A Phenomenological Model of Inflation from Quantum Gravity

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Quant. Gravitational Inflation

- Fund. IR gravity: $G_{\mu\nu} = -\Lambda g_{\mu\nu}$
- $\Lambda \sim [10^{12} \text{ GeV}]^2$ starts inflation
  - $ds^2 = -dt^2 + a^2(t) \, dx^2$ with $a(t) = e^{Ht}$
- QG "friction" stops inflation
  - $\rho_1 \sim +\Lambda^2$
  - $\rho_2 \sim -G\Lambda^3 \ln[a(t)]$
  - $\rho_L \sim -\Lambda^2 \left[ G\Lambda \ln(a) \right]^{L-1}$
- Hence $p \sim -\rho \sim \Lambda^2 \, f[G\Lambda \ln(a)]$
Only Causality Stops Collapse!

- IR gravitons ⇒ \( \rho_1 \sim +\Lambda^2 \)
- w/o causality ⇒ \( \rho_2 \sim -G\Lambda^3 a^2(t) \)
  - \( R(t) \sim a(t)/H \) and \( M(t) \sim H a^3(t) \)
  - \( \Delta E(t) = -GM^2/R \sim -GH^3 a^5(t) \)
- Causality changes powers of \( a(t) \) to powers of \( \ln[a(t)] \)
- But grav. Int. E. still grows w/o bound
Need Phenomenological Model

- Advantages of QG Inflation
  - Natural initial conditions
  - No fine tuning
  - Unique predictions

- But tough to USE!

- Try guessing most cosmologically significant part of effective field eqns
\[ G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu}[g] \]

- \[ T_{\mu\nu}[g] = p \ g_{\mu\nu} + (\rho + p) \ u_\mu u_\nu \]
  - Posit \( p[g] \)
  - Infer \( \rho \) and \( u_\mu \) from conservation

- Getting \( p[\text{de Sitter}] = \Lambda^2 \ f[\Lambda \ G \ A \ \ln(a)] \)
  - [...] must be nonlocal because
  \[ R_{\mu\nu\rho\sigma} = \Lambda/3 \ [g_{\mu\rho} \ g_{\nu\sigma} - g_{\mu\sigma} \ g_{\nu\rho}] \]
  - Simplest is \( X = 1/\square R \)
\[ R & \square \equiv (-g)^{-\frac{1}{2}} \partial_\mu [(g)^{\frac{1}{2}} g^{\mu \nu} \partial_\nu] \]

- \[ R = 6 \frac{dH}{dt} + 12 H^2 \] for flat FRW
- \[ \square f(t) = -a^{-3} \frac{d}{dt} [a^3 \frac{df}{dt}] \]
  - Hence \( \frac{1}{\square} f = -\int^t du a^{-3} \int^u dv \ a^3 \ f(v) \)
- For de Sitter \( a(t) = e^{Ht} \) and \( \frac{dH}{dt} = 0 \)
  - \[ \frac{1}{\square} R = -4 \ Ht + \frac{4}{3} [1 - e^{-3Ht}] \sim -4 \ln(a) \]
Spatially Homogeneous Case

- \( G_{\mu\nu} = (p-\Lambda)g_{\mu\nu} + (\rho+p) u_\mu u_\nu \)
  - \( X = \frac{1}{8\pi} R = -\int^t \! \! \! \! \! \! \! \! \! du \frac{a^{-3}}{a^3} \int^v \! \! \! \! \! \! \! \! \! dv \frac{a^{3}}{a^3} [12H^2 + 6dH/dv] \)
  - \( p = \Lambda^2 f(-G\Lambda X) \)
  - \( \rho+p = a^{-3} \int^t \! \! \! \! \! \! \! \! \! du \frac{a^{3}}{a^3} \frac{dp}{du} \) and \( u_\mu = \delta_\mu^0 \)

- Two Eqns
  - \( 3H^2 = \Lambda + 8\pi G \rho \)
  - \( -2dH/dt - 3H^2 = -\Lambda + 8\pi G \rho \) (easier)

- Parameters
  - 1 Number: \( G\Lambda \) (nominally \( \sim 10^{-12} \))
  - 1 Function: \( f(x) \) (needs to grow w/o bound)
Numerical Results for \( G\Lambda = 1/300 \) and \( f(x) = e^{x-1} \)

- \( X = -\int^tu a^{-3}\int^u dv a^3R \)
- Criticality
  \( p = \Lambda^2f(-G\Lambda X) = \Lambda/8\pi G \)
- Evolution of \( X(t) \)
  - Falls steadily to \( X_c \)
  - Then oscillates with constant period and decreasing amplitude
  - For all \( f(x) \) growing w/o bound
Inflation Ends, \( H(t) \) goes < 0, \( R(t) \) oscillates about 0
Analytic Treatment \((ε ≡ GΛ)\)

- \[2 \frac{dH}{dt} + 3 H^2 = \Lambda[1 - 8\piεf(-εX)]\]
- \[X(t) = X_c + \Delta X(t)\]
  - \[f ≈ f_c - ε\Delta X f'_c\]
  - \[2dH/dt + 3 H^2 ≈ 24\piε^2 f'_c \Delta X\]
- Use \(R = 6 \frac{dH}{dt} + 12 H^2\)
  - L.H.S. = \(R/3 - H^2\)
  - \(\Delta X = 1/R - X_c\)
- \(\text{Act } = -[d/dt + 3H]d/dt\) to localize
  - \[([(d/dt)^2 + 2H(d/dt) + \omega^2]R ≈ 0\]
  - \(R(t) ≈ \sin(\omega t)/a(t)\)
  - \(\omega^2 = 24\piε^2\Lambda f'_c\) (agrees with plots!)
Tensor Perturbations

- No change from usual eqn
  \[ \ddot{x} + 3H \dot{x} + \frac{k^2}{a^2} x = 0 \]
- Of course \( a(t) \) is unusual . . .
  - Oscillations in \( H(t) \)
  - And \( H(t) \) drops below zero!
- But this happens at the end of inflation
  - Little effect on far super-horizon modes
Origin of Scalar Perturbations

1. In Fundamental QG Inflation
   - $\mathcal{L} = 1/16\pi G (R - 2\Lambda)(-g)^{1/2}$
   - Two $h_{ij}$'s can make a scalar!
     E.g. Graviton KE: $\dot{h}_{ij} \dot{h}_{ij} + \nabla h_{ij} \nabla h_{ij}$
   - Usually negligible but if IR logs make homogeneous $\sim O(1)$ maybe perts $\sim O(G\Lambda)$

2. In Phenomenological Model
   - $T_{\mu\nu}[g] = p \, g_{\mu\nu} + (\rho+p) \, u_\mu u_\nu$
   - $p = \Lambda^2 \, f(-G\Lambda/\Box \, R)$ fixed by retarded BC
   - But $\rho$ and $u_i$ at $t=0$ not fixed by $D^\mu T_{\mu\nu} = 0$
Analysis (in conformal coords)

- 0th order: \( \frac{2a''}{a^3} - \frac{a'''}{a^4} = \Lambda [1 - 8\pi\epsilon f(-\epsilon X_0)] \)
- \( h_{\mu\nu} dx^\mu dx^\nu = -2\phi d\eta^2 - 2B_{,i} dx^i d\eta - 2[\psi \delta_{ij} + E_{,ij}] dx^i dx^j \)
  - \( \Phi = \phi - \frac{a'}{a} (B - E') - (B' - E'') \)
  - \( \Psi = \psi + \frac{a'}{a} (B - E') \)
- \( G_{ij} \) Eqn \( \Rightarrow \) \( \Psi = \Phi \) and
  \[ 2/a^2 \Phi'' + 6a'/a^3 \Phi' + [4a''/a^3 - 2a''/a^4] \Phi = -8\pi\epsilon^2 \Lambda f'(-\epsilon X_0) \]
  \[ \times 1/\Box_0 [\nabla^2/a^2 \Phi - 6/a^2 \Phi'' - 24 a'/a^3 \Phi' - 4/a^2 X_0' \Phi'] \]
\[ \frac{d^2 \Phi}{dt^2} + 4H \frac{d\Phi}{dt} + (2dH/dt + 3H^2)\Phi = -8\pi \varepsilon^2 \Lambda f'(-\varepsilon X(t)) \text{ NL} \]

- Early \( \Rightarrow f'(-\varepsilon X(t)) \ll 1 \)
  - + de Sitter \( \Rightarrow \Phi_1 = 1/a \) and \( \Phi_2 = 1/a^3 \)
  - Same for all k’s
- Late \( \Rightarrow f'(-\varepsilon X(t)) \approx f_c' \)
  - Oscillates with constant frequency \( \omega \)
    \[ \frac{d^2 \Phi}{dt^2} \approx -\omega^2 \frac{1}{\Box} [\frac{d^2 \Phi}{dt^2}] \]
  - Amplitude seems constant (numerically)
- Energy transfer to matter crucial
After Inflation

- Model driven by $X = 1/R$
  - Oscillations & $H < 0 \Rightarrow$ efficient reheating
  - $H = 1/2t \Rightarrow R = 6 \frac{dH}{dt} + 12 H^2 = 0$
- QG ends inflation, reheats & then turns off for most of cosmological history
  - $X(t) = -\int^t du a^{-3} \int^u dv a^3 R \rightarrow X_c$
Two Problems at Late Times

Eventually matter dominates

- $H(t)$ goes from $1/(2t)$ to $2/(3t)$
- $R = 6dH/dt + 12H^2$ from $0$ to $3/(4t^2)$
- $X = 1/□ R$ from $X_c$ to $X_c - 4/3 \ln(t/t_{eq})$

1. The Sign Problem:
   This gives further screening!

2. The Magnitude Problem:
   \[ p \approx -\Lambda/G (\Lambda^2 f'_c \Delta X \approx -10^{86} p_0 \times f'_c \Delta X \]
Magnitude Problem: Too many $\Lambda$’s

- $p = \lambda^2 f(-G\lambda \frac{1}{\Box} R)$
  - Dangerous changing initial $\lambda^2$
  - But can do $-G\lambda \frac{1}{\Box}[R] \rightarrow -G/\Box[\ "\lambda" R]$

Properties of “$\lambda$”

- Approximately $\lambda$ during inflation
- Approx. $R$ by onset of matter domination
- No change to initial value problem
- Invariant functional of metric

- Many choices but “$\lambda$” $= R(t/10)$ works
  - Can specify invariantly
Same as before with

\[ \Lambda = \frac{1}{4} R(t/10) \]
Sign Problem: $R(t) > 0$

- $p = \Lambda^2 f(-G/\Box[\ "\Lambda" \ R])$
- Need to add term to "\Lambda" R inside [ ]
  - Nearly zero during inflation & radiation
  - Comparable to $R^2$ after matter
  - Opposite sign
- Many choices but $\Box R$ works
  - $R = 4/(3t^2) \rightarrow \Box R = -8/(3t^4)$
Conclusions

- Advantages of QG Inflation
  1. Based on fundamental IR theory $\Rightarrow$ GR
  2. $\Lambda$ not unreasonably small!
  3. $\Lambda$ starts inflation naturally
  4. QG back-reaction stops
     Simple idea: Grav. Int. E. grows faster than V
  5. 1 free parameter: $\Lambda$

- But tough to use $\Rightarrow$ Phenom. Model
\( T_{\mu\nu}[g] = p \, g_{\mu\nu} + (\rho + p) \, u_\mu \, u_\nu \)

- Guess \( p[g] = \Lambda^2 \, f(-G\Lambda \, X) \)
  - \( X_1 = 1/\Box \, R \)
  - Infer \( \rho \) and \( u_i \) from conservation
- Homogeneous evolution: (generic \( f \))
  - \( X \) falls to make \( p \) cancel \(-\Lambda/8\pi G\)
  - Then oscillate with const. period & decreasing amp.
- Reheats to radiation dom. (\( R=0 \))
  - Matter dom. \( \Rightarrow R\neq0 \)
  - \( \Lambda X_2 = 1/\Box \, ["\Lambda" \, R + \Box R] \) can give late acceleration
- Perturbations
  - Little change to observable tensors
  - Scalars differ but still not clear