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Special and General Relativity I

Black Hole Thermodynamics

Introduction

A black hole's event horizon can only increase, given most transformations of the black hole (Hawking 1975). And the universe's total entropy can only increase, given most thermodynamic processes of the universe as a system. What if entropy, or the missing energy that cannot do mechanical work, is just as dark as the missing information hidden within a black hole?

At the beginning of the 1970's, Jacob Bekenstein, was the first to realize a remarkable parallel that the black hole horizon plays the role of entropy, and from there, other parallels were drawn to formulate 4 laws of black hole mechanics that resemble the thermodynamic laws. Bekenstein's idea took off and extensive progress was made thereafter in the topic of black hole thermodynamics. The topic is fascinating in that it implies a connection between gravity, quantum mechanics, and statistical mechanics because its coherence relies on them. The history of black hole thermodynamics will be explained from the Horizon Area Theorem onward, and then the focus will be narrowed to more recent attempts to use thermodynamic geometry to probe the mysterious microscopic structure of a black hole.

History of Black Hole Mechanics

The Bekenstein-Hawking formula for the entropy of a black hole, where S is entropy and A is the event horizon area, is given here (Page 2004):

$$S_{BH} \equiv 1/4 \, A \quad (1)$$

This can be thought of as the second law of black hole mechanics. The other three laws of black hole mechanics will be explained after summarizing some flaws in Bekenstein's analogy. Namely, the temperature of a black hole vanishes, the horizon area is not dimensionless like entropy, and every black hole as its own individual system has an increasing area, while the universe's total entropy is only increasing, and its smaller systems are not necessarily (Jacobson 1996). J.A. Wheeler further emphasized that when matter is dropped into a black hole, by general relativity principles, it will disappear into a spacetime singularity, where information is lost and there is no known compensation in the form of entropy gain for the universe, so the regular second thermodynamic law fails (Wald 2001). The information loss is known as the information paradox, which is controversial because it violates quantum mechanical principals where information must be preserved by a wave function.

On more classical aspects of black hole thermodynamics, the zeroth law is that a stationary black hole's surface gravity κ is constant over the event horizon just as temperature is constant for a system in thermal equilibrium (Jacobson 1996). An extended form includes that the angular velocity and electrostatic potential are also constant over this black hole boundary. The no-hair theorem states that the simple characteristics of a black hole include mass, charge, and angular momentum. The first law relates these variables to the change in area of a black hole, providing an overall conservation of energy law where M, Ω, J, Φ, and Q are mass, angular velocity, angular momentum, electrostatic potential, and charge respectively.

$$\delta M = 1/8\pi\ \kappa\delta A + \Omega\delta J + \Phi\delta Q \quad (2)$$

Lastly, like the temperature of a system cannot reach zero in a bounded number of processes, nor can the surface gravity of a black hole, which is the third law.

Hawking Radiation

One of the flaws in classical theory was that black holes could absorb particles but could not emit them. In 1975, Steven Hawking considered quantum mechanical effects that would cause black holes to emit particles, treating black holes like a radiating blackbody. The general idea is to see a black hole as an excited energy state of a gravitational field that should decay quantum mechanically, and due to small fluctuations of the metric like in a vacuum, energy should be able to tunnel through a black hole's gravitation potential well (Hawking 1975). Through vacuum fluctuations across the event horizon, there is a means for a black hole to radiate energy. The term for a black hole's thermal emission was coined "Hawking radiation."

A black hole's mass continues to get smaller under Hawking radiation and its fate is to evaporate (Hawking 1975). This violates the horizon area theorem, but ironically, goes back to a similar parallelism that Bekenstein saw and courageously pursued. Recognizing the internal energy thermodynamics relation and relating it back to the first two terms of equation (2):

$$dU = TdS + pdV \quad (3)$$

Hawking was able contribute to some of Bekenstein's equations. At first, Bekenstein believed that in some sense, some multiple of A *is* the entropy of a black hole, and that some multiple of the surface gravity *is* the temperature of a black hole (Bekenstein 1973). Realizing that entropy, however, can be lost down black holes, he polished the second law to a generalized form where entropy plus a multiple of the horizon area never reduces (Bekenstein 1973). Adding that particles can be emitted from a black hole, Hawking fixed the problem of a black hole violating the regular second law because the entropy would not only be returned to the universe, but blanks were filled in the generalized second law. An understanding is reached where the laws of black hole mechanics are regular thermodynamical laws applied to black holes (Wald 2001). The quantum mechanical approach to the generalized second law of black hole mechanics quantifies the factor in front of surface gravity to be π/2 and confirms that of the horizon to be 1/4th.

Unruh Radiation

One notable experiment that tested Hawking radiation was based on a clever idea that an accelerating observer would observe thermal radiation like that emitted by a black body. Considering the strangeness of a vacuum that it is not really a vacuum due to quantum fluctuations, it is not surprising that in the reference frame of a comet moving through a vacuum, the background would appear to be warm simply from accelerating. Unruh used the equivalence principle to test the results of Hawking radiation. The gravitational field near a black hole's event horizon should be equivalent to acceleration in a flat spacetime, and so Unruh compared the measured temperature seen by an inertial versus uniformly accelerating observer and found a thermal flux of particles in the same form as the Hawking temperature, except where acceleration is just *a*, rather than *g*, backed by the equivalence principle (Carlip 2014). It became known as the Unruh effect, and it has been debated whether Unruh radiation is real.

Hawking vs. Unruh Radiation

The Hawking and Unruh effects differ in the quantum field states they refer to and their differences can be seen prominently with a Kerr blackhole (Wald 2001). In the Unruh effect, the "Hartle-Hawking vacuum state" is the quantum state in question, which is globally nonsingular, and the quantum field modes are all thermally populated, while in Hawking radiation, only the "UP modes" are thermally populated (Wald 2001). The "UP modes" as opposed to the "IN modes" that come from infinity, come from the white hole region of the continuous spacetime. In the case of the Kerr black hole, there is no Hartle-Hawking vacuum state on Kerr spacetime, but there is no problem with deriving the Hawking effect, where temperature is measured from an observer from infinity (Wald 2001).

Attempts to Calculate the Entropy of a Black Hole and the Importance

By the time of Hawking radiation, it was known that incorporating quantum theory resolved flaws in the horizon analogy and so further studies sought to study black holes within a fully quantum theory of gravity. Therefore, some approaches were taken to calculate the entropy of a black hole because a big question became what those microstates associated with black hole entropy are, and so the research concerning this question will be discussed extensively.

I. Gibbons and Hawking

Gibbons and Hawking did the first calculation in the context of Euclidean quantum gravity by evaluating an integral expression for the canonical ensemble partition function in the classical approximation for a black hole, and the findings were equivalent to the classical derivation (Wald 2001). Although this shows agreement between Euclidean quantum gravity and black hole thermodynamics and an important relation between entropy and geometry since the sum is over smooth Riemann geometries, it did not identify the quantum dynamical degrees of freedom for a better understanding of why black hole entropy is A/4 and how many degrees of freedom there are for a black hole.

II. Entanglement Entropy

One calculation that sought to answer the entropy question was one that used entanglement entropy from quantum field correlations across the event horizons, where a short distance cutoff of the order of the Planck scale for the von Neumann entropy resulted in an entropy of the order of the horizon area (Wald 2001). However, gravity and field equations played no part in this analysis. Some approaches instead tried to attribute the degrees of freedom to horizon shape, to causal links crossing the horizon, or to the entropy of the black hole's surrounding atmosphere (Wald 2001). Relating the degrees of freedom to the entropy in the black hole's atmosphere agreed well with the von Neumann entropy because it localizes the entropy near the event horizon at a scale of order of the Planck length. This is necessary since temperature diverges at the horizon and so does entropy, but an energy can be measured far away even though it may be high locally. Additionally, Witten showed in 1998 that the entropy of a Schwarzschild black hole in Anti-de Sitter (AdS) space equaled that of thermal radiation of the matter or fields on its boundary by taking advantage of AdS mapping to boundary conformal field theory (Witten 1998).

III. String Theory

Entanglement entropy seemed promising for understanding black hole entropy's origin on a microscopic scale and the information paradox since information cannot be communicated to observers from inside to outside the horizon, so it can be thought of as an entangling surface, and entanglement and Bekenstein-Hawking entropy both scale with the area of the black hole horizon (Cadoni and Melis 2010). Hooft conjectured that black hole entropy could be understood in terms of quantum entanglement in black hole geometry and showed that the modes of a quantum field near the horizon should be cut off due to effects from strong gravity (Hooft 1985).

The most successful calculations of black hole entropy came from string theory, including those for weak coupling states and calculations for the entropy of charged black holes, because they came into agreement with Hawking-Bekenstein entropy. Their agreement further highlights the connection between pillars of physics. It remained unclear, however, how the weak coupling states related to the picture of local physics of the black hole. In string theory, black hole entropy is thought to count the number of fundamental string states.

A 10-dimensional supergravity theory is what string theory reduces to at low energies, where if one takes it as a classical theory with classical fields like the spacetime metric, one can find black hole solutions; but in weak coupling, the states are treated as perturbations and there is no notion of a black hole just like in linearized general relativity (Wald 2000). Leonard Susskind suggested that for a large coupling value, strings form black holes because strings produce a higher gravitational force and the radius becomes smaller, while for a small coupling value, a flat spacetime metric can be used and the black hole becomes an excited string state (Horowitz 1997).

IV. Entropy Counting Matter States

Entropy counting matter states within a black hole was considered by Mukhanov and Bekenstein (1995) by looking at the spectroscopy of a quantum black hole. It tested the idea that at the horizon the black hole should emit a spectrum consistent with a characteristic set of broad emission lines with energy degeneracy that make up black hole entropy. They found that fluctuations of the horizon expected in quantum gravity were evident in that the radiation spectrum was discretized, where no quanta corresponded to a wavelength of or larger than the black hole size (Mukhanov and Bekenstein 1995). The argument was made that Hawking's idea of quanta hovering over the event horizon at a specific distance had to be tweaked since the location of the horizon should fluctuate. The mass spectrum of a black hole displayed discreteness just like any other quantum system of a finite size.

The results of the black hole entropy studies are good evidence that stationary black holes relate to the gravitational field as localized thermal equilibrium states and that black hole mechanics laws are those of thermodynamics simply applying to a system with a black hole (Wald 2001).

The Information Paradox

But another current unresolved issue in black hole thermodynamics aside from what and where are the degrees of freedom associated with black hole entropy, is whether a pure quantum state transforms into a mixed state during the formation and eventual evaporation of a black hole, suggesting lost information. The information paradox conflicts with quantum mechanics, which suggests that information must always be preserved, given that the evolution operator for a wave function has an inverse and that the evolutionary operator determines changes in a wave function. AdS/CFT correspondence or the holographic principle demonstrate that information is preserved, and Hawking agreed in 2004 that he may have been wrong about Hawking radiation not preserving information.

Since Hawking radiation suggested the loss of information, several publications tried to address this problem including the information being encoded in the radiation or within distortions of event horizon, leaking out at the end of evaporation, or the possibility of the information going to a different universe behind the black hole (Preskill 1992). AdS/CFT correspondence was pivotal in suggesting to physicists that there was a means for a black hole to preserve information. It was just a matter of finding out how. An early 1990s study proposed "complementarity," that information bounces back off the event horizon, but the main flaw was that one observer could not detect the information propagating in opposite directions (Robson 2014). In 2013, the firewall paradox added a firewall to "complementary", or a layer of high energy quanta at the horizon, but its contradiction lied in general relativity's equivalence principle the observer should feel nothing aside from tidal effects when falling through the horizon (Robson 2014). The aforementioned "fuzzball" proposal in string theory accepting the microstates of black holes as the excitation states of strings or branes allowed information to leak out.

How a Quantum Extremal Surface May Solve the Information Paradox Problem

A more recent work that is believed to be the best solution of solving the information paradox is a 2019 study by Almheiri and others on “the entropy of bulk quantum fields and the entanglement wedge of an evaporating black hole”. A quantum extremal surface (QES) arises when considering evaporating black holes from gradients in the bulk entropy from boosts, and bulk quantum fields are what contribute to generalized black hole entropy (Almheiri et al. 2019). Quantum entanglement happens across the horizon in the entanglement wedge just inside the horizon, and the particles inside the black hole QES do not contribute to the entropy anymore, while the other particles in the form of radiation leave the black hole, explaining why the black hole’s entropy can decrease as a thermodynamic system despite it being originally thought of as an analog to the universe’s entropy always increasing. Since the particles inside the QES are not a part of the black hole anymore, it answers the question of where the information goes only partially. It remains a mystery what exactly happens inside of a black hole and the nature of the innermost QES. The results provide how to calculate the Page curve and the Page time in the context of an evaporating black hole from a phase transition and reveals how information gets out of a black hole (Almheiri et al. 2019). The entropy of Hawking radiation follows the Page curve, where the entropy increased in the first half of the black hole’s life and for the latter half, it decreases until it evaporates (Page 1993). The Page time is known as the midway point of the Page curve (Page 2013).

Ruppeiner Geometry and Probing the Microscopic Structure of a Black Hole

Now to transition into a more specific topic that relates back to the microstates implied by the entropy of a black hole puzzling physicists for years; and that is, how Ruppeiner geometry is being studied to probe the microscopic structure of a black hole. While quantum gravity can help understand the microscopic structure because geometry can be applied to thermodynamical properties and phase transitions, the main issue is that there is no complete theory of quantum gravity. As previously explored, the statistical description of black hole microstates is not fully understood.

I. Ruppeiner Geometry

Ruppeiner geometry has a history of its own. Weinhold was the first to introduce the geometrical concept to thermodynamics by suggesting that a Riemann metric can be defined as the second derivatives of internal energy with respect to entropy and other thermodynamic quantities (Shao-Wen Wei et al. 2019). But a metric with significant physical meaning was still needed. So Ruppeiner introduced one analogous to Weinhold's where the thermodynamic potential is the entropy instead of the internal energy and where the metrics were conformal to each other with the conformal factor temperature (Shao-Wen Wei et al. 2019).

The Ruppeiner metric[1] measures the difference between two neighboring fluctuating states because it includes the line element of the geometry[2]. When fluctuations are more probable between two neighboring states, they have a shorter separation distance. Ruppeiner geometry led to several investigations of different types of black holes as thermodynamic systems, and the key physical meaning of the Ruppeiner geometry is that it carries phase transition information. The Ruppeiner approach gives a systematic way to calculate the Ricci curvature scalar of the Ruppeiner metric (Wang et al. 2020). The Ricci curvature scalar gives an idea of strength and dominant interaction between particles in a system.

In 1904, from Einstein's work came thermodynamic fluctuation theory. The relation between the number of states, entropy, and Boltzmann's constant in logarithmic form has a

[1] See equation 6
[2] See equation 7

gaussian approximation that Ruppeiner added "covariance, conservation, and consistency" to (Ruppeiner 2010). The Guassian approximation of thermodynamic fluctuation theory gives the probability of being in a thermodynamic state. If a closed system with constant volume and internal energy is divided by an open barrier enclosing a smaller subsystem with fluctuating internal energy, then the outer subsystem becomes the reservoir and energy is exchanged through fluctuations between the two, and the microcanonical ensemble means that the probability of the internal energy being between some values is proportional to the number of microstates in this range (Ruppeiner 2010). In statistical mechanics, the energy is thought to be divided between two subsystems in a way that maximizes the entropy or brings it to equilibrium. So, going through the gaussian approximation derivation with entropy as a function of internal energy and volume, taking advantage of entropy being additive for both the reservoir and the partitioned subsystem, Ruppeiner, does the taylor expansion of the entropy densities corresponding to these subsystems about the maximum, and applies conservation of energy between the two partitions, leading to the Gaussian approximation in thermodynamic fluctuation theory (Ruppeiner 2010):

$$P_G(u,V)du = \sqrt{\frac{V}{2\pi}} \exp\left[-\frac{V}{2} g(u_0)(\Delta u)^2\right] \sqrt{g(u_0)}\, du \quad (4)$$

Considering that a decreasing change in volume would increase the internal energy density, Ruppeiner made the argument that that the third order term should not be thrown away in the entropy expansion once energy conservation is introduced, unlike thermodynamic fluctuation theory considered at the time (Ruppeiner 2010). The energy conservation he applied was between the internal energy of the reservoir and the other partition in the hypothetical close system discussed.

Ruppeiner's covariant argument regarding the gaussian approximation of fluctuation theory was the aspect that added "consistency". He realized that under a coordinate transformation, a transformation rule could be added to make it covariant, and he noted that it was significant because the transformation rule needed to be valid under small fluctuations; He also realized that the normalization factor varied with internal energy, more specifically that it picked up the Jacobian since that varied, while entropy as a function of state did not (Ruppeiner 2010). Naturally, after the internal energy – entropy hypothetical, he started to consider other thermodynamic variables to examine how it could change the normalization factor of the gaussian approximation of entropy or the higher order terms of the entropy expansion.

Ruppeiner proposed a hypothetical larger partition that was concentric with the first, so that the reservoir was external to both concentric closed systems, and after expanding the entropy for this case, he found that the fluctuations for this system were smaller and that the Gaussian approximation could be improved by adding more concentric partitions and doing a path integral over all the intermediate states (Ruppeiner 2010). The next step, of course, was his attempt to add energy conservation and covariance, which was more difficult for this hypothetical where he added more layers, loosely and literally speaking!

Ruppeiner considered pressure this time as a function of internal energy and volume. He used the diffusion equation as a workable partial differential equation to limit the expression of his modified thermodynamic fluctuation theory (Ruppeiner 2010). Once he proved its covariance, conservation, and consistency, he found that the scalar curvature arose from considering two entropy-related fluctuation variables. The Gaussian approximation with two fluctuation variables instead of one becomes equation (5):

$$P_G\, da^1 da^2 = \Big(\frac{V}{2\pi}\Big) \exp\Big(-\frac{V}{2} g_{\mu\nu} \Delta a^\mu \Delta a^\nu\Big) \sqrt{g}\, da^1 da^2 \quad (5)$$

$$g_{\alpha\beta} \equiv -\frac{1}{k_B} \frac{\partial^2 s}{\partial a^\alpha \partial a^\beta} \quad (6)$$

$$\Delta \ell^2 \equiv g_{\mu\nu} \Delta a^\mu \Delta a^\nu \quad (7)$$

where V is volume, the metric is defined in equation (6) where s is the function of the internal energy and density that the additive entropy is dependent on in the surroundings plus reservoir partitions, and where equation (7) shows the Riemannian metric that results, which shows the distance between thermodynamic fluctuation states (Ruppeiner 2010). This provides a mathematical picture for what has been described earlier.

The scalar curvature was pivotal and a useful tool to better understand the microscopic structure of black holes. More on the scalar curvature will be explained before diving into those studies.

II. Recent Studies using Ruppeiner Geometry to probe black hole microstructure

Non-zero scalar curvature means there is an underlying interacting statistical system. Choosing a suitable ensemble helps to study different black hole's microscopic behavior. The scalar curvature is thought to probe microscopic structure because for different physical systems like the ideal gas, for example, curvature is negative, and for Bose and Fermi ideal gases, it can be either positive or negative, and so the curvature gives insight into what kind of particle force interactions are going on, repulsive or attractive. It has been in interest to study black hole thermodynamics and phase transitions in phase space where the cosmological constant has been taken to be the thermodynamic pressure (Mansoori and Mirza 2019). The idea of using the scalar curvature to study the phase transitions of black holes is still one of speculation. Ruppeiner proposed that scalar curvature is proportional to the correlation length of the thermodynamic system to the power of the dimensionality, meaning scalar curvature near critical point approaches infinity (Ruppeiner 1995), but this is not expected for a thermodynamical system, and other studies show that scalar curvature does not diverge at critical point.

Normally thermodynamic phenomena from phase transitions are backed by microscopic models, but if heat capacity diverges in a black hole, we are not sure if an inappropriate choice of thermodynamic variables was made or which divergence curves correspond to true phase transitions (Ruppeiner 2013). Ruppeiner looked into the scalar curvature of Bañados, Teitelboim, and Zanelli (BTZ), Kerr, and AdS black holes among others, and discussed physical implications. For the BTZ and Reissner-Nordström black holes, the scalar curvature is zero, for example, and he discusses a possibility that it could be referring to particles on the horizon experiencing gravity as a non-statistical force where other particle interactions may be independent of each other, and at the singularity, of course, the particle interactions are shrunk to a zero volume (Ruppeiner 2013).

The scalar curvature has been calculated for several thermodynamic systems including ideal gases and ising models, and thermodynamic geometry has been studied for different types of black holes. Åman and Pidokrajt (2006) studies geometries for some families of black holes and finds flat Ruppeiner geometry for any dimension Reissner–Nordström black holes, as well as Kerr black hole curvature singularities. Charged AdS black holes were probed for microstructure using Ruppeiner geometry and phase transitions, and findings show that attractive interactions between the particles dominate (Shao-Wen Wei et al. 2019). In more detail, they reviewed the thermodynamics of a van der Waals fluid finding that the scalar curvature is always negative and finding the critical exponent relating the scalar curvature, reduced temperature, and the heat capacity. They applied a similar, but new and normalized, curvature to charged AdS black holes of higher dimensions, since instead of having a finite constant value the scalar curvature should diverge, and they found agreement with the critical exponent related to the small to large black hole phase transition. One distinguishment, though, was that for small, charged AdS black holes, the scalar curvature can be positive meaning that the particles are dominated by repulsive forces for these black holes (Shao-Wen Wei et al. 2019).

In 2018, Miao and Xu studied the feasibility of the intermolecular interaction potential of molecules of the Swarzschild anti-de Sitter black hole in terms of the thermodynamic scalar curvature. Since this black hole is thermally stable, it is physically appropriate to try to probe its microscopic structure. Doing it in the context of its molecules and their interaction potentials is more philosophical, being experience or phenomena based, which is why they studied its validity in comparison to the more well-accepted scalar curvature. They found that attractive interaction force increases as Hawking temperature increases, and the scalar curvature also increases, which verified that both interaction force descriptions agreed with each other (Miao and Xu 2018). They further found small thermo-correction terms to the equation of state.

A 2020 study by Wang found that there may be a connection between black hole microstates and the boundary condition. Certain black holes in a cavity can be thermally stable allowing exploration of how much thermodynamic geometry might be related to black hole boundary conditions (Wang et al. 2020). To do this, they focused on the (RN-AdS) black hole and an RN black hole in a cavity. Both black holes agree with the critical exponent of a Van der Waals fluid as well from mean field theory predictions, and boundary conditions considered were Dirichlet and the asymptotically AdS boundary to display the charge-temperature space which showed similarities.

Information about phase transitions comes from the scalar curvature of phase space geometry. Phase spaces represents all possible phases of a system. Mass and pressure, for example, can be used as the parameter space for a charged black hole, which would provide possible values for mass and pressure that the black hole could have. Phase space geometry is concerned with the shape and spatial relationships of the phase space because the intrinsic Riemannian geometry can give some physical meaning to statistical fluctuations since fluctuations connecting two equilibrium states are related to the invariant distance. The second order Taylor expansion of thermodynamic entropy with respect to the state-space variables makes up the “state-space” metric tensor (Belucci and Tawari 2012). It reduces to the Ruppeiner metric when the variables are all thermodynamic in the low energy limit of string theory.

Now that thermodynamic geometry and phase space is defined, specifically the charge and the potential phase space of the RN-AdS black hole played similar roles to that of pressure and volume in a Van der Waals fluid (Wang et al. 2020). And studying the free energy-Ruppeiner invariant space helped determine what temperature a thermally stable versus unstable small, intermediate, or large black hole could exist at for a RN-AdS black hole. The charge-temperature diagram showed a phase transition from a small to large black hole.

Likewise, the thermodynamical geometries of certain black hole solutions to Einstein’s equations can be studied and compared with that of normal fluid systems if their scalar curvature is similar. It goes back to another old analogy that black holes resemble fluid lumps.

Like a bubble in static equilibrium minimizes its surface area, a black hole maximizes the area of its horizon and fluid lumps and black holes share the same fate of evaporating (Caldarelli et al. 2009). AdS/CFT correspondence shows this duality well for a specific class of black holes in AdS space, explaining why Wang et al. (2020) use an RN-AdS black hole. But other good reasons are because large AdS black holes have a positive specific heat capacity and they do not disappear by Hawking radiation; One of the flaws in the analogy, unfortunately, is that the dual fluids to these black holes do not have a surface that bounds it (Caldarelli et al. 2009).

Dual fluid/gravity correspondence was used to study the dual fluid system of a slowly rotating Kerr black hole which was found to be a generalization of the incompressible Navier-Stokes equation that when modified has a Killing-like solution similar to the incompressible fluid on a sphere (Lysov 2015). So, the dual fluid to a rotating black hole geometry obeys their modified Navier-Stokes equation. Interestingly because the rotating black hole fluid dual shows stability since the flow is Killing-like, the paper notes that it would be useful to study the gravity dual of turbulence, and goes on to explore it.

One weakness in trusting the dual nature between black holes and fluid lumps is that scale invariance is not present in a fluid like it is for black holes scaling uniformly with mass because fluids come with a length scale already from surface tension which sets fluid lumps of different sizes apart and is one extra parameter considered in parameter space than for a black hole (Lysov 2015). Although the Planck length helps to distinguish between small and large black holes, large black holes and large fluid balls exhibit very different properties. For example, the entropy cost of breaking up a very large plasma ball is very small, whereas breaking up a large black hole has a constant entropy cost independent of the black hole size. Lysov suggested through calculations that the property that solves this issue is the dimensionality of the black hole, or that the number of spacetime dimensions distinguishes between large and small black holes, analogous to how the surface tension distinguishes between large and small fluid balls, the former which breaks up more easily (Lysov 2015).

Black holes as fluid lumps is just another clever idea that resembles the same type of thinking that black hole thermodynamics originated from, and in trying to phenomenologically probe the microscopic nature of a black holes, it is natural for us to use well studied thermodynamics as a start, because it is what we are familiar with, and it is difficult to target a subject with no current, backing observations from a perfectly objective viewpoint, as we have only recently just seen the first picture of a black hole.

Where it stands now, more progress needs to be made to probe the microscopic structure of a black hole, but Ruppeiner geometry has proven reliable in studying well-known thermodynamic systems and fluid systems, and further relating these systems to black hole solutions helped provide an unprecedented understanding of at least the phase transitions of black holes as well as the intermolecular forces between their particles.

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