1. Calculate the electric field and scalar potential for a conducting sphere of radius $R$ for all $r$ when a charge $Q$ is distributed over its surface. [Hint for those who have forgotten E&M: a conductor has $\phi = \text{const.}$ everywhere inside and on its surface.]

2. Show using Maxwell’s equations that in free space, the electric field $\vec{E}$ satisfies a wave equation with wave speed equal to the speed of light, $c = 3.0 \times 10^8$ m/s. You may need

$$\mu_0 = 1.26 \times 10^{-6} \text{ Henry/m (SI)} \quad (1)$$
$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m (SI)} \quad (2)$$

3. Supposing you apply a static magnetic field parallel to the surface of a metal, $B = B_0 \hat{x}$ (the normal to the surface is $\hat{z}$). Make the choice of gauge corresponding to vector potential $\vec{A} = -B_0 \hat{z} \hat{y}$, and verify that $\vec{\nabla} \times \vec{A} = \vec{B}$. Then assume that the metal in question is a superconductor, which means there is a very special relation between the current and the applied vector potential,

$$\vec{j} = K \vec{A}, \quad (3)$$

where $K$ is a constant $< 0$. Verify using Maxwell’s equations that the magnetic field $B$ decays exponentially into the bulk of the metal, $B = B_0 \exp(-z/\lambda)$, and find $\lambda$.

4. Boas Problem 6.11.16, “What is wrong...”

5. The scalar potential due to a charge distribution, and the vector potential due to a current distribution, may be written as

$$\Phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_\tau \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau' \quad \text{and} \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_\tau \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} d\tau',$$ 

where the integration variable $\vec{r}'$ runs over some volume $\tau$ to which the charges and currents are confined (could be all space). Show that

$$\vec{\nabla} \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) = -\vec{\nabla}' \left( \frac{1}{|\vec{r} - \vec{r}'|} \right) = -\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^2}, \quad (5)$$
where

\[ \vec{r} - \vec{r}' = \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|}. \] (6)

If the notation is confusing, ASK! Now calculate the fields \( \vec{E}(\vec{r}) \) and \( \vec{B}(r) \) from the potentials using \( \vec{E} = -\vec{\nabla}\Phi \) and \( \vec{B} = \vec{\nabla} \times \vec{A} \).