1. This question concerns a one-dimensional harmonic operator described by the Hamiltonian \( H = P^2/2m + m\omega^2X^2/2 \), which has eigenkets \( |n\rangle \) with energies \( (n + \frac{1}{2})\hbar\omega \) \((n = 0, 1, 2, \ldots)\). Use the Heisenberg picture in answering all parts of the question.

(a) Starting from the Heisenberg equation, \( d\Omega/dt = (i/\hbar)[H, \Omega] + \partial\Omega/\partial t \), derive equations of motion for the position operator \( X \) and the momentum operator \( P \). (These equations should contain no commutators.) \([15]\)

(b) Show that the lowering (annihilation) operator \( a \) obeys an equation of motion of the form \( da/dt \propto a \). Integrate this equation of motion to obtain \( a(t) \). \([10]\)

(c) Let \( |\phi\rangle = c_0|0\rangle + c_1|1\rangle \) be the normalized linear combination of \( |0\rangle \) and \( |1\rangle \) that has the greatest value of \( \langle P(t = 0) \rangle \). Find the coefficients \( c_0 \) and \( c_1 \). \([20]\)

(d) Find \( \langle P(t) \rangle \) for the state \( |\phi\rangle \). Eliminate \( c_0 \) and \( c_1 \) from your final answer. \([10]\)

(e) Find \( \langle P(t) \rangle \) for the mixed state described by the state operator \( \rho = |c_0|^2|0\rangle\langle 0| + |c_1|^2|1\rangle\langle 1| \), where \( c_0 \) and \( c_1 \) are the coefficients defined in part (c). Eliminate \( c_0 \) and \( c_1 \) from your final answer. \([10]\)

2. The two parts of this question are unrelated to one another.

(a) Suppose that a particle in two dimensions is described by a wave function

\[ \psi(x, y) = (2x^2 - y^2)f(x^2 + y^2), \]

where \( f(s) \) is an unspecified function. What are the possible values that could be obtained when \( l_z \) (the z component of the orbital angular momentum) is measured? What is the probability of obtaining each \( l_z \) value? \([15]\)

(b) Find the transformed orbital angular momentum operator, \( \mathbf{L}' = U^{-1}\mathbf{L}U \), under the action of the symmetry operator \( U \) corresponding to each of the following symmetry transformations: (i) spatial displacement by \( a \); (ii) clockwise rotation by an angle \( \omega \) about the \( \hat{z} \) axis; (iii) spatial inversion through the origin; (iv) time reversal.

Note: The four symmetry transformations are to be applied separately, not in combination; separate answers are required for (i)–(iv). \([20]\)