Back-Reaction, Relaxation of $\Lambda$ & Dark Energy

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Quant. Gravitational Inflation

- Fund. IR gravity: $G_{\mu\nu} = -\Lambda g_{\mu\nu}$
- $\Lambda \sim [10^{12} \text{ GeV}]^2$ starts inflation
  - $ds^2 = -dt^2 + a^2(t) \, dx^2$ with $a(t) = e^{Ht}$
- QG “friction” stops inflation
  - $\rho_1 \sim +\Lambda^2$
  - $\rho_2 \sim -G\Lambda^3 \ln[a(t)]$
  - $\rho_L \sim -\Lambda^2 [G\Lambda \ln(a)]^L$
- Hence $p \sim -\rho \sim \Lambda^2 \, f[G\Lambda \ln(a)]$
Only Causality Stops Collapse!

- IR gravitons $\Rightarrow \rho_1 \sim +\Lambda^2$
- w/o causality $\Rightarrow \rho_2 \sim -G\Lambda^3 a^2(t)$
  - $R(t) \sim a(t)/H$ and $M(t) \sim H a^3(t)$
  - $\Delta E(t) = -GM^2/R \sim -G\Lambda^3 a^5(t)$
- Causality changes powers of $a(t)$ to powers of $\ln[a(t)]$
- But grav. Int. E. still grows w/o bound
Need Phenomenological Model

- Advantages of QG Inflation
  - Natural initial conditions
  - No fine tuning
  - Unique predictions

- But tough to USE!

- Try guessing most cosmologically significant part of effective field eqns
\[ G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g] \]

- \[ T_{\mu\nu}[g] = p \ g_{\mu\nu} + (\rho + p) \ u_\mu u_\nu \]
  - Posit \( p[g] \)
  - Infer \( \rho \) and \( u_\mu \) from conservation

- Getting \( p[\text{de Sitter}] = \Lambda^2 \ f[\Lambda \Lambda \ln(a)] \)
  - [...] must be nonlocal because

\[ R_{\mu\nu\rho\sigma} = \frac{\Lambda}{3} \left[ g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho} \right] \]

- Simplest is \( X = 1/\Box R \)
\[ R \& \quad \Box \equiv (-g)^{-1/2} \partial_{\mu} [(-g)^{1/2} g^{\mu\nu} \partial_{\nu}] \]

- \( R = 6 \frac{dH}{dt} + 12 H^2 \) for flat FRW
- \( \Box f(t) = -a^{-3} \frac{d}{dt} [a^3 \frac{df}{dt}] \)
  - Hence \( \frac{1}{\Box} f = -\int^t du a^{-3} \int^u dv a^3 f(v) \)
- For de Sitter \( a(t) = e^{Ht} \) and \( \frac{dH}{dt} = 0 \)
  - \( \frac{1}{\Box} R = -4 Ht + 4/3 [1 - e^{-3Ht}] \sim -4 \ln(a) \)
- Also \( D_P = \Box^2 + 2 D_{\mu} [R^{\mu\nu} - g^{\mu\nu} R/3]D_{\nu} \)
  - On \( f(t) \Rightarrow a^{-3} \frac{d}{dt} a \frac{d}{dt} a \frac{d}{dt} a \frac{d}{dt} a \)
  - Hence \( \frac{1}{D_P} [\alpha R^2 + \beta \Box R] \) also possible
Spatially Homogeneous Case

- $G_{\mu\nu} = (p-\Lambda)g_{\mu\nu} + (\rho+p)u_\mu u_\nu$
  - $X = \frac{1}{\Box} R = -\int^t du a^{-3}\int^u dv a^3 \left[12H^2 + 6dH/dv\right]$
  - $p = \Lambda^2 f(-G\Lambda X)$
  - $\rho + p = a^{-3}\int^t du a^3 dp/du$ and $u^\mu = \delta^\mu_0$

- Two Eqns
  - $3H^2 = \Lambda + 8\pi G \rho$
  - $-2dH/dt - 3H^2 = -\Lambda + 8\pi G p$ (easier)

- Parameters
  - 1 Number: $G\Lambda$ (nominally $\sim 10^{-12}$)
  - 1 Function: $f(x)$ (needs to grow w/o bound)
Numerical Results for $\Gamma / 300$ and $f(x) = e^{x-1}$

- $X = -\int^t du \ a^{-3} \int^u dv \ a^3 R$
- Criticality
  $p = \Lambda^2 f(-G \Lambda X) = \Lambda / 8\pi G$
- Evolution of $X(t)$
  - Falls steadily to $X_c$
  - Then oscillates with constant period and decreasing amplitude
  - For all $f(x)$ growing w/o bound
Inflation Ends, $H(t)$ goes $< 0$, $R(t)$ oscillates about 0
Analytic Treatment ($\epsilon \equiv G\Lambda$)

- $2 \frac{dH}{dt} + 3 H^2 = \Lambda[1 - 8\pi\epsilon f(-\epsilon X)]$
- $X(t) = X_c + \Delta X(t)$
  - $f \approx f_c - \epsilon\Delta X f'_c$
  - $2\frac{dH}{dt} + 3 H^2 \approx 24\pi\epsilon^2 f'_c \Delta X$
- Use $R = 6 \frac{dH}{dt} + 12 H^2$
  - L.H.S. = $R/3 - H^2$
  - $\Delta X = 1/R - X_c$
- Act $\Box = -[d/dt + 3H]d/dt$ to localize
  - $[(d/dt)^2 + 2H(d/dt) + \omega^2]R \approx 0$
  - $R(t) \approx \sin(\omega t)/a(t)$
  - $\omega^2 = 72\pi\epsilon^2 f'_c$ (agrees with plots!)
Origin of Scalar Perturbations

1. In Fundamental QG Inflation
   - \( \mathcal{L} = \frac{1}{16\pi G} (R - 2\Lambda)(-g)^{1/2} \)
   - Two \( h_{ij}'s \) can make a scalar!
     - E.g. Graviton KE: \( h_{ij} h_{ij} + \nabla h_{ij} \nabla h_{ij} \)
   - Usually negligible but if IR logs make homogeneous \( \sim O(1) \) maybe perts \( \sim O(G\Lambda) \)

2. In Phenomenological Model
   - \( T_{\mu\nu}[g] = p \, g_{\mu\nu} + (\rho+p) \, u_{\mu} u_{\nu} \)
   - \( p = \Lambda^2 \, f(-G\Lambda/\square R) \) fixed by retarded BC
   - But \( \rho \) and \( u_{i} \) at \( t=0 \) not fixed by \( D^{\mu} T_{\mu\nu} = 0 \)
Analysis (in conformal coords)

- 0th order: \( 2\alpha''/a^3 - a'/a^4 = \Lambda[1 - 8\pi\epsilon f(-\epsilon X_0)] \)
- \( h_{\mu\nu}dx^\mu dx^\nu = -2\phi d\eta^2 - 2B_{,i}dx^i d\eta - 2[\psi\delta_{ij} + E_{,ij}]dx^i dx^j \)
  - \( \Phi = \phi - a'/a (B-E') - (B'-E'') \)
  - \( \Psi = \psi + a'/a (B-E') \)
- \( G_{ij} \text{ Eqn } \Rightarrow \Psi = \Phi \text{ and } \Phi = \phi - a'/a (B-E') - (B'-E'') \)
- \( \frac{2}{a^2}\Phi'' + 6a'/a^3 \Phi' + [4a''/a^3 - 2a^2/a^4] \Phi = -8\pi\epsilon^2\Lambda f'(-\epsilon X_0) \times 1/\Box_0 [\nabla^2/a^2 \Phi - 6/a^2 \Phi'' - 24 a'/a^3 \Phi' - 4/a^2 X_0' \Phi'] \)
- Divide by \( f'(-\epsilon X_0) \) & act \( \Box_0 = a^{-2}[\partial_0^2 + 2a'/a \partial_0 - \nabla^2] \)
  - 4th order eqn \( \Rightarrow \) 2 physical & 2 unphys.
  - If \( X_0(t) \Rightarrow X_c \) then \( \Phi(t,x) \Rightarrow \text{const} \)
Late Time Acceleration

- Model driven by $X = 1/R$
  - Oscillations & $H < 0 \Rightarrow$ efficient reheating
  - $H = 1/2t \Rightarrow R = 6 \frac{dH}{dt} + 12 H^2 = 0$
- QG ends inflation, reheats & then turns off for most of cosmological history
  - $X(t) = -\int^t du \ a^{-3} \int^u dv \ a^3 \ R \rightarrow X_c$
  - $H = 2/3t \Rightarrow R = 4/3t^2 \neq 0$
    - $X(t) \approx X_c - 4/3 \ln(t/t_{eq})$
    - But this gives FURTHER screening!
- $X_2 = -1/D_p \ [\alpha R^2 + \beta \Box R]$
  - $\alpha > 0 \Rightarrow$ ends inflation
  - $\beta > 2/3 \alpha \Rightarrow$ late acceleration
Conclusions

- Advantages of QG Inflation
  1. Based on fundamental IR theory \(\Rightarrow\) GR
  2. \(\Lambda\) not unreasonably small!
  3. \(\Lambda\) starts inflation naturally
  4. QG back-reaction stops
     - Simple idea: Grav. Int. E. grows faster than \(V\)
  5. 1 free parameter: \(\Lambda\)

- But tough to use \(\Rightarrow\) Phenom. Model
\[ T_{\mu\nu}[g] = p \, g_{\mu\nu} + (\rho+p) \, u_{\mu} u_{\nu} \]

- Guess \( p[g] = \Lambda^2 \, f(-G\Lambda \, X) \)
  - \( X_1 = 1/\Box \, R \) or \( X_2 = -1/D_p \left[ \alpha R^2 + \beta \Box R \right] \)
  - Infer \( \rho \) and \( u_i \) from conservation

- Homogeneous evolution: (generic \( f \))
  - \( X \) falls to make \( p \) cancel \(-\Lambda/8\pi G\)
  - Then osc. with const. period & decr. amp.

- Reheats to radiation dom. (R=0)
  - Matter dom. \( \Rightarrow R \neq 0 \)
  - \( X_2 \) can give acceleration

- Perturbations look good