Should we consider modified gravity?

Dark Matter, Dark Energy

Versus

Modify gravity

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CECS internal workshop
\[ G_{\mu\nu} \neq 8\pi G(T_{\mu\nu}) \text{ known} \]

- Two distinct problems to get Einstein equation to work
  - Motions of large gravitationally bound object. E.g. galaxies, galactic cluster
  - Expansion of the current universe
- Two ways to fix them
  - Change stress tensor
  - Change gravity
Some Observational Evidence

- Galactic Rotation Curves
- Velocity Dispersion of Galaxies
- Anisotropies in the CMB
- Gravitational Lensing
- Structure Formation
- BBN only allows 4% ordinary matter
- Supernovae Observation
- Large scale structure surveys
- Integrated Sachs-Wolfe Effect
- Missing Energy 74%
Dark Matter

- MACHO’s
- Axion
- Sterile Neutrino
- WIMP’s
  ~SUSY Particles
  e.g. Neutralino
Change Gravity

- **Modified Newtonian dynamics**

  \[ \left\| \vec{F} \right\| = m\mu \left( \frac{a}{a_0} \right) a \to m \frac{a^2}{a_0}, \quad \frac{a}{a_0} \ll 1 \]  
  \[ \frac{GM}{r^2} = \frac{a^2}{a_0} = \frac{v^4}{r^2 a_0} \implies v = \frac{4}{\sqrt{GMa_0}} \]

- Fail to predict gravitational lensing

  \[ ds^2 = -B(r)c^2 dt^2 + A(r)dr^2 + r^2 d\Omega^2 \]

  \[ B'(r) = \frac{2}{r} \left( \frac{v}{c} \right)^2 = \frac{2\sqrt{GMa_0}}{rc^2} \]

- Gravitational lensing need \( A(r) \) and \( B(r) \)

- **Relativistic version:** TeVeS

  \[ \text{(Bekenstein)} \]
FLRW Cosmology

- Homogeneous, isotropic and spatially flat metric:
  \[ ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \]

- Scale factor: \( a(t) \)
- Physical distance: \( a(t) d\vec{x} \)
- Physical momentum: \( \vec{p} = \frac{\vec{k}}{a(t)} \)
- Redshift: \( z = \frac{a_0}{a(t)} - 1 \)
- Hubble parameter: \( H(t) = \frac{\dot{a}}{a} \)
- Deceleration parameter: \( q(t) = -\frac{\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} \)
History of the Universe

$2q = 1 + 3W$

$q(t)$

+1

+0.5

-1

radiation domination

matter domination

primordial inflation

2300
The current universe is accelerating

- **Type Ia Supernovae**
  \[ d_L = \sqrt{\frac{L}{4\pi F}} = a_0(1 + z)r \]

- **Hubble plot**: \( d_L \) versus \( z \)
  - Slope \( \sim \) Hubble parameter
  - Curvature \( \sim \) deceleration
Dark Energy

- **Cosmological Constant**
- **Quintessence**

\[
\mathcal{L}_s = -\frac{1}{2} \partial_\mu \varphi \partial_\nu \varphi g^{\mu \nu} \sqrt{-g} - V(\varphi) \sqrt{-g}
\]

\[
3H^2 = 8\pi G \left( \frac{1}{2} \varphi_0^2 + V(\varphi_0) \right)
\]

\[
-2 \dot{H} - 3H^2 = 8\pi G \left( \frac{1}{2} \dot{\varphi}_0^2 - V(\varphi_0) \right)
\]

\[
-2 \dot{H} = 8\pi G \varphi_0^2 \quad \Rightarrow \quad \varphi_0(t) = \varphi_I \pm \int_{t_I}^t dt' \sqrt{\frac{-2 \dot{H}(t')}{8\pi G}} \quad \Rightarrow \quad T(\varphi)
\]

\[
V(\varphi) = \frac{1}{8\pi G} \left\{ \dot{H}(t) + 3H^2(t) \right\}
\quad \text{t} = T(\varphi)
\]
Metric-based Modifications of Gravity

- Invent $F(R)$ to fit the data
- but not from Fundamental Theory
  - Carroll, Duvvuri, Trodden and Turner
  - Sergei Odintsov
- Why $F(R)$? How about the other
  - possibility?
  - e.g. $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$
Canonical Formalism for lower derivative theories

\[ L = L(q, \dot{q}) \implies \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0 \]

\[ \ddot{q} = \mathcal{F}(q, \dot{q}) \implies q(t) = Q(t, q_0, \dot{q}_0) \]

\[ Q \equiv q, \quad P \equiv \frac{\partial L}{\partial \dot{q}} \]

- **Non-degeneracy**: \[ \dot{q} = v(Q, P) \]

\[ H(Q, P) \equiv P \dot{q} - L = P v(Q, P) - L(Q, v(Q, P)) \]

- **Hamiltonian generates time evolution**

\[ \dot{Q} = \frac{\partial H}{\partial P} \implies \dot{q} = v \]

\[ \dot{P} = -\frac{\partial H}{\partial Q} \implies \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \]
Ostrogradski Theorem

- A linear instability in Hamiltonian if the associated Lagrangian has more than one time derivative which cannot be eliminated by partial integrations

\[ L = L(q, \dot{q}, \ddot{q}) \quad \Rightarrow \quad \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} = 0 \]

\[ q^{(4)} = \mathcal{F}(q, \dot{q}, \ddot{q}, q^{(3)}) \quad \Rightarrow \quad q(t) = z(t, q_0, \dot{q}_0, \ddot{q}_0, q_0^{(3)}) \]
• # of initial values = # of canonical coordinates

\[ Q_1 \equiv q, \quad P_1 \equiv \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}} \]

\[ Q_2 \equiv \dot{q}, \quad P_2 \equiv \frac{\partial L}{\partial \ddot{q}} \]

• Non-degeneracy:

\[ \ddot{q} = a(Q_1, Q_2, P_2) \]

\[ H(Q_1, Q_2, P_1, P_2) = P_1 Q_2 + P_2 a(Q_1, Q_2, P_2) - L(Q_1, Q_2, a(Q_1, Q_2, P_2)) \]

• Check if they generate time evolution

\[ \frac{\partial H}{\partial P_1} = \dot{Q}_1 \implies \dot{q} = \dot{q} \]

\[ \frac{\partial H}{\partial P_2} = \dot{Q}_2 \implies \ddot{q} = a(Q_1, Q_2, P_2) \]

\[ \frac{\partial H}{\partial Q_1} = -\dot{P}_1 \implies \frac{\partial L}{\partial q} = \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} \]

\[ \frac{\partial H}{\partial Q_2} = -\dot{P}_2 \implies P_1 = \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}} \]
Ostrogradskian Instability

- Linear in canonical momentum
- Positive energy alternates with negative energy
- Instability comes from kinetic term
  - Constant field configuration doesn’t probe this instability
- Dynamical variables carry both positive, negative energy creation and annihilation operators
- The system excites to states with Max. entropy
- Quantization doesn’t save this instability
- Imposing constraints can eliminate the negative energy degree of freedom in the interaction terms but typically loses *causality, Unitarity and Lorentz invariance* on non-perturbative level (for a continuum and interacting theory).
Any hope for a higher derivative, interacting, continuum theory?

- The only way to avoid Ostrogradskian instability is to violate the only assumption: Non-degeneracy
- Any theory which possesses a continuous symmetry is degenerate.
  
  E.g. gauge symmetry
- A chance to avoid: \(\#\) of gauge constraints are more than (or equal to) \(\#\) of unstable directions in canonical phase space
- But only help some fixed number of higher derivatives
diffeomorphism : 4 gauge constraints

- E.g. $g_{00} = -1$, $g_{0i} = 0$

$$R \sim g^{ij} \ddot{g}_{ij} + \cdots$$

$$R_{\mu \nu} R^{\mu \nu} \sim \left( g^{ij} \ddot{g}_{ij} \right)^2 + g^{ik} g^{jl} \ddot{g}_{ij} \ddot{g}_{kl} + \cdots$$

- The lower derivative degree of freedom is Newtonian potential $\sim$ negative energy but fixed by $g_{00}$ constraint

- The higher derivative degree of freedom is positive

- The only local, stable, metric-based modification of gravity is $F(R)$ in $D=4$. 
Summary

- G.R. + ordinary matter doesn’t work on galaxy scales and larger
- The usual fix for this is to add lots of exotic stress energy
- This works but it strains credulity that so much of the universe has never seen in the Lab
  - 4% normal, 22% dark matter, 74% dark energy
- An equally valid alternative is to modify gravity
- The only local, stable, metric based modification of gravity is changing the Lagrangian from R to F(R)
The problems with MOND(TeVeS)

- Original MOND cannot predict Cosmology and gravitational lensing.
- At the classical level, all models of this kind also have to include ev-range neutrino to explain velocity dispersion near the cores of galaxy clusters.
- TeVeS is complicated
- TeVeS is not stable in the Vector field sector.(gr-qc/0104103 by Clayton)
- Bullet Cluster problem(0704.0381 by Angus and McGaugh )
- Far away from the galaxy, Rotational Curve \(\sim 1/r\) fall off by Dark Matter Model; not fall off by MOND
- Problem in 3rd Doppler Peak of CMB, from 5th year data release, 2nd and 3rd peaks are about the same height.
Viable modify gravity for Dark Matter, Dark Energy?

- MOND (TeVeS) for Dark Matter
- Einstein-Aether by Jacobson and Mattingly?
- Dipolar Dark Matter and Dark Energy by Blanchet
- $-\mu^4 / R, \mu = 10^{-33}$ eV for dark energy
- Possibility purely based on metric?
- No go Theorem by Woodard and Soussa
- Can nonlocal quantum corrections to the effective Field Equation play a role in Cosmology?
Need more data to break the degeneracy between Modified gravity and Dark Matter/Energy

- Joint Dark Energy Mission by NASA and DOE.
- What kinds of data becoming available to break this degeneracy. E.g. arXiv:0909.3853