Λ-Driven Inflation


- \( \mathcal{L} = \frac{1}{16\pi G} (R-2\Lambda)\sqrt{-g} \) for IR gravity
  - \( G\Lambda \sim 10^{-10} \) NOT \( 10^{-122} \)
- No scalars, no fine tuning, no special IC’s
- \( \Lambda \) starts inflation & QG back-reaction stops

**Mechanism of back-reaction**
- Inflation rips gravitons from vacuum
- Their self-gravitation slows expansion
- Gravity is weak \( \rightarrow \) long phase of inflation
Features of QG Back-Reaction

- It starts at 2 loops
  - Gravitons produced at 1 loop
  - Self-interactions require another loop
- It’s slow
  \[ \varepsilon_L \sim \Lambda^2 \cdot (G\Lambda \ln[a(t)])^{L-1} \text{ with } a(t) = e^{Ht} \]
- It’s nearly (negative) vacuum energy
  - \[ \frac{d\varepsilon_L}{dt} = -3H (\varepsilon_L + p_L) \]
  - \[ \ln[a(t)] = Ht \gg 1 \Rightarrow |\frac{d\varepsilon_L}{dt}| \ll H|\varepsilon_L| \]
  - Hence \[ p_L \sim -\varepsilon_L \]
Tedious Arguments against QG Back-Reaction

- It’s not causal
  - Factors of $\ln[a(t)]$ from past light-cone
- $R = 4\Lambda$ from the Einstein equations
  - Gravitational COLLAPSE obeys this too
- IR gravitons can’t do anything
  - Small $\neq$ Zero and (big) $\times$ (small) $\neq$ small
- Effect must be self-limiting
  - Late effect from early times
- There ought to be a classical picture
  - There is!
Worries about Fluctuations

No Large Spatial Fluct.

- Nearby points share most past L-Cone
- Especially far past!

What about $\delta \varepsilon(t,x)$?

- Past light-cones not quite identical
- Small fluctuations in local $H(t,x)$
Back-Reaction CERTAIN in \( \phi \)-Driven Inflation

- Inflation from \( V_{\text{eff}} \neq V \)
  - \( \lambda \phi^4 \to \lambda \phi^4 + \lambda^2 \phi^4 \ln(\phi) \)
- Secular at 2 loops
- Can have either sign
  - Bosons: \( +\lambda^2 \phi^4 \ln(\phi) \)
  - Fermions: \( -\lambda^2 \phi^4 \ln(\phi) \)
- \( \pm \lambda^2 \phi^4 f(\lambda \phi/H) \) in de Sit.
  - Not even local generally
- No gauge issue
But we still have to prove it for Quantum Gravity

- Back-reaction requires $\geq 2$ loops
  - Even 1 loop is tough in de Sitter QG!
- Real interest after perturbative regime
  - $\varepsilon_L \sim \Lambda^2 [\Lambda \ln(a)]^{L-1} \sim \Lambda^2$ for $\ln(a) \sim 1/G\Lambda$
  - Still $\ll \Lambda/8\pi G \sim \Lambda^2/G\Lambda$
- Thin edge: prove it perturbatively
- What constitutes a proof? ($\sim 90\%$ political)
  - Critics tried $\langle g_{\mu\nu}(t,x) \rangle = \#\bar{g}_{\mu\nu}$ from dS invariance
  - But won’t accept computing $\langle g_{\mu\nu}(t,x) \rangle$
Case of φ-Driven Inflation
(Geshnizjani & RHB: gr-qc/0204074)

- \( \phi(t,x) = \phi_0(t) + \delta \phi(t,x), \quad |\phi_0| << |\delta \phi| \)
  ➞ Gradient of \( \phi \) is timelike

- \( u_{\mu}[\phi,g](t,x) = -\partial_{\mu} \phi(t,x)/[-g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi]^{1/2} \)

- Fix surfaces of simultaneity with \( T[\phi](t,x) \)
  ➞ \( \phi(T(t,x),x) = \phi_0(t) \)

- \( H[\phi,g](t,x) = 1/3 \, D^\mu u_{\mu}(T,x) = H(t) + \ldots \)
  ➞ No secular back-reaction at one loop

- Different result for \( \phi \to \Phi \) (spectator scalar)
  - hep-th/0310265
What about Quantum Gravity?

- Same given scalar “clock” \( \Phi[g](t,x) \)
  - \( u_\mu[g](t,x) = -\partial_\mu \Phi(t,x)/[-g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi]^{1/2} \)
  - \( T[g](t,x) \) such that \( \Phi[g](T,x) = \Phi[\bar{g}](t,x) \)
  - \( \mathcal{H}[g](t,x) = 1/(D-1) D^\mu u_\mu(T,x) \)

- But what to use for \( \Phi[g](t,x) \)?
  - Must have timelike gradient
  - And \( \mathcal{H}[g](t,x) \) must be UV & IR finite
“Expansion” is ambiguous even for classical de Sitter!

\[ ds^2 = -dt^2 + \cosh(Ht) d\chi^2 \]

- \( \Phi(t, \chi) = t \)
- \( u_\mu = -\delta^{\mu}_0 \)
- \( \mathcal{H} = H \tanh(Ht) \)

\[ ds^2 = -dt^2 + e^{2Ht} d\chi^2 \]

- \( \Phi(t, \chi) = t \)
- \( u_\mu = -\delta^{\mu}_0 \)
- \( \mathcal{H} = H \)
Our Solution: Exploit Causality & the Initial Value Surface

- \( \mathcal{V}[g](x) = \int d^Dx' \sqrt{-g(x')} \Theta[-\ell^2[g](x;x')] \)
  - New UV \( \infty \)'s from geodesic parameter ints
  - & \( \langle h_{\mu\nu}(t',x) h_{\rho\sigma}(t'',x) \rangle \rightarrow \infty \) on light-cone
- \( 1/D_4 \) for \( D_4 = \Box^2 + 2D_\mu[R_{\mu\nu} - g_{\mu\nu}R/3]D_\nu \)
  - Gives \( \mathcal{V}[\bar{g}]/8\pi \) for ANY FRW
  - Ok, but complicated
- \(-1/\Box \) for \( \Box = 1/\sqrt{-g} \partial_{\mu}[\sqrt{-g} g^{\mu\nu}\partial_{\nu}] \)
  - \( \int_0^t dt'/a^D-1 \int_0^{t'} dt'' a''^D-1 \) monotonically increasing
  - Simple
Schwinger-Keldysh Realization
(For the experts)

- $\Phi[g] = -[(\Box^{-1})_{++} - (\Box^{-1})_{+-}] 1$

- $\Box = \Box_0 + \Delta \Box$
  $\Phi = \Phi_0 - [((\Box_0^{-1})_{++}(\Delta \Box)_+ \Phi_0 - (\Box_0^{-1})_{+-}(\Delta \Box)_- \Phi_0] + \ldots$

- $g_{\mu\nu} = a^2 [\eta_{\mu\nu} + h_{\mu\nu}]$
  - $\Box_0 = a^{-D} \partial_\mu [a^{D-2} \partial^\mu]$
  - $\Delta \Box = \frac{1}{2} a^{-2} h_{\mu\nu} \partial_\mu - a^{-D} \partial_\mu [a^{D-2} h^{\mu\nu} \partial_\nu] + O(h^2)$

- $\Phi_0(\eta) = H^{-2}/(D-1) \left[ \ln(a) - (1-a^{-(D-1)})/(D-1) \right]$
  - $\Phi_0'(\eta) = a H^{-1}/(D-1) \left[ 1 - a^{-(D-1)} \right]$
Implementation at 1 Loop

\[ g_{\mu\nu} = a^2 \left[ \eta_{\mu\nu} + h_{\mu\nu} \right] \]

- \( \Phi[g](\eta,x) = \Phi_0(\eta) + \Phi_1(\eta,x) + \Phi_2(\eta,x) + \ldots \)
  - \( \Phi_1(x) = \int d^Dx' \ i\Delta(x;x') \{-\frac{1}{2} a^{D-2} h' \Phi_0' + \partial_\mu [h_0^{\mu} a^{D-2} \Phi_0']\} \)
  - \( \Phi_2 \) not needed
- \( T[g](\eta,x) = \eta - \Phi_1(\eta,x)/\Phi_0'(\eta) + \ldots \)
  - \( T_2 \) not needed because \( H_0 = H \)
- \( u_\mu[g] = -a([1-\frac{1}{2}h_{00}+\Phi_0'/\Phi_0'+\ldots]\delta_\mu^0 + \partial_\mu \Phi_1/\Phi_0' + \ldots) \)
- \( \mathcal{H}[g] = H + \mathcal{H}_1 + \mathcal{H}_2 + \ldots \)
  - \( \mathcal{H}_1 = \frac{1}{2}Hh_{00} + [\frac{1}{2}h_{ii}' - h_{0i,i} - \nabla^2 \Phi_1/\Phi_0']/(D-1)a \)
  - \( \mathcal{H}_2 = 3/2 H(h_{00})^2 + \ldots + h_{00,i} \partial_i \Phi_1/\Phi_0' + \ldots - \nabla^2 \Phi_2/\Phi_0' \)
  - \( \langle \mathcal{H}_1 \rangle \rightarrow 1 \ h + 1 \ \text{vertex} \) (gr-qc/0506056)
  - \( \langle \mathcal{H}_2 \rangle \rightarrow \text{just propagators} \)
Infrared & Ultraviolet

- **Infrared**
  - \( \mathbb{R} \times \mathbb{R}^{D-1} \rightarrow \mathbb{R} \times T^{D-1} \) & release at \( t=0 \)
  - IR finite, but possibly secular
  - Must perturbatively correct initial state

- **Ultraviolet**
  - \( \mathcal{H}[g](t,x) \) is local for \( T[g](t,x) = t \) gauge
  - Likely (composite operator) renormalizable
Conclusions

- $\Lambda$-Driven Inflation would be great
  - Solves (old) $\Lambda$ Problem
  - Provides unique model of inflation
- But must prove QG stops inflation!
  - First step: show it slows perturbatively
  - Need invariant measure of “expansion”
- Prediction: if we’re right
  - Everyone will use noninvariant measures
  - And VERY dirty approximations
Our Proposal ➔ Same form as \( \varphi \)-Driven Inflation

- \( \Phi[g](t,x) \rightarrow T[g](t,x), u_\mu[g](t,x) \) & \( H[g](t,x) \)
  - \( \Phi[g] = [(\Box^{-1})_{++} - (\Box^{-1})_{+-}]1 \)
- Implementation at 1 loop
  - \( H = H + H_1 + H_2 + \ldots \)
  - \( H_1 \rightarrow 1 \) vertex, \( H_2 \rightarrow \) just propagators
- Implementation at 2 loops
  - \( H = H + H_1 + H_2 + H_3 + H_4 + \ldots \)
  - \( H_k \rightarrow (4 - k) \) vertices
  - Need state corrections at order \( h^1 \) & order \( h^2 \)
- IR finite but perhaps secular at 2 loops
- UV renormalizable because gauge-local