Gravity Gets There First with Dark Matter Emulators

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Dark Matter vs Mod. Gravity

- $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ works for solar system
- But not for galaxies
- Theory: $v^2 = GM/r$
- Obser: $v^2 \sim (a_0 GM)^{1/2}$
- Maybe missing Mass
- Or modified gravity
MOND (Milgrom 1983)

- $\rho(x,y,z) \equiv$ mass in stars and gas
  $\Rightarrow g_N^i \equiv$ Newtonian acceleration
- $g^i \equiv$ actual acceleration
  $\Rightarrow g^i \mu(|g|/a_0) = g_N^i$
- $a_0 \sim 10^{-10}$ m/s$^2$
- GR regime: $\mu(x) = 1$ for $x \gg 1$
- MOND regime: $\mu(x) = x$ for $x \ll 1$
  $\Rightarrow$ Eg. $\mu(x) = x/(1+x)$, or $\tanh(x)$, . . .
MOND Successes for Rotationally Supported Systems

- Asymp. flat curves
- Milgrom’s Law: need dark mat. for $g \sim a_0$
- Freeman’s: $\Sigma < a_0 / G$
- Sancisi’s: bumps trace baryons
cf. NCG 1560 (LSB) & NGC 2903 (HSB)
Baryonic Tully-Fisher Relation

$$GMa_0 = (v_\infty)^4$$
MOND Successes for Pressure-Supported Systems

- Isothermal: $\sigma^2 \sim p/\rho = \text{const}$
- Equipartition: $\frac{1}{2} \rho \sigma^2 = \rho \int dr \ g$
- Hydro Equil: $p'(r) = -\rho(r) \ g(r)$
- Newtonian: $\rho(r) \sim 1/r^2$
- MONDian: $\rho(r) \sim 1/r^4$

  ➔ Faber-Jackson: $\sigma^2 = 4 \ (GMa_0)^{1/2}$
  ➔ Size: $r = (GM/a_0)^{1/2}$
\[ \sigma^2 = a_0 r \] over six orders of magnitude in size!

- Molecular clouds
- Globular clusters
- Dwarf spheroidals
  - Compact dwarfs
- Elliptical galaxies
- Galaxy clusters
Potentials and Data Sets

- $ds^2 = -B(r) c^2 dt^2 + A(r) dr^2 + r^2 d\Omega$
- $\Delta B(r) \equiv B(r) - 1$
  - Controls rotation curves
  - $\Delta B'(r) = 2v^2/c^2$
- $\Delta A(r) \equiv A(r) - 1$
  - Affects light deflection (lensing)

$\Delta \phi = \int_0^{\pi/2} d\theta \left\{ \Delta A(R_0 \sec \theta) - \csc^2 \theta [\Delta B(R_0) - \Delta B(R_0 \sec \theta)] \right\}$
Galactic Potentials for $\rho(r)$

- $ds^2 = -B(r)c^2 dt^2 + A(r) dr^2 + r^2 d\Omega$

  \[
  \frac{A'}{rA} + \frac{(A - 1)}{r^2} = \frac{8\pi G \rho A}{c^2 B}
  \]

  \[
  \frac{B'}{rB} - \frac{(A - 1)}{r^2} = 0
  \]

- $\Delta A \equiv A - 1$ and $\Delta B \equiv B - 1$

  \[
  \Delta A(r) = \frac{8\pi G}{c^2 r} \int_0^r dr' r'^2 \rho(r')
  \]

  \[
  \Delta B(r) = -\int_r^\infty dr' \frac{\Delta A(r')}{r'}
  \]

- N.B. $\Delta A > 0$ and $\Delta B \leq -\Delta A$
Common Dark Matter Profiles

- **Isothermal Halo** \( r < r_c \)
  \[
  \rho(r) = \rho_0 r_0^2 \left[ \frac{1}{r^2 + r_0^2} - \frac{1}{r_c^2 + r_0^2} \right]
  \]

- **NFW**
  \[
  \rho(r) = \frac{\rho_0 r_0^3}{[r(r^2 + r_0^2)]}
  \]

- **Moore 99**
  \[
  \rho(r) = \frac{\rho_0 r_0^3}{[r^{3/2}(r + r_0)^{3/2}]}
  \]

<table>
<thead>
<tr>
<th>Profile</th>
<th>(8\pi G \rho_0 r_0^3/c^3)</th>
<th>(r_0)</th>
<th>(r_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isothermal</td>
<td>3.98 days</td>
<td>4.00 kpc</td>
<td>219 kpc</td>
</tr>
<tr>
<td>NFW</td>
<td>60.8 days</td>
<td>16.7 kpc</td>
<td>N.A.</td>
</tr>
<tr>
<td>Moore</td>
<td>51.8 days</td>
<td>29.5 kpc</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

**TABLE I:** Dark Matter Profile Parameters for the Milky Way Galaxy from Ascasibar et al. [60].

<table>
<thead>
<tr>
<th>Profile</th>
<th>(8\pi G \rho_0 r_0^3/c^3)</th>
<th>(r_0)</th>
<th>(r_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isothermal</td>
<td>1.88 days</td>
<td>1.47 kpc</td>
<td>117 kpc</td>
</tr>
<tr>
<td>NFW</td>
<td>48.6 days</td>
<td>12.5 kpc</td>
<td>N.A.</td>
</tr>
<tr>
<td>Moore</td>
<td>45.8 days</td>
<td>25.0 kpc</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

**TABLE II:** Dark Matter Profile Parameters for the Andromeda Galaxy from Tempel et al. [61].
Eg. NFW Potentials

\[ \Delta A(r) = \frac{8\pi G\rho_0 r_0^3}{c^2} \left\{ \frac{\ln(1 + r/r_0)}{r} - \frac{1}{r + r_0} \right\} \]

\[ \Delta B(r) = \frac{8\pi G\rho_0 r_0^3}{c^2} \left\{ -\frac{\ln(1 + r/r_0)}{r} \right\} \]
No-Go Thm for metric models
With Soussa, astro-ph/0307358

- $\mathcal{E}_{\mu\nu} = 8\pi G T_{\mu\nu} \sim GM$ for $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- $h_{\mu\nu} \sim v^2 \sim (a_0 GM)^{1/2} \Rightarrow$ some $\mathcal{E}_{\mu\nu} \sim h^2$
- If all $\mathcal{E}_{\mu\nu} \sim h^2 \Rightarrow$ unstable!

- Distinguished subsets
  - Divergence (0 to all orders)
  - Trace $\Rightarrow$ conf. invariant for $\mathcal{E}_{\mu\nu} \sim h_{\mu\nu}$

- Extra force in conf. factor $\Rightarrow$ no lensing!
Five Assumptions

1. Gravity carried by $h_{\mu\nu}$ with source $T_{\mu\nu}$
2. General coordinate invariance
3. Extra force in ultra-weak field regime
4. Stability (forbids all $\mathcal{E}_{\mu\nu} \sim h^2$)
5. Light couples conformally
   - Known models violate (1) & (5)
   - Violating (4) may also work
Known Models

TeVeS (Bekenstein)
- astro-ph/0403694
- Cosmology (astro-ph/0505519, 0606216, 0608602, 0611255)

SVTG (Moffat)
- gr-qc/0506021
- astro-ph/0506370

- Fields: $g_{\mu\nu}$, $A_\mu$, $\phi$
- Extra force from $\phi$
- Matter couples to $\hat{g}_{\mu\nu}$
  $= e^{-2\phi} g_{\mu\nu} + \sinh(2\phi)A_\mu A_\nu$
- R term for solar system
- Gravitons couple to $g_{\mu\nu}$
- $\hat{g}_{\mu\nu}$ from GR with D.M.
- $g_{\mu\nu}$ from GR w/o D.M.
Null Geodesics $\chi^\mu(\tau)$

- **SOURCE:** $x_0^\mu = (0, x_0^i)$ to **US:** $x_1^\mu = (ct, x_1^i)$
- $d^2\chi^\mu/d\tau^2 + \Gamma^\mu_{\rho\sigma}(\chi)(d\chi^\rho/d\tau)(d\chi^\sigma/d\tau) = 0$
  - $\chi^\mu(0) = x_0^\mu$ and $\chi^i(1) = x_1^i$
  - $g_{\mu\nu}(\chi)(d\chi^\mu/d\tau)(d\chi^\nu/d\tau) = 0$
- **0th order:** $\chi_0^\mu(\tau) = x_0^\mu + \Delta x^\mu \tau$
  - $\Delta x^0 \equiv ||x_1 - x_0||$ and $\Delta x^i \equiv x_1^i - x_0^i$
  - $ds^2 \equiv (\eta_{\mu\nu} + h_{\mu\nu}) \, dx^\mu \, dx^\nu$
  - $h_{00} = -\Delta B$, $h_{0i} = 0$, $h_{ij} = \Delta A \, x^i \, x^j/r^2$
  - $\chi_0^0(1) = \frac{1}{2\Delta x} \int_0^1 d\tau \, h_{\mu\nu}(x_0 + \Delta x\tau) \Delta x^\mu \Delta x^\nu$
  - $\chi_1^0(1) = \frac{1}{2\Delta x} \int_0^1 d\tau \, h_{\mu\nu}(x_0 + \Delta x\tau) \Delta x^\mu \Delta x^\nu$
1\textsuperscript{st} order $\Delta t$ just from $\rho_{DM}$!

- $c\Delta t = \frac{1}{2\Delta x} \int_{0}^{1} d\tau h_{\mu\nu}(x_0 + \Delta x\tau) \Delta x^\mu \Delta x^\nu$

- $h_{\mu\nu} \Delta x^\mu \Delta x^\nu = -\Delta B \Delta x^2 + \Delta A (\hat{r} \cdot \Delta \vec{x})^2 > 0$

- Gravity Gets There First!

- $c\Delta t = \frac{\Delta \vec{x} \cdot \vec{x}_0}{2\Delta x} \Delta B(r_0) - \frac{\Delta \vec{x} \cdot \vec{x}_1}{2\Delta x} \Delta B(r_1)$
  
  $$+ \int_{r_1}^{r_0} dr \frac{2GM(r)}{c^2r} \sqrt{1 - \left(\frac{C \Delta x}{r}\right)^2}$$

- $M(r) \equiv 4\pi \int_{0}^{r} dr' r'^2 \rho(r')$

- $C \equiv \frac{1}{\Delta x^2} \sqrt{r_0^2 \Delta x^2 - (\vec{x}_0 \cdot \Delta \vec{x})^2}$
Three Sources

- **GRB 070201**
  - Short hard $\gamma$-ray burst
  - Binary Merger?
  - In Andromeda

- **SN 1987a**
  - Core Collapse Supernova
  - $\nu$'s and $\gamma$'s observed

- **Sco-X1**
  - Low mass X-ray binary
  - Close and bright

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<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Milky Way</td>
<td>17h 45m 40s</td>
<td>$-29^\circ 09' 28''$</td>
<td>7.94 kpc</td>
</tr>
<tr>
<td>Andromeda</td>
<td>00h 42m 44s</td>
<td>$+41^\circ 16' 09''$</td>
<td>778 kpc</td>
</tr>
<tr>
<td>GRB 070201</td>
<td>00h 44m 32s</td>
<td>$+42^\circ 14' 21''$</td>
<td>780 kpc</td>
</tr>
<tr>
<td>SN 1987a</td>
<td>05h 35m 28s</td>
<td>$-69^\circ 16' 12''$</td>
<td>51.4 kpc</td>
</tr>
<tr>
<td>Sco-X1</td>
<td>16h 19m 55s</td>
<td>$-15^\circ 38' 24''$</td>
<td>2.80 kpc</td>
</tr>
</tbody>
</table>

**TABLE III:** Angular coordinates and distances for the Milky Way and Andromeda galaxies and for the three sources of this study.

<table>
<thead>
<tr>
<th>Profile</th>
<th>GRB 070201</th>
<th>SN 1987a</th>
<th>Sco-X1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isothermal</td>
<td>742 days</td>
<td>78.2 days</td>
<td>4.98 days</td>
</tr>
<tr>
<td>NFW</td>
<td>804 days</td>
<td>74.8 days</td>
<td>4.88 days</td>
</tr>
<tr>
<td>Moore</td>
<td>811 days</td>
<td>74.5 days</td>
<td>4.97 days</td>
</tr>
</tbody>
</table>

**TABLE IV:** Time delays from three dark matter profiles for each of the three sources of this study.
How Reliable Is $\Delta t$?

- Different Profiles
  - 9% for GRB 070201
  - 5% for SN 1987a
  - 2% for Sco-X1
- Also profile parameters
  - 75 vs 110 for SN 1987a
- Angular Coords Known
  - 2% for GRB 070201
  - Due to M31!
- Distances NOT!

<table>
<thead>
<tr>
<th>R. Ascension</th>
<th>Declination</th>
<th>$\Delta t_{MW}$</th>
<th>$\Delta t_{M31}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>00h 44m 32s</td>
<td>42° 14' 21&quot;</td>
<td>407 dy</td>
<td>335 dy</td>
</tr>
<tr>
<td>00h 46m 18s</td>
<td>41° 56' 42&quot;</td>
<td>407 dy</td>
<td>337 dy</td>
</tr>
<tr>
<td>00h 41m 51s</td>
<td>42° 52' 08&quot;</td>
<td>407 dy</td>
<td>322 dy</td>
</tr>
<tr>
<td>00h 42m 47s</td>
<td>42° 31' 41&quot;</td>
<td>407 dy</td>
<td>330 dy</td>
</tr>
<tr>
<td>00h 47m 14s</td>
<td>41° 35' 35&quot;</td>
<td>407 dy</td>
<td>338 dy</td>
</tr>
</tbody>
</table>

TABLE V: Shapiro Delays for GRB 070201 from the Isothermal Profiles of the Milky Way ($\Delta t_{MW}$) and Andromeda ($\Delta t_{M31}$) at the central value of the angular position and at the four vertices of the error box. In all cases the distance to the burst was taken to be 780 kpc.
Conclusions I

- Mod. Gravity may explain rot. curves
- But unstable if pure metric
- Otherwise new fields and two metrics
  - Gravitons couple to $g_{\mu\nu}$ of GR w/o D.M.
  - Matter couples to $\hat{g}_{\mu\nu}$ of GR with D.M.
- Gravity gets there first!
Conclusions II

- Big time lag for $\nu$'s and $\gamma$'s
  - By $> 2$ years from Andromeda
  - By $> 2$ months from the LMC
  - Generally $\Delta t \sim (v^2/c^2) \times \Delta x/c$
- $\Delta t$ not accurate (& little LIGO directionality)
  - Need many null detections from plausible sources
- Simultaneous pulses rule out DM Emulators
  - Maybe not modified gravity