

Homework E

PHY3513

Due: Monday, April 1, 2019

1. (16.1) Try and do these without looking up the derivations in your textbook.
2. (16.2) Justify each step in your derivations (i.e., reciprocity, Maxwell relation, etc.)
3. (a) (16.3) Use the relations: $dU = dQ - pdV = (\partial U / \partial T)_V dT + (\partial U / \partial V)_T dV$, $C_p = (\partial Q / \partial T)_V$, and $\beta_p = V^{-1} (\partial V / \partial T)_p$.

(b) Use your answer to show that for an ideal gas, $C_p - C_v = R$

4. (16.4) For part (b), you have to first show $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$ from $dU(T, V)$.

Then, show $\frac{1}{T^2} \left(\frac{\partial U}{\partial V}\right)_T = \left(\frac{\partial}{\partial T}\right)_V \left(\frac{p}{T}\right)$.

5. The internal energy of an ideal gas can be derived using statistical mechanics as

$$U = U(S, V) = \alpha N k_B \left(\frac{N}{V}\right)^{2/3} e^{2S/3Nk_B}.$$

where α is a constant. Show that this expression leads to the equation of state for an ideal gas $pV = Nk_B T$. (What is dU ?)

6. (17.1)
7. (17.2) (In 17.1 and 17.2, use the proper Maxwell's relations: $-p \rightarrow f$, $V \rightarrow L$)
8. (17.3) Express N_+ and N_- in terms of N , L , and a . Then, Sterling's formula.
9. (17.6) (The problem says the tension f is proportional to T with a fixed L . This means that $\left(\frac{\partial f}{\partial T}\right)_L = \text{const} > 0$)

10. Show the following relation:

$$\kappa_T - \kappa_S = T \frac{V \beta_p^2}{C_p}.$$

When we derived a similar relation involving C_p and C_v , we started from dU . Here it is reasonable to start from dV as $V = V(p, T)$.