In the figure below right, a small circular hole of radius $R = 3.00$ cm has been cut in the middle of an infinite, flat, nonconducting surface that has uniform charge density $\sigma = 6.00$ pC/m$^2$. A $z$ axis, with its origin at the hole’s center, is perpendicular to the surface. We are going to find out the electric field at point $P$ at $z = 5.00$ cm in a following way:

a) Applying Gauss’s law, find the electric field at point $P$ due to the nonconducting surface before the hole was cut.

We take our Gaussian surface to be a cylinder which has end caps of area $A$ parallel to the flat surface. Then the electric field is perpendicular to the end caps and parallel to the round surface of the cylinder. Gauss’s law yields

$$\varepsilon_0 E(2A) = \sigma A \Rightarrow E = \frac{\sigma}{2\varepsilon_0} = 0.339 \text{ N/C}$$

b) If charge $q$ is uniformly distributed around a ring of radius $r$, the electric field due to the ring along its central axis is given by $E = kqz/(z^2 + r^2)^{3/2}$ $\hat{k}$. Use this fact to find out the electric field at point $P$ due to a disk of radius $R$ that is centered at the origin and has uniform charge density $\sigma$.

The charge on a ring with a radial width $dr$ is expressed in terms of $\sigma$ as $q = \sigma(2\pi rd)dr$ and it produces an electric field $dE = k\sigma(2\pi rd)/(z^2 + r^2)^{3/2}$. We consider a disk to be made out of concentric thin rings and add electric fields due to individual rings. Therefore, the electric field due to the disk of radius $R$ is

$$E_d = \int [k\sigma(2\pi rd)/(z^2 + r^2)^{3/2}2\pi rd] dr = 2\pi k\sigma(1/s^{3/2})(ds/2) = 2\pi k\sigma(1/z)\left[1 - (1/z^2 + r^2)^{3/2}\right]_0^R = 2\pi k\sigma((1/z) - (1/(z^2 + r^2)) = (\sigma/2\varepsilon_0)\left[1 - 1/(1 + (r/z)^2)\right] = 0.0483 \text{ N/C}$$

c) Use the superposition principle to obtain the electric field due to the nonconducting surface after the hole was cut.

According to the superposition principle, the electric field due to a flat surface w/o a hole is given by the vector sum of the field due to the surface with a hole and that due to the removed disk. Therefore, after the hole was cut, the electric field is

$$E' + E_d = E \Rightarrow E = E - E_d = 0.339 - 0.0483 = 0.291 \text{ N/C}$$