

## Homework A

PHY3513

Due: Wednesday, Jan 22

1. (a)  $2 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots = ??$   
(b)  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \dots = ??$

2. Expand  $(1 - 2tz + t^2)^{-1/2}$  in powers of  $t$  assuming that  $t$  is small. Collect the coefficients of  $t^0$ ,  $t^1$ , and  $t^2$ .

3. The displacement  $z$  of a particle of rest mass  $m_0$ , resulting from a constant force  $m_0g$  along the  $z$ -axis is

$$z = \frac{c^2}{g} \left\{ \left[ 1 + \left( g \frac{t}{c} \right)^2 \right]^{1/2} - 1 \right\}$$

including relativistic effect. Find the displacement  $z$  as a power series in time  $t$ . Compare with the classical result,

$$z = \frac{1}{2} g t^2.$$

4. A magnetic system has specific heat which is a function of applied magnetic field,  $H$  and temperature,  $T$ . It can be represented theoretically by

$$C(T, H) = C(H/T) = N \frac{(H/T)^2}{\cosh^2(H/T)}$$

where  $N$  is constant.

- (a) Expand  $C(T, H)$  in power of  $(H/T)$  up to  $(H/T)^4$  assuming  $H \ll T$ .  
(b) Sketch (freehand) the behavior of your answer as a function of  $x = H/T$ .

5. (1.4) For parts (a) and (b) express answers as whole numbers to a precision of  $\pm 1$ , for (c) use Stirling's formula and express answer as 10 raised to some power.

6. The probability  $P(n)$  that an event characterized by a probability  $p$  occurs  $n$  times in  $N$  trials is given by the binomial distribution

$$P(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}.$$

Consider a case where  $p \ll 1$  and  $N \gg 1$ .

(a) Show that  $(1 - p)^{N-n} \approx e^{-Np}$  using  $\ln(1-p) \approx -p$ .

(b) Show that  $N!/(N-n)! \approx N^n$ .

(c) Therefore, you can show that  $P(n) = \frac{\lambda^n}{n!} e^{-\lambda}$  where  $\lambda = Np$ . You have just demonstrated that the binomial distribution for small  $p$  and large  $N$  turns into the Poisson distribution!

7. (2.5)

8. (3.3)

9. (3.4)

10. (3.5) Do (a) through (g), skip (h) and (i), and do (j).