1. If \( u = f(x - ct) + g(x + ct) \), show that

\[
\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}.
\]  
(1)

What can you say about the possible physical interpretation of \( u \)?

2. Given \( s(v, T) \) and \( v(p, T) \), define \( c_p \equiv T(\partial S/\partial T)_p \), \( c_v \equiv T(\partial s/\partial T)_v \). Show that

\[
c_p - c_v = T \left( \frac{\partial s}{\partial v} \right)_T \left( \frac{\partial v}{\partial T} \right)_p
\]  
(2)

[Hint: you need \( s(p, T) \) to calculate \( c_p \), i.e. find \( dS = (\ldots)dp + (\ldots)dT \).]

3. Find the point on the curve \( x^2 - 2\sqrt{3}xy - y^2 = 2 \) which is closest to the origin \( 0, 0 \).

4. The temperature of a rectangle bounded by the lines \( x = \pm 1 \), \( y = \pm 2 \) is given by \( T = x^2 - 4y^2 + y - 5 \). Find the hottest and coldest point.

5. Transform the differential equation

\[
x^2 \left( \frac{d^2 y}{dx^2} \right) + 2x \left( \frac{dy}{dx} \right) - 5y = 0
\]  
(3)

with the help of the substitution \( x = e^z \) into another differential equation in \( d^2 y/dz^2 \), \( dy/dz \) and \( y \) which has only constant coefficients of the derivative terms.