31. Consider the magnetic field produced by a current $I$ in an infinitely long wire lying on the $z$ axis $-\infty < z < \infty$.

   a) Use symmetry arguments and Ampère’s law to obtain the $B$ field everywhere outside the wire. Express the Cartesian components of $B$ as explicit functions of $x, y, z$.

   b) By direct integration of each component of $\nabla \times A = B$, find the vector potential $A$ for this $B$ in Coulomb gauge, $\nabla \cdot A = 0$.

   c) Since $\nabla \times B = 0$ “almost everywhere” we should be able to find a scalar potential such that $B = -\nabla \phi$ “almost everywhere”. By explicitly integrating the components of this equation, find a candidate for $\phi$ as an explicit function of $x, y, z$.

   d) In view of the fundamental theorem of calculus

   $$\phi(y) - \phi(x) = \int_x^y d\ell \cdot \nabla \phi = -\int_x^y d\ell \cdot B,$$

   explain how your result for part c) does not run afoul of Ampère’s law.

32. J, Problem 5.7.