A Non-Technical (Would I Lie?) Discussion of the Problem of Quantum Gravity

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Seven Questions

1. Why is QGR so bad & CGR so good?
2. Why must we quantize gravity?
3. Why do QFT’s have ∞’s?
4. Why are those of QGR worse?
5. How bad is the problem?
6. What are the main approaches to it?
7. What would we do with QGR?
1. Gravitational Field: $g_{\mu\nu}(t, x)$
   - $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$
2. $g_{\mu\nu}$ affects fields thru Minimal Coupling
   - $-(\partial/\partial ct)^2 + \nabla^2 \rightarrow \partial_\mu [(-g)^{1/2} g^{\mu\nu} \partial_\nu]$
3. Other fields affect $g_{\mu\nu}$ thru Einstein Eqn
   - $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
   - Key Principle: Energy gravitates
4. Eqn also predicts gravitational radiation
   - Purely grav. DOF’s not fixed by matter
A Solution versus the GENERAL Solution

QM eqns same as in CM, eg $\ddot{x} + \omega^2 x = 0$

- $x(t) = x_0 \cos(\omega t) + \dot{x}_0 / \omega \sin(\omega t)$ \text{ BUT}
- $x_0 = 0 \& \dot{x}_0 = 0$ OK in CM
- $x_0 = 0 \& \dot{x}_0 = 0$ not OK in QM

Classic CGR tests have most IVD = 0

- $ds^2 = -[1-2GM/r]dt^2 + dr^2/[1-2GM/r] + r^2 d\Omega$
- Need $g_{\mu\nu}(t,x)$ for general IVD
- Lots of DOF’s with 0-pt motion
Some Quantization Unavoidable

Dual role of force fields:
- Mediate interactions
- Harbor new quanta

Cf. EM
\[
\text{div}(E) = \frac{\rho}{\varepsilon_0} \quad \text{div}(B) = 0 \\
\text{curl}(B) - c^{-2} \frac{\partial E}{\partial t} = \mu_0 J \\
\text{curl}(E) + \frac{\partial B}{\partial t} = 0
\]

\[
\vec{B}(t, \vec{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \int_0^t dt' \frac{\sin[ck(t-t')]}{ck\varepsilon_0} i\vec{k} \times \vec{J}(t', \vec{k}) \\
+ \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \left\{ \vec{B}_0(\vec{k}) \cos(ckt) - \frac{i}{ck} \vec{k} \times \vec{E}_0(\vec{k}) \sin(ckt) \right\}
\]

1st term quantized because of matter, whether or not there are photons
Matter quantized whether or not photons & gravitons are!

- Cf Hydrogen: \( H = \frac{p^2}{2m} - e \Phi(q) \)
  \( \Phi(q) = \frac{e}{4\pi \epsilon_0 q} \) an operator from \( q \)

- Same for General Relativity
  \( g_{\mu\nu} = (\text{functional of } T_{\mu\nu}) + (\text{gravitons}) \)
  Fields in \( T_{\mu\nu} \) are certainly quantum!

- Allowing quantum matter to interact gravitationally causes problems with or without gravitons
Asymptotic Series

Are Your Friends

- Impossible to find general solution
  ➔ MUST approximate

- Typical Asymptotic series
  \[ E_1(x) = \int_x^\infty \frac{\ln t}{t} e^{-t} \rightarrow e^{-x}/x \sum_{n=0}^{\infty} (-1/x)^n n! \]
  - Great for small \(1/x\) at fixed \(n\)
  - But diverges for large \(n\) at fixed \(x\)
  - Hence use out to \(n \sim x\) and no further
  - Not exact but often good enough
Should Be Great for QGR

- **QED:** \( \alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \sim \frac{1}{137} \)
  
  Results = (0th order)[1 + a_1\alpha + a_2\alpha^2 + ...]
  
  - Begins diverging at \( L \sim 430 \)
  - Best experiments sensitive to \( L \sim 4 \)

- **QGR:** \( \kappa = \frac{G E^2}{\hbar c^5} \sim \left( \frac{E}{10^{19}\text{Gev}} \right)^2 \)
  
  Results = (0th order)[1 + b_1\kappa + b_2\kappa^2 + ...]
  
  - Same factorials & \( E = 1 \text{ TeV} \Rightarrow \kappa \sim 10^{-32} \)
  - But the coefficients \( b_n \) diverge!
Physics behind the $\infty$'s:
Recall the QM Harmonic Osc.

- $H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$
  $\Rightarrow q(t) = q_0 \cos(\omega t) + \frac{p_0}{m\omega} \sin(\omega t)$

- $\cos(\omega t) = \frac{1}{2} (e^{i\omega t} + e^{-i\omega t})$
  $\Rightarrow q(t) = \frac{1}{2}(q_0 + \frac{ip_0}{m\omega})e^{-i\omega t} + \frac{1}{2}(q_0 - \frac{ip_0}{m\omega}) e^{i\omega t}$
  $= \frac{1}{2} \left[ a e^{-i\omega t} + a^\dagger e^{i\omega t} \right] / (2\omega)^{1/2}$

- Mode coordinate: $a = (\omega/2)^{1/2} (q_0 + \frac{ip_0}{m\omega})$

- Mode function: $\varepsilon(t) = e^{-i\omega t} / (2\omega)^{1/2}$
  $\Rightarrow q(t) = a \varepsilon(t) + a^\dagger \varepsilon^*(t)$
Field Theories have $\infty$ Modes
E.g., EM for $0 \leq x_i \leq L$

\[ E_1(t,x) = \sum_k [a_k \, \varepsilon_k(t,x) + a_k^\dagger \, \varepsilon_k^*(t,x)] \]

where $k = \pi/L \, (n_1,n_2,n_3)$ & $\omega = \pi nc/L$

\[ \varepsilon_k = [\pi^3/2\omega L^3]^{1/2} \sin(k_1 x_1)\cos(k_2 x_2)\cos(k_3 x_3)e^{-i\omega t} \]

\[ a_k = \int^L dx_1 \int^L dx_2 \int^L dx_3 \, \varepsilon_k^*(0,x) \{\omega \, E_1(0,x) \]

\[ + \, ic \left[ \partial_2 B_3(0,x) - \partial_3 B_2(0,x) \right] \}\]

Two modes for every $(n_1,n_2,n_3)$
Renormalization 1: Classical EM in a medium

- **Dumb way:** \( \epsilon_0 \text{div}(E) = \rho_{\text{free}} + \rho_{\text{bnd}} \)
  - \( \rho_{\text{bnd}}(x) = \Sigma_{\text{atm}} \rho_{\text{atm}}(x-x_{\text{atm}}) \)
- **Local Representations**
  - \( \rho(x) = [Q - p^i \partial_i + \frac{1}{2} Q^{ij} \partial_i \partial_j - ...] \delta^3(x) \)
  - Atoms have \( Q = 0 \) but (with \( E \)) \( p^i \neq 0 \)
  - \( \rho_{\text{bnd}}(x) \approx -\text{div}[\Sigma_{\text{atm}} \rho_{\text{atm}} \delta^3(x-x_{\text{atm}})] = -\text{div}(P) \)
- **Smart way:** (for \( P = \Delta \epsilon \ E \))
  - \( [\epsilon_0 + \Delta \epsilon] \text{div}(E) \approx \rho_{\text{free}} \)
Renormalization 2:
\[ \varepsilon_0 \text{ div}(E) = \rho_{\text{exc}} + \rho_{\text{opt}} \text{ in QED} \]

\[ e^+ \text{ w. } \varepsilon(k) = [(\hbar c k)^2 + m^2 c^4]^{1/2} \text{ & } e^- \text{ w. } \varepsilon(p-k) \]

live \( \Delta t \sim \hbar / [\varepsilon(k) + \varepsilon(p-k)] \)

They polarize \( e\Delta x \), where

\[ \frac{d}{dt} \left[ \varepsilon/c^2 \Delta \dot{x} \right] = eE \Rightarrow \Delta x \sim c^2 \Delta t^2 eE/2\varepsilon \]

Sum over \( k \) to get total polarization

\[ \vec{P}(\vec{k}) = \frac{8}{3} \int \frac{d^3k}{(2\pi)^3} \frac{e^2 \hbar^2 c^2 \vec{E}(\vec{p})}{\varepsilon(k) + \varepsilon(\vec{p} - \vec{k})^3} \]

Renormalization: \( \Delta \varepsilon(p) = \Delta \varepsilon(0) + \text{finite} \)
Renorm 3: $c^4/8\pi G \left[ G_{\mu\nu} + \Lambda g_{\mu\nu} \right]$

$$= (T_{\mu\nu})_{\text{exc}} + (T_{\mu\nu})_{\text{0pt}}$$

Cf. $-\epsilon_0 \nabla^2 \Phi = \rho_{\text{exc}} + \rho_{\text{0pt}}$

$$\left( \rho(\vec{x}) \right)_{\text{opt}} = -\int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \int \frac{d^3k}{(2\pi)^3} \frac{\frac{8}{3} e^2 \hbar^2 c^2 p^2 \Phi(\vec{p})}{[\mathcal{E}(\vec{k}) + \mathcal{E}(\vec{p} - \vec{k})]^3}$$

$$= A\alpha\epsilon_0 \ln(K^2) \nabla^2 \Phi(\vec{x}) + \text{Finite}$$

$$\left( T_{\mu\nu}(\vec{x}) \right)_{\text{opt}} = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \int \frac{d^3k}{(2\pi)^3} \frac{\mathcal{E}^4 h(\vec{p})}{[\mathcal{E}(\vec{k}) + \mathcal{E}(\vec{p} - \vec{k})]^3}$$

$$= \hbar c \left\{ AK^4 h(\vec{x}) + BK^2 \nabla^2 h(\vec{x}) + C \ln(K^2) \nabla^4 h(\vec{x}) + \text{Finite} \right\}$$

A $K^4 \rightarrow \Lambda$, B $K^2 \rightarrow G$ but C $\ln(K^2)$ new
What’s wrong with higher ∂’s?

- **Newton:** $m\ddot{x} = F(x,\dot{x})$
  - $L = L(x,\dot{x}) \Rightarrow Q = x\text{ and } P = \partial L/\partial \dot{x}$
  - $H(Q,P) = P\dot{x}(Q,P) - L(Q,\dot{x}(Q,P))$

- **Ostrogradsky:** $\frac{d^4x}{dt^4} = f(x,\dot{x},\ddot{x},\frac{d^3x}{dt^3})$
  - $L = L(x,\dot{x},\ddot{x}) \Rightarrow Q_1 = x, Q_2 = \dot{x}, P_2 = \partial L/\partial \ddot{x}$
  - $P_1 = \partial L/\partial \dot{x} - \frac{d}{dt} \partial L/\partial \ddot{x}$
  - $H(Q_1,Q_2,P_1,P_2) = P_1 Q_2 + P_2 \ddot{x}(Q_1,Q_2,P_2) - L(Q_1,Q_2,\ddot{x}(Q_1,Q_2,P_2))$

- Why physics is based on 2nd order eqns!
Pert. QGR worse because

- Other forces couple to charges
  - Same for all $k$
- GR couples to energy
  - Grows with $k$
- Absorbing extra $\infty$'s requires adding $(\partial/\partial t)^4$ terms to Einstein eqn
  - Makes the universe blow up instantly
How Bad Is It?

- Quantum matter is the first problem
  - Gravity + most matter diverges at 1\textsuperscript{st} order
  - Gravitons don’t diverge until 2\textsuperscript{nd} order
- Fermi statistics can help
  - Bosons have $+\frac{1}{2}\hbar\omega$
  - Fermions have $-\frac{1}{2}\hbar\omega$
- But local theories still diverge eventually
Conspiracy of Four Principles

1. Continuum Field Theory $\Rightarrow \infty$ Modes
2. Q. Mechanics $\Rightarrow$ Can’t have $q_0=p_0=0$
   - Each mode has $\pm \frac{1}{2} \hbar \omega +$ interactions
   - Changes shift energies (Casimir & Lamb)
3. General Relativity $\Rightarrow$ Energy gravitates
4. Perturbation theory $\Rightarrow$ shifts add
   “Too many modes interacting too strongly”
Divergent Opinions

1. Relativists (love General Relativity)
   - “Perturbation theory is wrong!”
   - Nonlinear grav. ints cancel the ∞’s

2. Particle Theorists (love Pert. Theory)
   - “General Relativity is wrong!”
   - Superstrings have ± 0-point energies and interact more weakly at large k
Repartee with Relativists

Fact: GR is weak & QGR unobs. small

Question: How can PT be wrong?

1. Correct series nonanalytic in G
   - Eg (0th) \{1 + \frac{GE^2}{\hbar c^5} \ln\frac{GE^2}{\hbar c^5} + \ldots\}

2. 0th order may diverge for G \to 0
Charged shell of radius $R \to 0$ (ADM 1960)

- Without GR: $mc^2 = m_0c^2 + q^2/8\pi\varepsilon_0 R$
  
  "renormalize" with $m_0c^2 = m_{\text{obs}}c^2 - q^2/8\pi\varepsilon_0 R$

- With GR: $mc^2 = m_0c^2 + q^2/8\pi\varepsilon_0 R - Gm^2/2R$

$$m = \frac{Rc^2}{G} \left[ -1 + \sqrt{1 + \frac{2Gm_0}{Rc^2} + \frac{Gq^2}{4\pi\varepsilon_0 R^2 c^2}} \right] \to \sqrt{\frac{q^2}{4\pi\varepsilon_0 G}}$$

- Perturbative Result:
  
  $\Rightarrow$ Oscillating series of ever-higher $\infty$'s
The Other Problem ➔ Experiments!

- Signal is down by $10^{-32}$ at 1 TeV
- Study weak ints using unique features
  - $\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$ doesn’t occur in QED
- Unique features of gravity
  - Negative interaction energy
  - $M=0$ gravitons without conformal invariance
    - Photons can’t distinguish $g_{\mu\nu}(x)$ from $\Omega^2(x) g_{\mu\nu}(x)$
- Cosmology is a natural venue
  - $ds^2 = -dt^2 + a^2(t) \, dx^2 = a^2 \left[-d\eta^2 + dx^2\right]
  - Gravity is long range and knows about $a(t)$
Seven Questions Answered

1. CGR works when almost all DOF’s off, QGR hurt by 0-point motion of DOF’s
2. Must quantize because part of $g_{\mu\nu}$ from quantum matter
   - This has been seen in CMB anisotropies!
3. QFT $\infty$’s from infinite modes
4. QGR is worse because it couples to energy instead of charge
Seven Questions Answered

5. GR + most matter fails a one loop, pure GR fails at two loops

6. Divergent Opinions
   - Relativists: “GR is right & PT is wrong!”
   - Particle Theorists: “PT is right & GR is wrong!”

7. Phenomenology of QGR exploits unique
   - Negative interaction energy
   - $M = 0$ gravitons w/o conformal invariance