Nonlocal Cosmology

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Problem: what is making the universe accelerate?

- FRW: $ds^2 = -dt^2 + a^2(t) \, dx \, dx$
  - $H(t) = d\ln(a)/dt \Rightarrow H_0 \sim 67 \text{ km/(s Mpc)}$
  - $q(t) = -1 - \dot{H}/H^2 \Rightarrow q_0 \sim -0.54$

- General Relativity with $1+z = a_0/a$
  - $3H^2 = 3H_0^2 \left[ \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_{\text{vac}} \right]$
  - $-2\dot{H}-3H^2 = 3H_0^2 \left[ \frac{1}{3} \Omega_r(1+z)^4 + 0 - \Omega_{\text{vac}} \right]$

- $\Lambda$CDM works
  - $\Omega_r \sim 8.5 \times 10^{-5}, \quad \Omega_m \sim 0.306, \quad \Omega_{\text{vac}} \sim 0.692$
  - But why is $G\Lambda \sim 10^{-122}$ & why NOW?
Quintessence works

\[ \mathcal{L} = -\frac{1}{2} \partial_{\mu} \varphi \partial_{\nu} \varphi g^{\mu\nu} \sqrt{-g} - V(\varphi) \sqrt{-g} \]

- \[ 3H^2 = 8\pi G \left[ \frac{1}{2} \dot{f}^2 + V(f) \right] \]
- \[ -2\dot{H} - 3H^2 = 8\pi G \left[ \frac{1}{2} \dot{f}^2 - V(f) \right] \]

**Given a(t) \Rightarrow Reconstruct V(f)**

- \[ -2\dot{H} = 8\pi G \dot{f}^2 \Rightarrow f(t) = f_0 \pm \int_0^t dt' \left[ -\dot{H}(t')/4\pi G \right]^{1/2} \]
  - Monotonic \Rightarrow t(f)
  - \[ \dot{H} + 3H^2 = 8\pi G V \Rightarrow V(f) = \left[ \dot{H}(t[f]) + 3H^2(t[f]) \right]/8\pi G \]

**But who ordered that!**

- Why homogeneous?
- Why so small?
- Why no new scalar force?
f(R) models do not really work

- $\mathcal{L} = f(R)\sqrt{-g}/16\pi G$
- Unique solution for $\Lambda$CDM is . . .
  - $f(R) = R - 2\Lambda$
  - Dunsby et al, arXiv:1005.2205
- Hence deviations occur at zeroth order!
- Other problems
  - Why now? $\Rightarrow$ new scales
  - New scalar DoF $\Rightarrow$ need screening
Modifications of Gravity

- Only local, stable, metric-based is $f(R)$
- Nonlocal modifications proposed for
  - Summing quantum IR effects from inflation
  - Explaining late time acceleration w/o DE
  - Explaining galaxies & clusters w/o DM
Isaac Newton in 1692/3

“that one body may act upon another at a distance thro' a Vacuum, without the Mediation of any thing else, by and through which their Action and Force may be conveyed from one to another, is to me so great an Absurdity that I believe no Man who has in philosophical Matters a competent Faculty of thinking can ever fall into it.”
Was Newton wrong about action-at-a-distance?

- We don’t think so
  - Fundamental theory is local
  - But quantum effective field eqns are not
  - M=0 loops could give big IR corrections
- Primordial inflation $\rightarrow$ IR gravitons
  - $N(t,k) \sim \left[\frac{Ha(t)}{2kc}\right]^2$ for every $k$
  - Perhaps their attraction stops inflation
  - Late modifications from vacuum polarization
  - Would affect large scales most
- But for now, just model-building
Late-Time Acceleration (arXiv:0705.0153 with Deser)

- Nonlocality via \( \Box^{-1} \) for \( \Box = (-g)^{-1/2} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \)
  - Retarded BC \( \{\Box^{-1} & \partial_t \Box^{-1}\} = 0 \) at \( t=0 \)
- Act it on \( R \) \( \Box^{-1}R \) dimensionless
- \( \mathcal{L} = \sqrt{-g} R[1 + f(\Box^{-1}R)]/16\pi G \)
  - \( f(X) \) the “Nonlocal distortion function”
- \( G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi GT_{\mu\nu} \)
  \[ \Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu} \Box - D_\mu D_\nu] (f + \Box^{-1}[Rf']) \]
  \[ + (\delta_\mu (^\rho \delta_\nu - 1/2 g_{\mu\nu} g^{\rho\sigma}) \partial_\rho (\Box^{-1}R) \partial_\sigma (\Box^{-1}[R f'])) \]
Specialization to FRW
\[ ds^2 = -dt^2 + a^2(t) \, dx \cdot dx \]

- \( R = 6\dot{H} + 12 \, H^2 \)
- \( \Box^{-1} = -\int_0^t dt'/a^3 \int_0^{t'} dt'' a^3 \)
- Two Built-In Delays
  - \( R=0 \) during Radiation Dom. (\( H = 1/2t \))
    - No modification until \( t \sim 10^5 \) years
  - \( \Box^{-1}R \sim -4/3 \ln(t/t_{eq}) \) during Matter Dom.
    - \( \Box^{-1}R \sim -15 \) at \( t = 10^{10} \) years
Reconstructing $f(X)$ for $\Lambda$CDM (arXiv:0904.0961 with Deffayet)
How It Works

for slowly varying $a(t)$

- $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$, $\Delta G_{\mu\nu} \sim G_{\mu\nu}(f + \Box^{-1}[Rf'])$
- Just rescale $G! \Rightarrow G_{\text{eff}} = G/(1 + f + \Box^{-1}[Rf'])$
- Friedman Eqn: $3H^2 \sim 8\pi G_{\text{eff}} \rho_0/a^3(t)$
  - Growth of $G_{\text{eff}}$ balances $1/a^3(t)$
- But $G_{\text{eff}}$ strengthens gravity!
  - Not relevant for solar system
  - But should increase structure formation
- Dodelson & Park (arXiv:1209.0836)
  - Not purely $G_{\text{eff}}(t)$ when space dependent
  - Delayed so late that only $\sim 10\text{-}30\%$ effect
Screening

- f(R) models have a problem
  - R > 0 for cosmology AND solar system
  - Need “screening mechanism” to suppress response inside solar system

- f(□⁻¹R) models avoid this problem
  - □ ~ -∂ₜ² + ∂ₓ²
    - □⁻¹R < 0 for cosmology
    - □⁻¹R > 0 for gravitationally bound systems
  - f(X) = 0 for X > 0 means NO solar system change
Local Version Is Haunted
(Nojiri & Odintsov, arXiv:0708.0924)

- \[ R[1+f(\Box^{-1}R)] \Rightarrow R[1+f(\Phi)] + \Psi[\Box\Phi - R] \]
  - Varying wrt \( \Psi \) enforces \( \Box\Phi = R \)
  - NB both scalars have 2 pieces of IVD

- \( \Psi\Box\Phi \Rightarrow -\partial_\mu \Psi \partial_\nu \Phi g^{\mu\nu} \)
  \[ \Rightarrow -\frac{1}{2} \partial_\mu (\Psi + \Phi) \partial_\nu (\Psi + \Phi) g^{\mu\nu} \]
  \[ + \frac{1}{2} \partial_\mu (\Psi - \Phi) \partial_\nu (\Psi - \Phi) g^{\mu\nu} \]

- \( \Psi - \Phi \) has negative KE
No new initial value data for the original nonlocal version

- Synch. gauge: $ds^2 = -dt^2 + h_{ij}(t,x) \, dx^i dx^j$
- IVD for GR: $h_{ij}(0,x) \& \dot{h}_{ij}(0,x) = 6 + 6$
  - 4+4 for constrained fields
  - 2+2 for dynamical gravitons
- IVD in nonlocal cosmo $\Rightarrow$ count the $\partial_t$'s
  - $R \sim \partial_t^2 \& \Box^{-1} \sim \partial_t^{-2} \Rightarrow \Box^{-1}R \sim \partial_t^0$
  - $\Delta G_{\mu\nu}$ has up to $\partial_t^2 \Box^{-1}$
- Hence $h_{ij}(0,x) \& \dot{h}_{ij}(0,x)$, but what are they?
t=0 Constraints same as GR

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  
  $\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu} \square - D_{\mu} D_{\nu}] (f + \square^{-1}[Rf'])$
  
  $+ (\delta_{\mu}^{(\rho} \delta_{\nu^{\sigma})} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma}) \partial_{\rho} (\square^{-1}R) \partial_{\sigma} (\square^{-1}[Rf'])$

- Retarded BC $\Rightarrow [\square^{-1} & \partial_{t} \square^{-1}] = 0$ at $t=0$
  
  - $f(X)$ also vanishes at $X=0$
  
  - Only $[g_{\mu\nu} \square - D_{\mu} D_{\nu}] \{f(\square^{-1}R) + \square^{-1}[Rf'(\square^{-1}R)]\} \neq 0$

- Synchronous constraints $\Rightarrow \Delta G_{00} & \Delta G_{0i}$
  
  - $g_{00} \square - D_{0} D_{0} = \frac{1}{2} h^{ij} h_{ij} \partial_{t} - \Delta \Rightarrow 0$ at $t=0$
  
  - $g_{0i} \square - D_{0} D_{i} = -\partial_{0} \partial_{i} + \frac{1}{2} h^{jk} h_{ki} \partial_{j} \Rightarrow 0$ at $t=0$
No Ghosts $\implies$ check $\partial_t^2$ terms

- Recall $G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$
  $$\Delta G_{\mu\nu} = [G_{\mu\nu} + g_{\mu\nu} \Box - D_i D_j] \left( f + \Box^{-1}[Rf'] \right) + (\delta_{\mu}^{(\rho} \delta_{\nu}^{\sigma}) - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma}) \partial_{\rho} (\Box^{-1}R) \partial_{\sigma} (\Box^{-1}[Rf'])$$

- Dynamical eqns $\implies$ $G_{ij} + \Delta G_{ij} = 8\pi G T_{ij}$
  - $g_{ij} \Box - D_i D_j \rightarrow -h_{ij} \partial_t^2 + O(\partial_t) \rightarrow +h_{ij} \Box + O(\partial_t)$
  - $\Delta G_{ij} = 2 h_{ij} R f'(\Box^{-1}R) + O(\partial_t)$
  - $R_{ij} = \frac{1}{2} \ddot{h}_{ij} + O(\partial_t)$ & $R = h^{kl} \dddot{h}_{kl} + O(\partial_t)$

- $G_{ij} + \Delta G_{ij} \rightarrow \frac{1}{2} \dddot{h}_{ij} - \left[ \frac{1}{2} - 2 f'(X) \right] h_{ij} h^{kl} \dddot{h}_{kl} + O(\partial_t)$
  - No graviton becomes a ghost!
  - Still might have a potential energy instability
Conclusions about nonlocal modifications of gravity

- Last chance for gravity based on $g_{\mu\nu}$
- Not fundamental (we think)
  - From QG corrections during primordial inflation
  - Purely phenomenological for now
- Simplest models based on $f(\Box^{-1}R)$
  - Built-in delays explain cosmic coincidence
  - Simple $f(X)$ reproduces $\Lambda$CDM without $\Lambda$
- Model seems ok
  - No problem for gravitationally bound systems
  - Localized version has scalar ghost
  - But no new DoF’s in nonlocal version
  - Initial value constraints identical to GR
  - No graviton DoF’s become ghosts