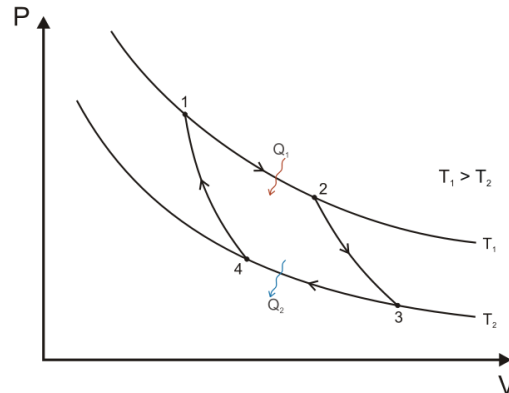


## Homework D

PHY3513

Due: Mar. 18, 2019

1. A Carnot cycle using 1 mole of an ideal gas (not necessarily a monatomic gas) as a working substance is composed of two isothermal and two adiabatic reversible processes (See figure).



The thermodynamic states are defined as **1** ( $V_1, p_1$ ), **2** ( $V_2, p_2$ ), **3** ( $V_3, p_3$ ), and **4** ( $V_4, p_4$ ). It is straightforward to show using the first law that the work produced in one cycle is,

$$|\Delta W| = Q_1 - Q_2 = R \ln(V_2 / V_1)(T_1 - T_2).$$

Show that you get the identical result by directly calculating (and then summing) the work done on all legs of the cycle.

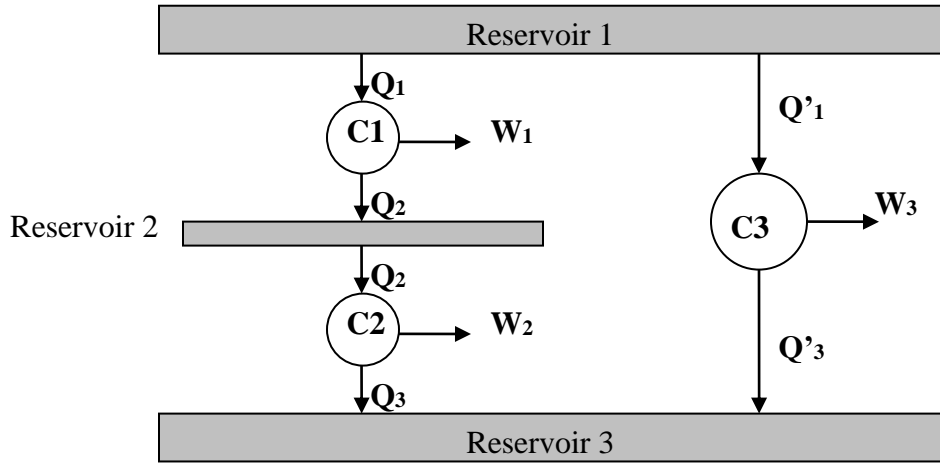
2. (13.4) Let's label three points as **1** ( $p_1, V_1, T_1$ ), **2** ( $p_1, V_2, T_2$ ), and **3** ( $p_2, V_2, T_3$ ). In  $1 \rightarrow 2$ ,  $\Delta Q_{12}$  is negative (why? The gas is cooled.) You can express  $\Delta Q_{12}$  in terms of the heat capacity and the temperature change. You should think what kind heat capacity you have to use here. In  $2 \rightarrow 3$ , heat  $\Delta Q_{23}$  is absorbed. You can do the same thing here. And in  $3 \rightarrow 1$ , it is adiabatic and back to the original point. Then use the definition of the efficiency  $\eta = W/Q_{abs}$  where  $Q_{abs}$  is the amount of heat absorbed by the system (or heat input to the engine). Then what is  $W$ ?

3. (13.6) The steady state means heat absorbed from the house  $Q_2$  should be the same as the heat leakage into the house  $Q$ . For this air conditioner (a type of Carnot engine), the 1<sup>st</sup> law should be applied (the electric power is used to do work on the engine) and the important relationship for a Carnot engine holds. You have three equations and should be able to answer after massaging these equations a little bit. Interpretation of the second part: you are running only 30% of the maximum power of your air conditioner and can maintain temperatures 20 C inside when 30 C outside. What would be the maximum temperature difference when you fully operate your air conditioner?

4. Show that the efficiencies of the three Carnot engines, operating between the three reservoirs as shown in the figure are related as

$$\eta_3 = \eta_1 + \eta_2 - \eta_1\eta_2.$$

Assign temperatures for each reservoir and consider energy conservation, the Carnot relations, and the efficiency of each engine.



5. (13.8)

6. Two equal quantities of water, of mass  $m$  and at temperatures  $T_1$  and  $T_2$ , ( $T_1 > T_2$ ) are mixed together with the pressure kept constant. Assume heat exchange occurs only between the two water containers.

(a) Show that the entropy change of the universe is

$$\Delta S = 2mc_p \ln \left\{ \frac{T_1 + T_2}{2\sqrt{T_1 T_2}} \right\}$$

where  $c_p$  is the constant pressure specific heat of water. (Assume that  $T_1$  and  $T_2$  are close and  $c_p$  is temperature independent within this temperature range. You should first know what would be the final temperature and then use  $\Delta S = \int \frac{C_p}{T} dT$ .)

(b) Show that  $\Delta S$  you derived in (a) is positive.

7. (14.3)

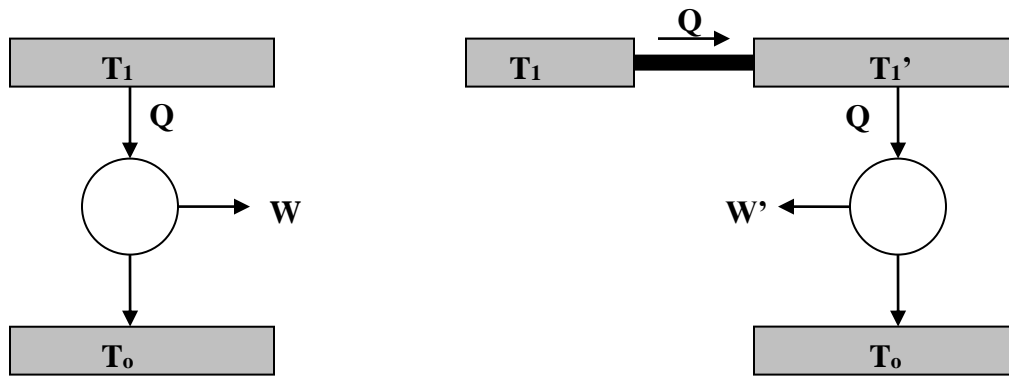
8. (14.5) ( $\Delta S = \int_{T_1}^{T_2} \frac{C_V}{T} dT$  You have to calculate entropy change and liquid bath separately. In doing so, be careful what  $T$  you should use inside the integral.)

**9. (The degradation of energy)** Let us consider a Carnot engine operating between two heat reservoirs at  $T_1$  and  $T_o$  ( $T_1 > T_o$ ). This engine absorbs heat  $Q$  from the hot reservoir. The efficiency of this engine (you better know this now) is  $1 - T_o/T_1$ .

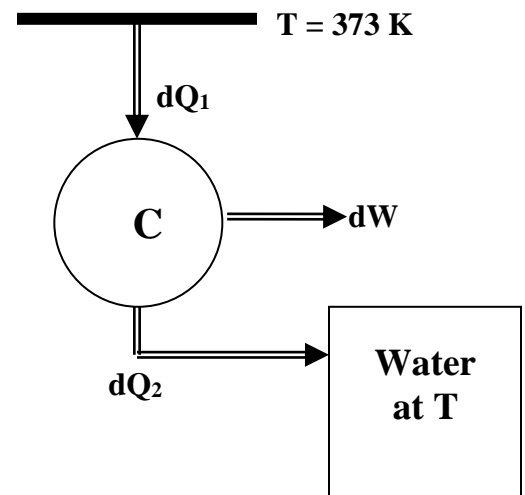
(a) Express the work produced by this engine in one cycle in terms of  $Q$ , and  $T_1$ , and  $T_o$ . What is the change in the total entropy  $\Delta S_{universe}$ ?

(b) Let us put the same amount of heat to the engine from a second reservoir at  $T_1'$  ( $< T_1$ ) as shown in the figure by allowing the heat  $Q$  conducted from the reservoir at  $T_1$  to the reservoir at  $T_1'$  through a conducting bar. Obviously there is an irreversible process inserted here. What is the work produced from the engine  $W'$  in a cycle? What would be the change in the total entropy  $\Delta S_{universe}$ ?

(c)\* Now you can see there is a direct relationship between the difference in work produced by these engines ( $\Delta W = W - W'$ ) and the difference in the entropy change. This result is often called the degradation of energy. Can you argue anything about the fate of the universe based on this simple result?



**10.** You want to boil a pot of water at  $20^\circ\text{C}$  heating to  $100^\circ\text{C}$ . There are two ways of doing this: (1) irreversibly by simply putting the pot in contact with  $100^\circ\text{C}$  heat reservoir (2) heating the pot of water in a reversible manner. The 2<sup>nd</sup> law of thermodynamics dictates that the total entropy of the system (pot of water + the heat reservoir) would increase in method (1). By simply inserting a Carnot engine between the reservoir and the pot (see the figure) one can heat the pot in a reversible way. The Carnot engine operates between two temperatures, absorbing heat  $dQ_1$  from the  $100^\circ\text{C}$  reservoir and rejecting  $dQ_2$  to the water at temperature  $T$  (heat capacity  $C_p$ ). At the same time, work  $dW$  has to be generated in this process. Starting from  $20^\circ\text{C}$  water temperature, delivering heat in this manner reversibly, the pot of water reaches  $100^\circ\text{C}$ .



(a) Show that the total entropy of the whole system (reservoir + water) remains the same in this process. Ignore the heat capacity of the pot and the temperature dependence of the water heat capacity.

(b) How much work is produced in this process?