Properties of Limits

Suppose that \( c \) is a constant and the limits \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) exist. Then

1. \[
\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)
\]

2. \[
\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)
\]

3. \[
\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)
\]

4. \[
\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)
\]

5. \[
\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \text{ if } \lim_{x \to a} g(x) \neq 0
\]

6. \[
\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n \text{ where } n \text{ is a positive integer}
\]

7. \[
\lim_{x \to a} c = c
\]

8. \[
\lim_{x \to a} x = a
\]

9. \[
\lim_{x \to a} x^n = a^n \text{ where } n \text{ is a positive integer}
\]

10. \[
\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a} \text{ where } n \text{ is a positive integer}
\]
    (if \( n \) is even, we assume \( a > 0 \).)

11. \[
\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} \text{ where } n \text{ is a positive integer}
\]
    (if \( n \) is even, we assume \( \lim_{x \to a} f(x) > 0 \).)
Limit Theorems

Direct Substitution Property: If $f$ is a polynomial or rational function and $a$ is in the domain of $f$, then

$$\lim_{x \to a} f(x) = f(a)$$

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided the limits exist.

1. THEOREM

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

2. THEOREM If $f(x) \leq g(x)$ when $x$ is near $a$ (except possibly at $a$) and the limits of $f$ and $g$ both exist as $x$ approaches $a$, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

3. THE SQUEEZE THEOREM If $f(x) \leq g(x) \leq h(x)$ when $x$ is near $a$ (except possibly at $a$) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$