Reading: Sections 4.1-4.4 from the textbook and class notes.

Problem 1. Allowed interactions. If $\phi$ is a real scalar field and $\psi$ is a Dirac field, which of the following terms can appear in the Lagrangian density of a sensible field theory in $3 + 1$ dimensions? Explain why or why not. Which ones can appear in principle, but become irrelevant at sufficiently low energies? In each case, the (real) coefficients $c_i$ are chosen to have dimensions such that the term in the Lagrangian density has mass dimension 4.

(a) $c_1 \phi^7(x)$;
(b) $c_2 \partial_\mu \phi(x) \bar{\psi}(x) \gamma^\mu \psi(x)$;
(c) $c_3 \phi(x) \psi^4(x) \psi(x)$;
(d) $c_4 \phi(x) i \bar{\psi}(x) \gamma^5 \psi(x)$;
(e) $c_5 (\bar{\psi}(x) \psi(x))^2$;
(f) $c_6 \phi^2(x) \partial_\mu \phi(x)$;
(g) $c_7 \phi(x) \bar{\psi}(x + \delta) \psi(x + \delta)$;
(h) $c_8 \phi(x) \gamma^\mu \partial_\mu \psi(x)$.

Problem 2. Symmetry factors in $\varphi^4$ theory. Consider the second-order term in the expansion (4.43)

$$\langle 0 | T \left\{ \phi(x) \phi(y) \frac{(-i)^2}{2!} \frac{\lambda}{4!} \int d^4 z \phi(z) \phi(z) \phi(z) \phi(z) \int d^4 u \phi(u) \phi(u) \phi(u) \phi(u) \right\} | 0 \rangle.$$

(a) Classify all topologically distinct classes of diagrams.

(b) By enumerating all possible contractions, calculate the symmetry factor for each class. Hint: Check that the sum of the symmetry factors is equal to $9!!/(2!4!4!) = 945/1152$.

(c) Check your answer from (b) by considering the symmetries of the diagrams and thus give an independent justification of your previous answers.

(d) Extra credit: Can you now handle the third-order term in (4.43)?