Relativistic Kinematics (sample problems)

Problem A

If neutrons were produced by the black hole in the center of our galaxy, with what energy would they need to be produced in order to reach the Earth before decaying? The distance between the center of our galaxy and the Sun is about 25,000 light years. Neutron’s mass is 940 MeV, and lifetime is 886 s.

Problem B

A particle of mass M at rest decays into two particles with masses m₁ and m₂. Find energies of these two particles.

Problem C

Consider a high energy proton interacting with a photon of the cosmic microwave background (cosmic microwave background photons are approximately in thermal equilibrium with T=3 K). Find the minimum energy the proton would need for the following reaction to occur (Δ-resonance is the lowest excited state of quarks inside a proton):

\[ p + \gamma \rightarrow \Delta^+ \rightarrow p + \pi^0. \]

Masses: proton—938 MeV, neutral pion—135 MeV, Δ-resonance—1232 MeV, photon—0

If kinematically allowed, this reaction has a very large probability. After such scattering, the outgoing proton emerges with smaller energy (some energy goes into pion’s mass and the rest is shared between the proton and the pion, proton’s share being dependent on the angle of scattering). Therefore, high energy protons should lose their energy via such interactions. As the result, this process limits the maximum energy of cosmic rays reaching the Earth, assuming such high energy protons are not produced in close vicinity of our galaxy. Recent searches for cosmic rays with energy higher than the threshold have been giving controversial results and, hence, very exciting. The threshold is known as GZK-cutoff (Greisen, Zatsepin, Kuzmin)
Earlier results by AGASA experiment indicated no sign of the cosmic ray energy cutoff.

More recent HiRes experiment results clearly show the GZK-cutoff.
AGASA results would agree with HiRes, if AGASA’s energy calibration is rescaled.
The case now seems to be closed (however, die-hard proponents of AGASA results are still out there).