Remember to work each problem as fully as possible. Partial credit will be given so SHOW ALL OF YOUR WORK!!

1. (a) (15 points) Which of the following cannot possibly be a valid electric field?
   i)  \[ E(r, \theta, \phi) = \left(2k \cos \frac{\theta}{r^3}\right) r + \left(k \sin \frac{\theta}{r^3}\right) \theta \]
   ii) \[ E(x, y, z) = 2kxyz^3 x + x^2z^3 y + 3x^2y^2z^2 z \]

   (b) (18 points) Suppose the potential in some region of space is given by:
   \[ V(s, \phi, z) = \frac{-k \sin \phi}{s^2} \]
   Find the charge distribution that produces this potential.

2. A solid spherical shell with inner radius \( a \) and outer radius \( b \) carries a charge distribution given by \( \rho(r) = k/r^2 \).
   
   (a) (5 points) Find the total charge on the shell.

   (b) (12 points) Find the electric field for the regions i) \( r < a \), ii) \( a < r < b \), and iii) \( r > b \)

   (c) (16 points) Find the energy of the charge configuration, i.e., the energy required to assemble it.

3. An uncharged spherical conductor has an arbitrarily shaped hollow cavity cut out of its interior. Inside the cavity is placed a charge \( q \). (See figure.) Answer the following questions, carefully explaining your reasoning.

   (a) What is the induced charge density on the outer surface of the sphere?

   (b) What is the electric field at a point \( P \) outside the conductor?

   (c) In which direction do the electric field lines point at the outer surface of the conductor?
(d) If a charge Q were brought close to the conductor, how much total charge would be located on the inner surface?

(e) If a charge Q were brought close to the conductor, in which direction do the electric field lines point at the outer surface of the conductor?

(f) If a charge Q were brought close to the conductor, would the charge density on the outer surface of the conductor be uniform?

(g) If a second cavity were cut out of the conductor without placing any charge in it, would the answer to any of the above questions change? Which ones?

Useful information:

\[
\int_{a}^{b} \frac{(r-a)^2}{r^2}dr = \frac{b^2 - a^2 + 2ab(ln a - ln b)}{b^2}
\]