Nonlocal Cosmology

S. Deser (arXiv:0705.0153)
N. C. Tsamis (arXiv:0904.1151)
C. Deffayet and G. Esposito-Farese
(arXiv:1106.4989)
Modifications of Gravity

- Only local, stable, metric-based is $f(R)$
- Nonlocal modifications proposed for
  - Summing quantum IR effects from inflation
  - Explaining late time acceleration w/o DE
  - Explaining galaxies & clusters w/o DM
Isaac Newton in 1692/3

“that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it.”
Was Newton wrong about action-at-a-distance?

- We don’t think so
  - Fundamental theory is local
  - But quantum effective field eqns are not
  - M=0 loops could give big IR corrections

- Primordial inflation $\rightarrow$ IR gravitons
  - $N(t,k) \sim \left[ \frac{H_a(t)}{2kc} \right]^2$ for every $k$
  - Perhaps their attraction stops inflation
  - Late modifications from vacuum polarization
  - Would affect large scales most

- But for now, just model-building
Late-Time Acceleration (arXiv:0705.0153 with Deser)

- Nonlocality via $\Box^{-1}$ for $\Box = (-g)^{-1/2} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu)$
  - Retarded BC $\Rightarrow \{\Box^{-1} \& \partial_t \Box^{-1}\} = 0$ at $t=0$
- Act it on $R \Rightarrow \Box^{-1} R$ dimensionless
- $\mathcal{L} = \sqrt{-g} R[1 + f(\Box^{-1} R)]/16\pi G$
  - $f(X)$ the “Nonlocal distortion function”
- $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi GT_{\mu\nu}$
  $\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu} \Box - D_\mu D_\nu] (f + \Box^{-1}[Rf'])$
  $+ (\delta_\mu (^{\rho} \delta_\nu) - \textstyle{1/2} g_{\mu\nu} g^{\rho\sigma}) \partial_\rho (\Box^{-1} R) \partial_\sigma (\Box^{-1}[R f'])$
Specialization to FRW

\[ ds^2 = -dt^2 + a^2(t) \, dx \, dx \]

- \[ R = 6\dot{H} + 12 \, H^2 \]
- \[ \Box^{-1} = -\int_0^t dt'/a^3 \int_0^{t'} dt''a^3 \]
- Two Built-In Delays
  - \[ R=0 \text{ during Radiation Dom. (} H = 1/2t) \]
    - No modification until \( t \sim 10^5 \text{ years} \)
  - \[ \Box^{-1}R \sim -4/3 \, \ln(t/t_{eq}) \text{ during Matter Dom.} \]
    - \[ \Box^{-1}R \sim -15 \text{ at } t = 10^{10} \text{ years} \]
Reconstructing $f(X)$ for $\Lambda$CDM (arXiv:0904.0961 with Deffayet)
How It Works
for slowly varying $a(t)$

- $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$, $\Delta G_{\mu\nu} \sim G_{\mu\nu}(f + \Box^{-1}[Rf'])$
- Just rescale $G! \rightarrow G_{\text{eff}} = G/(1 + f + \Box^{-1}[Rf'])$
- Friedman Eqn: $3H^2 \sim 8\pi G_{\text{eff}} \rho_0/a^3(t)$
  - Growth of $G_{\text{eff}}$ balances $1/a^3(t)$
- But $G_{\text{eff}}$ strengthens gravity!
  - Not relevant for solar system
  - But should increase structure formation
- Dodelson & Park (arXiv:1209.0836)
  - Not purely $G_{\text{eff}}(t)$ when space dependent
  - Delayed so late that only $\sim 10-30\%$ effect
Local Version Is Haunted
(Nojiri & Odintsov, arXiv:0708.0924)

- $R[1+f(\Box^{-1}R)] \rightarrow R[1+f(\Phi)] + \Psi[\Box\Phi-R]$
  - Varying wrt $\Psi$ enforces $\Box \Phi = R$
  - NB both scalars have 2 pieces of IVD

- $\Psi \Box \Phi \rightarrow -\partial_{\mu} \Psi \partial_{\nu} \Phi g^{\mu\nu}$
  - $\rightarrow -\frac{1}{2} \partial_{\mu} (\Psi+\Phi) \partial_{\nu} (\Psi+\Phi) g^{\mu\nu}$
  - $\quad + \frac{1}{2} \partial_{\mu} (\Psi-\Phi) \partial_{\nu} (\Psi-\Phi) g^{\mu\nu}$

- $\Psi-\Phi$ has negative KE
No new initial value data for the original nonlocal version

- Synch. gauge: $ds^2 = -dt^2 + h_{ij}(t,x) \, dx^i dx^j$
- IVD for GR: $h_{ij}(0,x) \& \dot{h}_{ij}(0,x) = 6 + 6$
  - 4+4 for constrained fields
  - 2+2 for dynamical gravitons
- IVD in nonlocal cosmo $\Rightarrow$ count the $\partial_t$'s
  - $R \sim \partial_t^2 \& \Box^{-1} \sim \partial_t^{-2} \Rightarrow \Box^{-1}R \sim \partial_t^0$
  - $\Delta G_{\mu\nu}$ has up to $\partial_t^2 \Box^{-1}$
- Hence $h_{ij}(0,x) \& \dot{h}_{ij}(0,x)$, but what are they?
t=0 Constraints same as GR

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi GT_{\mu\nu}$
  $$\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu} \Box - D_\mu D_\nu] (f + \Box^{-1}[Rf']) + (\delta_{\mu}^\rho \delta_{\nu}^\sigma - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}) \partial_\rho (\Box^{-1}R) \partial_\sigma (\Box^{-1}[Rf'])$$

- Retarded BC $\Rightarrow [\Box^{-1} & \partial_t \Box^{-1}] = 0$ at $t=0$
  - $f(X)$ also vanishes at $X=0$
  - Only $[g_{\mu\nu} \Box - D_\mu D_\nu] \{f(\Box^{-1}R) + \Box^{-1}[Rf'(\Box^{-1}R)]\} \neq 0$

- Synchronous constraints $\Rightarrow \Delta G_{00} & \Delta G_{0i}$
  - $g_{00} \Box - D_0 D_0 = \frac{1}{2}h^{ij}h_{ij} \partial_t - \Delta \Rightarrow 0$ at $t=0$
  - $g_{0i} \Box - D_0 D_i = -\partial_0 \partial_i + \frac{1}{2}h^{jk}h_{ki} \partial_j \Rightarrow 0$ at $t=0$
No Ghosts at $t = 0$

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  
  \[
  \Delta G_{\mu\nu} = \left[ G_{\mu\nu} + g_{\mu\nu} \Box - D_{\mu} D_{\nu} \right] (f + \Box^{-1}[Rf']) \\
  + (\delta_{\mu}^{\rho} \delta_{\nu}^{\sigma} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma}) \partial_{\rho} (\Box^{-1}R) \partial_{\sigma} (\Box^{-1}[Rf'])
  \]

- Dynamical eqns $\Rightarrow G_{ij} + \Delta G_{ij} = 8\pi G T_{ij}$
  
  - $g_{ij} \Box - D_i D_j \rightarrow -h_{ij} \partial_t^2 + O(\partial_t)$
  - At $t = 0 \Rightarrow \Delta G_{ij} = 2f'(0) h_{ij} R$
  - $R_{ij} = \frac{1}{2} \ddot{h}_{ij} + O(\partial_t) \& R = h^{kl} \ddot{h}_{kl} + O(\partial_t)$
  - $G_{ij} + \Delta G_{ij} \rightarrow \frac{1}{2} \ddot{h}_{ij} - [\frac{1}{2}f'(0)]h_{ij} h^{kl} \ddot{h}_{kl} + O(\partial_t)$
  
  - $0 < f'(0) \ll 1 \Rightarrow$ No graviton becomes a ghost!
Avoid $G_{\text{eff}}$ with

$$T_{\mu\nu}[g] = p[g] \ g_{\mu\nu} + (\rho+p) \ u_\mu \ u_\nu$$

(arXiv:0904.1151 with Tsamis)

- $D^\mu T_{\mu\nu} = 0 \Rightarrow 4$ eqns
  - $p, \rho \ u_\mu \ (g_{\mu\nu} \ u_\mu \ u_\nu = -1) \Rightarrow 5$ variables
  - Pick $p[g] \Rightarrow \rho[g] \ & \ u_\mu[g]$ for $D^\mu T_{\mu\nu} = 0$

- Enforcing conservation about FRW + $\Delta g_{\mu\nu}$
  - $0^{\text{th}}$ order $u_\mu = \delta^\mu_0$
  - Get $\Delta u^0$ from $g_{\mu\nu} \ u_\mu \ u_\nu = -1$
  - $D^\mu[(\rho+p)u_\mu] = u.\partial p \Rightarrow \partial_t [a^3(\rho+p)] = \text{Known}$
  - $(\rho+p) \ u.D \ u_\nu = -(\partial_\nu + u_\nu \ u.\partial) \ p \Rightarrow \partial_t(u_i/a) = \text{Known}$
**Λ-Driven Inflation with QG back-reaction from \( p = \Lambda^2 f(-G\Lambda\Box^{-1}R) \)**

- \( G_{\mu\nu} = (p-\Lambda)g_{\mu\nu} + (\rho+p) u_{\mu} u_{\nu} \)
  - \( \Box^{-1}R = -\int^t dt' a^{-3} \int^{t'} dt'' a^3 [12H^2+6\dot{H}] \)
  - \( \rho+p = a^{-3} \int^t dt' a^3 \dot{p} \) and \( u_{\mu} = \delta_{\mu}^0 \)

- **Two Equations**
  - \( 3H^2 = \Lambda + 8\pi G \rho \)
  - \( -2\dot{H} - 3H^2 = -\Lambda + 8\pi G p \) (easier)

- **One Number:** \( G\Lambda \) (nominally \( \sim 10^{-6} \))

- **One Function:** \( f(x) \) (grows w/o bound)
Numerical Results for \( G\Lambda = 1/300 \) and \( f(x) = e^{x-1} \)

- \( X = -\int^t dt' a^{-3} \int t' dt'' a^3 R \)
- Criticality
  \( p = \Lambda^2 f(-G\Lambda X) = \Lambda/8\pi G \)
- Evolution of \( X(t) \)
  - Falls steadily to \( X_{cr} \)
  - Then oscillates with constant period and decreasing amplitude
  - Generic for any \( f(x) \) growing w/o bound
Inflation Ends, $H(t)$ goes $< 0$, $R(t)$ oscillates about 0
Dark Matter vs Mod. Gravity

- $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ works for solar system
- But not for galaxies
- Theory: $v^2 = GM/r$
- Obser: $v^2 \sim (a_0 GM)^{1/2}$
- Maybe missing Mass
- Or modified gravity
MOND (Milgrom 1983)

- $\rho(x,y,z) \equiv$ mass in stars and gas
  $\Rightarrow g_{Ni} \equiv$ Newtonian acceleration
- $g^i \equiv$ actual acceleration
  $\Rightarrow g^i \mu(|g|/a_0) = g_{Ni}$
- $a_0 \sim 10^{-10}$ m/s$^2$
- GR regime: $\mu(x) = 1$ for $x >> 1$
- MOND regime: $\mu(x) = x$ for $x << 1$
  $\Rightarrow$ Eg. $\mu(x) = x/(1+x)$, or $\tanh(x)$, . . .
Good agreement with galaxies but need relativistic model for

- Gravitational Lensing
- Recently disturbed systems
  - The Bullet Cluster!
- Cosmology

Previous models have new fields
- TeVeS (Bekenstein 2004)
- Another form of dark matter?

Our Goal: A purely metric version
Metric potentials for static, spherically symmetric

- \[ ds^2 = -B(r)c^2 dt^2 + A(r)dr^2 + r^2dΩ^2 \]
- \[ b(r) = B(r) - 1 \rightarrow Rotation\ curves \]
  - \[ rb'(r) = 2v^2/c^2 \rightarrow [4GMA_0/c^4]^{1/2} \]
- \[ a(r) = A(r) - 1 \rightarrow Lensing \]
  - Data \[ a(r) \sim + rb'(r) \]
GR vs MOND
for a MONDian \( \rho(r) \)

- \( M(r) = \frac{4\pi}{c^2} \int r \, dr' \, r'^2 \rho(r') \)
  - MONDian \( \Rightarrow \frac{GM(r)}{r^2} \ll a_0 \)
- GR \( \Rightarrow a(r) = rb'(r) = \frac{2GM(r)}{c^2r} \)
  - \( \frac{\delta S_{\text{GR}}}{\delta b} = \left( \frac{c^4}{16\pi G} \right) [(ra)' + O(h^2)] - \frac{1}{2} r^2 \rho \)
  - \( \frac{\delta S_{\text{GR}}}{\delta a} = \left( \frac{c^4}{16\pi G} \right) [-rb' + a + O(h^2)] \)
- MOND \( \Rightarrow a(r) = rb'(r) = \left[ \frac{4GM(r)a_0}{c^4} \right]^{1/2} \)
  - \( \partial_r (a^2) = \partial_r (rb')^2 = \left( \frac{16\pi Ga_0}{c^4} \right) r^2 \rho \)
\( \mathcal{L}_{\text{MOND}} \) to cancel \( h^2 \) from GR & add \( h^3 \) for MOND

- \( \mathcal{L}_{\text{GR}} = -\frac{1}{2} r^2 \rho b + (c^4/16\pi G)[-r a b' + \frac{1}{2} a^2] \)

- \( \mathcal{L}_{\text{MOND}} = r^2 (c^4/16\pi G)[a b'/r - \frac{1}{2} (a/r)^2 + c^2/a_0 [-1/6 (b')^3 + k (b' - a/r)^3 + \ldots ] \)

- \( h^3/r^2 \) of GR \( \ll \) \( c^2/a_0 (h/r)^3 \) of MOND for \( r \ll r_H \)

- \( S = \int dr \left[ \mathcal{L}_{\text{GR}} + \mathcal{L}_{\text{MOND}} \right] \)
  - \( \partial_r (r b')^2 - 6k \partial_r (r b' - a)^2 = (16\pi G a_0/c^4) r^2 \rho \)
  - \(-6k/r (r b' - a)^2 = 0 \)
Conclusions

- Last chance for modified gravity based on $g_{\mu \nu}$
- Not fundamental (we think)
  - From QG corrections during inflation
  - Purely phenomenological for now
- Models devised for
  - Summing QG corrections from inflation
  - Producing late acceleration w/o Dark Energy
  - Describing galaxies & clusters w/o Dark Matter
- Tools for nonlocal model building
  - Inverse covariant d’Alembertian
  - Invariant volume of past light-cone