ldots)$, it will be to your benefit to replace the exponential by $\exp(-a_2 x^2)(1 + a_3 x^3 + \ldots)$. Under what conditions is this approximation valid?

2. **Ideal gas in a gravitational field.**

Do Reif Problem 7.2.

3. **Classical picture of electrical resistivity.**

Do Reif Problem 7.8.