Homework B

PHY3513 **Due: Friday, Feb. 8, 2019**

1. (4.2)

2. (4.4) Note that there is a factor of two error in the equations stated in the text for this problem. Also, recognize that dT/dE is related to an important thermodynamic quantity.

3. (4.7) Hint, use the relation, $\dfrac{\sum_\alpha \alpha e^{-\alpha x}}{\sum_\alpha e^{-\alpha x}} = -\dfrac{\partial}{\partial x}\ln\left(\sum_\alpha e^{-\alpha x}\right)$

4. **Thermal Equilibrium of an Isolated System:** (This problem is optional and will not be graded. You should try it for practice.) Consider two macroscopic systems, *A* and *B*, in thermal contact an in equilibrium with each other but isolated from the surroundings. $P(E_A)dE$ is the probability that the system A has an energy between E_A and E_A+dE. Therefore, $P(E_A) = C\Omega_A(E_A)\Omega_B(E_B)$ ($E = E_A + E_B$ = constant). We are interested in the behavior of $P(E_A)$ near $E_A = \bar{E}$ where $P(\bar{E})$ is maximum. Show that

$$P(E) \approx P(\bar{E})\exp\left\{-\gamma(E-\bar{E})^2/2\right\}$$

where $\gamma = \gamma_A + \gamma_B$ and $\gamma_A = -\dfrac{\partial^2 \log_e \Omega_A(E_A)}{\partial E_A{}^2}$ (γ_B similarly defined). Use the Taylor expansion, not around $E = 0$ but around $E = \bar{E}$. Do you see that γ has to be positive? This problem shows that you can approximate a probability distribution near the maximum as a Gaussian distribution. (Think Central Limit Theorem)

5. (4.8)

6. (5.1) Get used to this type of integration!

7. (5.2) The escape velocity is $\sqrt{2GM/R}$ where G is gravitational constant, M is the mass of the earth, and R is the radius of the earth. Derive this expression first and then use constants in Appendix A..

8. **Equipartition Theorem** A very sensitive spring balance consists of a quartz spring suspended vertically from a fixed support. The spring constant is *k*. The balance is at temperature *T* in a location where the gravitational acceleration is *g*.
(a) When a small object of mass *m* is suspended from the spring, the spring will be stretched out by an amount *x* and will reach an equilibrium position. However, thermal

energy will cause fluctuations in x. Ignoring the mass of the spring, what is the average value of x, $\langle x \rangle$?

(b) What is the magnitude of thermal fluctuations of the object, $\left\langle \left(x - \langle x \rangle \right)^2 \right\rangle^{1/2}$?

(c) When the fluctuations become comparable to the average value, the measurement becomes impractical. How small a mass can this balance measure?

9. (5.4)

10. A particle can have an energy ε ranging continuously from 0 to infinity. The particle is subjected to temperature T.
(a) What is the probability of having energy ε, $P(\varepsilon)$ (with the correct normalization factor)?
(b) Show that $\langle \varepsilon \rangle = k_B T$. Compare this result with your answer for #3 of this HW assignment (i.e., problem 4.7 (b) of the text). In what condition does the result of 4.7 (b) reduce to this result?