Question 1

How much energy \( E \) (IN JOULES) is consumed in \( h \) hours by an electrical resistance \( R \) when the potential applied across it is \( V \)? (3 points) (Note: your answer must be in terms of the variables listed above to receive credit.)

Energy is power times time. The power dissipated by a resistance is,

\[
P = i^2 R = \frac{V^2}{R},
\]

(1)

and therefore in Joules,

\[
E = 3600hP = 3600\frac{V^2h}{R}.
\]

(2)

Question 2

(a) What is the equivalent resistance of the circuit above (1 point)?

The three resistors to the right are all in parallel and their equivalent resistance is in series with the remaining resistor \( R \),

\[
R_{eq} = R + \left( \frac{1}{R} + \frac{1}{r} + \frac{1}{R} \right)^{-1},
\]

(3)

\[
= \left( \frac{R + 3r}{R + 2r} \right) R.
\]

(4)

(b) In terms of \( V \), \( R \), and \( r \), what is the current through resistor \( r \) (1 point)?

The total voltage drop around a loop is zero, which we can express as,

\[
V - iR - i_r r = 0,
\]

(5)

where \( i \) is the current coming from the battery. We can use \( R_{eq} \) to express \( i \) in terms of \( R \), \( r \), and \( V \). Solving for \( i_r \),

\[
i_r = \frac{V}{r} \left( 1 - \frac{R}{R_{eq}} \right),
\]

(6)

\[
= \frac{V}{R + 3r}.
\]

(7)
(c) What is the current through the outer resistor $R$ (the furthest to the right) if one takes $r \to \infty$ (1 point)? If $r \to \infty$, then no current will pass through resistor $r$, and it is a simple task to solve the resulting circuit. Alternatively, we can simply solve for the current and take the limit formally. As we did in part (b),

\begin{align*}
V - iR - i_R R &= 0, \\
i_R &= \frac{V}{R} \left(1 - \frac{R}{R_{eq}}\right), \\
&= \frac{V}{R} \left(\frac{r}{R + 3r}\right) \xrightarrow{r \to \infty} \frac{V}{3R}.
\end{align*}