1. Show that
\[ \oint_C (z - z_0)^n \, dz = \begin{cases} 2\pi i & n = -1 \\ 0 & n \neq -1 \end{cases}, \] (1)
where the contour encircles the point \( z = z_0 \) in a positive (counterclockwise) sense. The exponent \( n \) is an integer.

2. Show that
\[ \frac{1}{2\pi i} \oint z^{m-n-1} \, dz = \delta_{mn}, \quad m \text{ and } n \text{ integers} \] (2)

3. Evaluate
\[ \oint \frac{dz}{z^2 - 1} \] (3)
where \( C \) is the circle \( |z| = 2 \).

4. Find the first three nonzero terms of the Laurent series of
\[ f(z) = (e^z - 1)^{-1} \] (4)

5. Determine the nature of the singularities of each of the following functions and evaluate the residues (take \( a > 0 \)).

(a) \[ \frac{1}{z^2 + a^2} \] (5)

(b) \[ \frac{z^2}{(z^2 + a^2)^2} \] (6)

(c) \[ \frac{\sin \frac{1}{z}}{z^2 + a^2} \] (7)

(d) \[ \frac{ze^{iz}}{z^2 + a^2} \] (8)