1. Consider four possible quantizations of the classical dynamical variable \( \omega = x^2p^2 \):

- \( \Omega_1 = x^2p^2 \)
- \( \Omega_2 = p^2x^2 \)
- \( \Omega_3 = \frac{1}{2}(x^2p^2 + p^2x^2) \)
- \( \Omega_4 = \frac{1}{4}(xp + px)^2 \)

Use the commutation relation between \( X \) and \( P \) to simplify as much as possible

(a) \( \Omega_1 - \Omega_2 \)
(b) \( \Omega_1 - \Omega_3 \)
(c) \( \Omega_2 - \Omega_4 \)

Here, simplification means expressing the result in terms of the lowest possible powers of the operators \( X \) and \( P \). Also, you should express each terms in such a way that any residual \( X \) operators are placed to the left of any \( P \) operators.

2. Based on Ballentine Problem 2.9: Let \( R = \begin{pmatrix} 6 & -2 \\ -2 & 9 \end{pmatrix} \) be a representation of the operator \( R \) corresponding to some dynamical variable \( r \), and \( |\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \) be an arbitrary, normalized state vector (with \( |a|^2 + |b|^2 = 1 \)). Suppose that \( f(R) \) is some function of \( R \). It is possible to calculate \( \langle f(R) \rangle \) in two ways: (i) Evaluate \( \langle f(R) \rangle = \langle \psi | f(R) | \psi \rangle \) directly. (ii) Find the eigenvalues and eigenvectors of \( R \), \( R|r_n\rangle = r_n|r_n\rangle \), expand the state vector as a linear combination of the eigenvectors, \( |\psi\rangle = c_1|r_1\rangle + c_2|r_2\rangle \), and evaluate \( \langle f(R) \rangle = f(r_1)|c_1|^2 + f(r_2)|c_2|^2 \).

(a) Use both methods (i) and (ii) to evaluate \( \langle R \rangle \).
(b) Use both methods (i) and (ii) to evaluate \( \langle R^2 \rangle \).
(c) Find the uncertainty \( \Delta R \).

3. Shankar Exercise 4.2.1. As Shankar says, this exercise is very important. Therefore, it will be graded even though the answers are given in the book.