Gravity in Large scale structure formation

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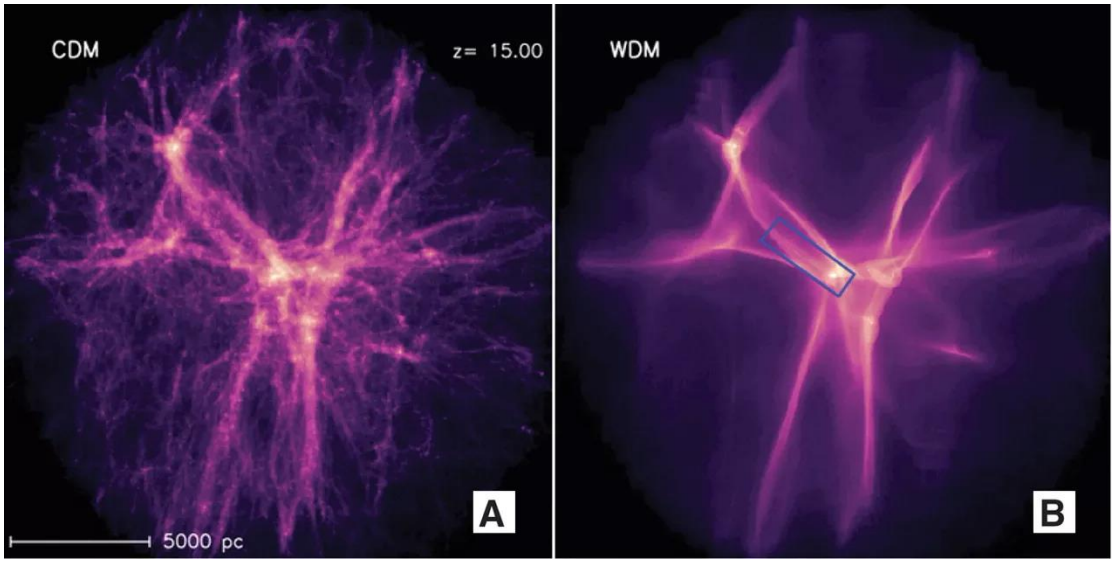
Chapter 1: Cosmological model

Various of modern experiments reveal our university is dominated by invisible matter-dark matter and dark emery. Furthermore, observations also reveal our universe is flat, isotropy and homogeneous in large scale, which makes it possible to take energy-mass fluctuation as perturbation on a flat background.

Dark matter and dark energy account for nearly 95% of energy-mass density in our universe. The visible baryon matter only occupy about 5% energy-mass density in universe. To get a precise explanation and simulation on university large structure formation, the influence of dark matter and dark energy should be included.

A wildly accepted cosmological model is ΛCDM model - lambda cold dark model. Λ(lambda) represents the cosmological constant, correspond to the dark energy, it drives the accelerating expansion of universe. Dark matter is another dominate composition of universe, they interact with other universe composition only through gravity interaction. In different dark matter models, scientists distribute it with different potential candidate particles, such as neutrinos, non-baryon particles, etc. In ΛCDM model, the dark matter is assumed

‘cold’, which means its velocity is far less than the speed of light after decoupled from other matters (Thus neutrinos are excluded, being non-baryonic but cold), they are also collisionless. In contrast, there is also an assumption, where dark matter is hot. In hot dark matter model, dark matter is active, its velocity is of the speed of light. Warm dark matter model assume dark matter running speed between CDM and HDM. Using different model, the simulation result are different, in this project, I will concentrate on the CDM model.



A and B are the simulation results of large scale structure based on CDM, and WDM, separately

Chapter 2: dynamics of perturbation

In the CDM domain, particles are assumed to be non-relativistic. Our investigation region is much smaller than the Hubble radius, thus the equations of motion reduce essentially to those of Newtonian gravity.

2.1: The Vlasov Equation- collisionless Boltzmann equation

In the limit that the number of particles N $\gg$ 1, collisionless dark matter obeys the Vlasov equation for the distribution function in phase space.

$$\frac{\mathrm{df}}{\mathrm{dt}} = \frac{\partial \mathrm{f}}{\partial \mathrm{t}} + \vec{\mathrm{v}} \cdot \overrightarrow{\nabla_{\mathrm{r}}}\mathrm{f} - \vec{\nabla}\Phi \cdot \overrightarrow{\nabla_{\mathrm{v}}}\mathrm{f} = 0$$

Where $f(\vec{x}, \vec{p}, t)$ is the particle number density in phase space. And ϕ here is the gravitational potential.

The Vlasov Equation can be easily derived:

$$\frac{\mathrm{d}f(\vec{x}, \vec{p}, t)}{\mathrm{dt}} = \frac{\partial f(\vec{x}, \vec{p}, t)}{\partial \mathrm{t}} + \frac{\mathrm{d}\vec{\mathrm{x}}}{\mathrm{dt}} \cdot \frac{\partial \mathrm{f}(\vec{x}, \vec{p}, t)}{\partial \vec{\mathrm{x}}} + \frac{\mathrm{d}\vec{\mathrm{v}}}{\mathrm{dt}} \frac{\partial \mathrm{f}(\vec{x}, \vec{p}, t)}{\partial \vec{\mathrm{v}}} = 0$$

The cold dark particles in universe only interact with other matters through gravity iteration, which gives them the equation of motion:

$$\mathrm{F} = -\mathrm{m} \cdot \vec{\nabla}\phi = \mathrm{m} \cdot \frac{\mathrm{d}\vec{\mathrm{v}}}{\mathrm{dt}}$$

Then, the Vlasov Equation writes:

$$\frac{\mathrm{d}f(\vec{x},\vec{p},t)}{\mathrm{dt}} = \frac{\partial f(\vec{x},\vec{p},t)}{\partial \mathrm{t}} + \vec{\mathrm{v}} \cdot \overrightarrow{\nabla_r} f - \vec{\nabla}\phi \cdot \overrightarrow{\nabla_v} f = 0$$

The phase space conservation makes them equal to zero.

2.1.2: Poisson equation

Poisson equation comes directly from newton's equation. By applying newton law of gravitation to many body system, we can get:

$$\vec{\mathrm{F}} = M\frac{d^2\vec{r}}{dt^2} = -\mathrm{M} \cdot \vec{\nabla}\phi = M\sum_i \frac{Gm_i(\vec{r}-\vec{r_i})}{|\vec{r}-\vec{r_i}|^3}$$

Since the number of particles N $\gg$ 1, apply the continuous condition to the summation.

$$\vec{\mathrm{F}} = M\frac{d^2\vec{r}}{dt^2} = -\mathrm{M} \cdot \vec{\nabla}\phi = MG\int \rho(\vec{r}',t)\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}d^3\vec{r}'$$

Taking the derivative to $\vec{r}$, we get the Poisson equation:

$$\nabla^2\phi = 4\pi G\rho(\vec{r}',t)$$

In principle, the large scale structure evolution can be fully solved and predicted by Vlasov equation and Poisson equation.

2.2 Eulerian dynamics

In practice however we are usually not interested in solving the full phase space dynamics, but rather the evolution of the spatial

distribution. This can be obtained by taking momentum moments of the distribution. Let's first define some important alternative variable, that can simplifies the calculation:

Conformal time τ, which relate to cosmic time by $d\tau = a\,dt$. Parameter a here is called cosmological scale factor, which relate the physical distance to commoving distance by $r = ax$. The peculiar velocity $\vec{u}(\vec{x},\tau)$, which is the velocity factor out the expansion of universe, it relate to the overall velocity by $\vec{v}(\vec{x},\tau) = H\vec{x} + \vec{u}(\vec{x},\tau)$. And momentum is $\vec{P} = ma\vec{u}$. Also define the perturbation density $\delta(\vec{x},\tau)$, which relatete to density by $\rho(\vec{x},\tau) = \bar{\rho}(\tau)[1 + \delta(\vec{x},\tau)]$, $\bar{\rho}(\tau)$ is the average density which is only the function of time.

Thus the Vlasov Equation can be rewritten as:

$$\frac{\mathrm{d}f(\vec{x},\vec{p},\tau)}{\mathrm{d}\tau} = \frac{\partial f(\vec{x},\vec{p},\tau)}{\partial\tau} + \frac{\vec{P}}{ma}\cdot\frac{\partial \mathrm{f}(\vec{x},\vec{p},\tau)}{\partial\vec{\mathrm{x}}} - ma\vec{\nabla}\phi\cdot\frac{\partial \mathrm{f}(\vec{x},\vec{p},\tau)}{\partial\vec{P}} = 0$$

Here, ϕ is also a modified gravitational potential, which factor out the expansion of unaversive.

First, take the zero order momentum, that is to integrate the Vlasov equation with $\vec{P}$ variable, and get the continuity equation:

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \nabla \cdot [1+\delta(\vec{x},\tau)]u(\vec{x},\tau) = 0$$

We can keep going to first order momentum. Multiply the equation by v_j, and integrate the Vlasov equation with $\vec{P}$ variable, and get the Euler equation:

$$\frac{\partial u(\vec{x},\tau)}{\partial \tau} + H(\tau)\cdot u(\vec{x},\tau) + u(\vec{x},\tau)\cdot \nabla u(\vec{x},\tau) = -\nabla\phi(\vec{x},\tau) - \frac{1}{\rho}\nabla_j(\rho\cdot\sigma_{ij})$$

Here σ_{ij} is the stress tensor, which is defined as $\sigma_{ij}^2 = \langle v_i\rangle\langle v_j\rangle - \langle v_i.v_j\rangle$.

2.2.1 Eulerian Linear Perturbation Theory

Taking the divergence of Euler equation and making Fourier transforms, we generate the Euler equation in Fourier space:

$$\frac{\partial \tilde{\delta}(\vec{k},\tau)}{\partial \tau} + \tilde{\theta}(\vec{k},\tau) = -\int d^3\overrightarrow{k_1}\, d^3\overrightarrow{k_2}\delta_D(\overrightarrow{k_1}+\vec{k}_2-\vec{k})\alpha(\overrightarrow{k_1},\vec{k}_2)\tilde{\theta}(\overrightarrow{k_1},\tau)\tilde{\delta}(\vec{k}_2,\tau)$$

$$\frac{\partial \tilde{\delta}(\vec{k},\tau)}{\partial \tau} + H(\tau)\tilde{\theta}(\vec{k},\tau) + \frac{3}{2}\Omega_m H^2(\tau)\tilde{\theta}(\vec{k},\tau) = -\int d^3\overrightarrow{k_1}\, d^3\overrightarrow{k_2}\delta_D(\vec{k}-\vec{k}_{12})\beta(\overrightarrow{k_1},\vec{k}_2)\tilde{\theta}(\overrightarrow{k_1},\tau)\tilde{\theta}(\vec{k}_2,\tau)$$

Where $\delta_D(\vec{k}-\vec{k}_{12})$ is 3 dimension Dirac delta function, and $\alpha(\overrightarrow{k_1},\vec{k}_2) = \frac{\vec{k}\cdot\vec{k}_1}{k_1^2}$, $\beta(\overrightarrow{k_1},\vec{k}_2) = \frac{k^2\cdot\vec{k}_1\cdot\vec{k}_2}{2k_1^2k_2^2}$.

At large scales, where we expect the universe to become smooth, the fluctuation fields can be assumed to be small compared to the homogeneous terms. Thus we can ignore the small terms and linearize the Fourier space continuity equation and Euler equation to get:

$$\frac{\partial \tilde{\delta}(\vec{k},\tau)}{\partial \tau} + \tilde{\theta}(\vec{k},\tau) = 0$$

$$\frac{\partial \tilde{\delta}(\vec{k},\tau)}{\partial \tau} + H(\tau)\tilde{\theta}(\vec{k},\tau) + \frac{3}{2}\Omega_m H^2(\tau)\tilde{\theta}(\vec{k},\tau) = 0$$

Combining the two equation then we get a second order differential equation on $\tilde{\delta}(\vec{k},\tau)$:

$$\frac{\partial^2 \tilde{\delta}\left(\vec{k},\tau\right)}{\partial \tau^2} + H(\tau)\cdot\frac{\partial \tilde{\delta}\left(\vec{k},\tau\right)}{\partial \tau} + \frac{3}{2}\Omega_m H^2(\tau)\tilde{\theta}\left(\vec{k},\tau\right) = 0$$

In the linear regime, the density perturbation can be obtained exactly:

$$\delta(\vec{x},a) = C^{(+)}(\vec{k})H(a)\int_0^a \frac{da'}{a'H^3(a')} + C^{(-)}(\vec{k})H(a)$$

The first term $D^{(+)}(\tau) = C^{(+)}(\vec{k})H(a)\int_0^a \frac{da'}{a'H^3(a')}$ is called growing mode while $D^{(-)}(\tau) = C^{(-)}(\vec{k})H(a)$ is decay mode.

These density growing modes depend on the particular underlying cosmology in such a way that, in different matter constitution

universes structures will grow in a different manner. Since by combining the universe constitution we can solve the $H(a)$ through Friedman equation.

Here are some important cases:

(1) when $\Omega_m = 1$ and $\Omega_\Lambda = 0$ we have:

$$D^{(+)}(\tau) = a \text{ and } D^{(-)}(\tau) = a^{-\frac{3}{2}}.$$

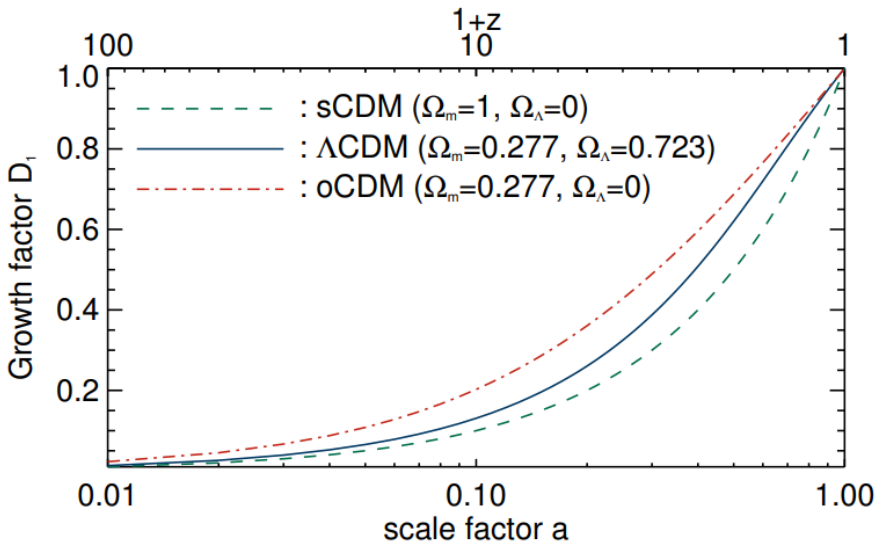
In which case the density fluctuation grows as the scale factor

(2) when $\Omega_m < 1$ and $\Omega_\Lambda = 0$ we have

$$D^{(+)}(\tau) = 1 + \frac{3}{x} + 3\sqrt{\frac{1+x}{x^3}}\ln\left(\sqrt{1+x} - \sqrt{x}\right)$$

$$D^{(-)}(\tau) = \sqrt{\frac{1+x}{x^3}}. \text{ Where } x = \frac{1}{\Omega_m} - 1.$$

The below picture shows the growing mode in different universe model.



2.2.2 Eulerian Non-Linear Perturbation Theory

In the non-linear regime, we will take the nonlinear terms into account. In this case, we cannot find a single mode that can describe the dynamics. What we get is a perturbative solution in which different modes coupled with each other. One assumption is that the general solution can be perturbative expanded as:

$$\widetilde{\delta}(\vec{k},\tau) = \sum_{n=1}^{\infty} \widetilde{\delta_n}(\vec{k},\tau) = \sum_{n=1}^{\infty} \int \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_{n-1}}{(2\pi)^3} d^3q_n \delta_D(\vec{k} - \sum_{n=1}^{\infty} \vec{q}_n) \cdot F_n^{(s)}(\overrightarrow{q_1}, \overrightarrow{q_2}_i \dots \overrightarrow{q_n}; \tau) \cdot \widetilde{\delta_1}(\vec{q}_1,\tau) \cdot \dots \cdot \widetilde{\delta_1}(\vec{q}_n,\tau)$$

$$\widetilde{\theta}(\vec{k},\tau) = \sum_{n=1}^{\infty} \widetilde{\theta_n}(\vec{k},\tau)$$

$$= -f(\tau)H(\tau) \sum_{n=1}^{\infty} \int \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_{n-1}}{(2\pi)^3} d^3q_n \delta_D\left(\vec{k} - \sum_{n=1}^{\infty} \vec{q}_n\right)$$

$$\cdot G_n^{(s)}(\overrightarrow{q_1}, \overrightarrow{q_2}_i \dots \overrightarrow{q_n}; t) \cdot \widetilde{\delta_1}(\vec{q}_1,\tau) \cdot \dots \cdot \widetilde{\delta_1}(\vec{q}_n,\tau)$$

Where $f(\tau) = \frac{\mathrm{d}\ln \mathrm{D}}{\mathrm{d}\ln \mathrm{a}}$, $\mathrm{D} = \frac{\mathrm{D}_+(\mathrm{a})}{\mathrm{D}_+(1)}$.) $F_n^{(s)}(\overrightarrow{q_1}, \overrightarrow{q_2}_i \dots \overrightarrow{q_n}; t)$ and $G_n^{(s)}(\overrightarrow{q_1}, \overrightarrow{q_2}_i \dots \overrightarrow{q_n}; t)$ are called symmetrized kernels, which characterize coupling between different wave modes. In the case of $n = 1$ which is the case of linear regime $F_n^{(s)} = G_n^{(s)} = 1$.

After put the expansion back to continuity equation and Euler equation we get:

$$\frac{1}{f(\tau)H(\tau)}\frac{\partial F_n(\vec{k},\tau)}{\partial t}+nF_n(\vec{k},\tau)-G_n(\vec{k},\tau)$$
$$=\sum_{m=1}^{n-1}\alpha(\overrightarrow{k_1},\vec{k}_2)F_{n-m}(\vec{k}_2,\tau)G_m(\overrightarrow{k_1},\tau)$$

$$\frac{1}{f(\tau)H(\tau)}\frac{\partial G_n(\vec{k},\tau)}{\partial t}+\left[\frac{3}{2}\frac{\Omega_m(\tau)}{f^2(\tau)}+n-1\right]G_n(\vec{k},\tau)-\frac{3}{2}\frac{\Omega_m(\tau)}{f^2(\tau)}F_n(\vec{k},\tau)$$
$$=\sum_{m=1}^{n-1}\beta(\overrightarrow{k_1},\vec{k}_2)G_{n-m}(\vec{k}_2,\tau)G_m(\overrightarrow{k_1},\tau)$$

In general, we can use the two recursion equation to solve F_m and G_n to any order, so that we can solve the density and velocity field.

2.2.3 example of Eulerian Non-Linear dynamics application

An important application of the non-linear Eulerian dynamics is the dynamics of density field in a spatially flat and matter-dominated universe, which is called the 'Einstein de-Sitter Universe'. In the Einstein de-Sitter Universe, the basic universe parameter are $\Omega_m = 1$ and $\Omega_\Lambda = 0$. By solving the Friedman equation, it is easy to get 'Einstein de-Sitter Universe' $H(\tau)\alpha\frac{2}{\tau}$ and $a(\tau)\alpha\tau^2$. Put these information back we can get $\frac{\Omega_m(\tau)}{f^2(\tau)} = 1$.

One assumption here is that kernel is insensitive to the time, so we

can neglect the terms of $\frac{\partial F_n(\vec{k},\tau)}{\partial t}$ and $\frac{\partial G_n(\vec{k},\tau)}{\partial t}$. In the case our kernel equation gives:

$$nF_n(\vec{k},\tau) - G_n(\vec{k},\tau) = \sum_{m=1}^{n-1} \alpha(\overrightarrow{k_1},\vec{k}_2)F_{n-m}(\vec{k}_2,\tau)G_m(\overrightarrow{k_1},\tau)$$

$$[2n-1]G_n(\vec{k},\tau) - 3(\vec{k},\tau) = \sum_{m=1}^{n-1} \beta(\overrightarrow{k_1},\vec{k}_2)G_{n-m}(\vec{k}_2,\tau)G_m(\overrightarrow{k_1},\tau)$$

By solving the algebraic equation, we can get the recursion relation of $F_n(\vec{k},\tau)$ and $G_n(\vec{k},\tau)$:

$$F_n(\overrightarrow{q_1},\dots,\overrightarrow{q_{n-1}}) = \sum_{n=1}^{n-1} \frac{G_m(\overrightarrow{q_1},\dots,\overrightarrow{q_{n-1}})}{(2n+3)(n-1)}\Big((2n+1)\alpha(\overrightarrow{k_1},\vec{k}_2)F_{n-m}(\overrightarrow{q_{m+1}},\dots,\overrightarrow{q_n}) + \beta(\overrightarrow{k_1},\vec{k}_2)G_{n-m}(\overrightarrow{q_{m+1}},\dots,\overrightarrow{q_n})\Big)$$

$$G_n(\overrightarrow{q_1},\dots,\overrightarrow{q_{n-1}}) = \sum_{n=1}^{n-1} \frac{G_m(\overrightarrow{q_1},\dots,\overrightarrow{q_{n-1}})}{(2n+3)(n-1)}\Big(3\alpha(\overrightarrow{k_1},\vec{k}_2)F_{n-m}(\overrightarrow{q_{m+1}},\dots,\overrightarrow{q_n}) + n\beta(\overrightarrow{k_1},\vec{k}_2)G_{n-m}(\overrightarrow{q_{m+1}},\dots,\overrightarrow{q_n})\Big)$$

In linear regime, we have $F_n^{(s)} = G_n^{(s)} = 1$ which gives us an initial condition, by using the initial condition, we can get any order solution. For example, the simplest case, the second order:

$$F_2(\overrightarrow{k_1},\overrightarrow{k_2}) = \frac{5}{7} + \frac{2}{7}\frac{(\vec{k}_1\cdot\overrightarrow{k_2})^2}{k_1^2k_2^2} + \frac{\vec{k}_1\cdot\overrightarrow{k_2}}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

$$G_2(\overrightarrow{k_1},\overrightarrow{k_2}) = \frac{3}{7} + \frac{4}{7}\frac{(\vec{k}_1\cdot\overrightarrow{k_2})^2}{k_1^2k_2^2} + \frac{\vec{k}_1\cdot\overrightarrow{k_2}}{2}\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$$

2.3 statistics

In previous sections, I introduced Euler linear and non-linear dynamics. By using Euler dynamics, we can calculate density fields and velocity field in any constitution universe. However As the direct observable of the cosmological observation is the statistical correlation function. In order to compare the non-linear solution to observation, which means to compare our simulation result and real universe observation we need calculate the statistical correlation function.

2.3.1 initial condition

Euler dynamics can be used to calculate the evolution of universe, however it can't determine the evolution solely. If we want to simulate the real universe, initial condition is also important. By applying different initial condition, our universe structure may different.

In practice, the primordial curvature perturbation obeys a nearly Gaussian statistics. Thus, the linear density field $\delta(\vec{x})$, when it is in linear regime, also obeys the same statistics. So we need

investigate Gaussian statistics in cosmology.

2.3.2 Gaussian statistics

In order to describe the statistics of a field, we need to introduce a probability function $P\big(\delta(\vec{x})\big)$, which describes the probability of having a configuration of density field δ whose value is $\delta(\vec{x})$ at a point $\vec{x}$. Then we can use the conclusion of statistics, for Gaussian distribution the probability function $P\big(\delta(\vec{x})\big)$ is given by:

$$P\big(\delta(\vec{x})\big) = \frac{1}{Z}\exp\left[-\frac{1}{2}\int d^3\vec{y}\int d^3\vec{z}\delta(\vec{y})K(\vec{y},\vec{z})\delta(\vec{z})\right]$$

Here Z is the normalization overall constant. And $K(\vec{y},\vec{z})$ is the inverse of covariance matrix ${C_{\vec{y}\vec{z}}}^{-1}$. Where $C_{\vec{y}\vec{z}} = E\big(\vec{y} - \overrightarrow{\mu_y}\big)(\vec{z} - \overrightarrow{\mu_z})$.

2.3.2 correlation function and density power spectrum

With the probability distribution known, we can calculate the important function-n point correlation function. Which is calculated as:

$$\langle\delta(\overrightarrow{x_1})\dots\delta(\overrightarrow{x_n})\rangle = \int\left[D\big(\delta(\vec{x})\big)\right]\delta(\overrightarrow{x_1})\dots\delta(\overrightarrow{x_n})P\big(\delta(\vec{x})\big)$$

Where $D\big(\delta(\vec{x})\big)$ is the integration measure in the Hilbert space.

The special case two point correlation function. The two point correlation function is defined as the joint ensemble average of the density at two different locations.

$$\xi(\vec{r}) = \langle \delta(\vec{x})\delta(\overrightarrow{x+r}) \rangle$$

In Fourier space:

$$\delta(\vec{x}) = \int d^3\vec{k}\delta(\vec{k})e^{i\vec{k}\cdot\vec{x}}$$

Because $\delta(\vec{x})$ is a real function, so $\delta^*(\vec{k}) = \delta(-\vec{x})$. Then,

$$\langle \delta(\overrightarrow{k_1})\delta(\overrightarrow{k_2}) \rangle = \int \frac{d^3\vec{x}\, d^3\vec{r}}{(2\pi)^3(2\pi)^3} \langle \delta(\vec{x})\delta(\overrightarrow{x+r}) \rangle e^{-i(\overrightarrow{k_1}+\overrightarrow{k_2})\cdot\vec{x} - i\overrightarrow{k_2}\cdot\vec{r}}$$

$$= \int \frac{d^3\vec{x}\, d^3\vec{r}}{(2\pi)^3(2\pi)^3} \xi(\vec{r}) e^{-i(\overrightarrow{k_1}+\overrightarrow{k_2})\cdot\vec{x} - i\overrightarrow{k_2}\cdot\vec{r}}$$

$$= \delta_D(\overrightarrow{k_1} + \overrightarrow{k_2}) \int \frac{d^3\vec{r}}{(2\pi)^3} \xi(\vec{r}) e^{-i\overrightarrow{k_2}\cdot\vec{r}} = \delta_D(\overrightarrow{k_1} + \overrightarrow{k_2}) P(k)$$

Here $P(k)$ is the density power spectrum.

Chapter 3: Cosmological simulation: N body simulation method

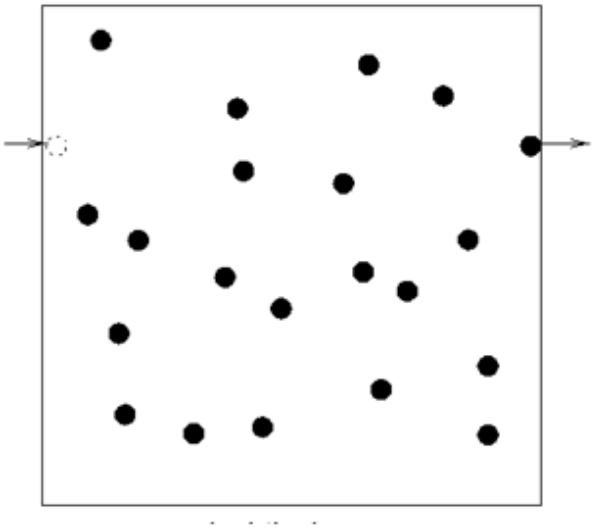
Cosmological simulation plays an extremely important role in testing our cosmological model, they are the only way to virtualize the evolution of our universe.

3.1: introduction

In practice, people only simulate certain part of universe. The task is to put N particles in an elementary volume, which is called simulation box. Distributing the N particles with initial condition, such as position and velocity.

Since amplitude of fluctuations at small scales is large and it drops at large scales. Fluctuations at scales much larger than 100 Mpc are generally not relevant for structure formation at the scale of galaxies. Thus the physical size corresponding to the periodic simulation box should be at least 100 Mpc,

The periodic boundary condition is adopted to solve the problem, that the simulation box have finite volume. The periodic boundary refers that "particles leaving the box on the one side are immediately entering the box again on the other side". As shown in the picture below.



3.2: Direct Summation method

Direct summation method is also called particle-particle method. Obviously, based on basic equation of motion, it is the method to calculate all the pairwise force and then make summation of all the force.

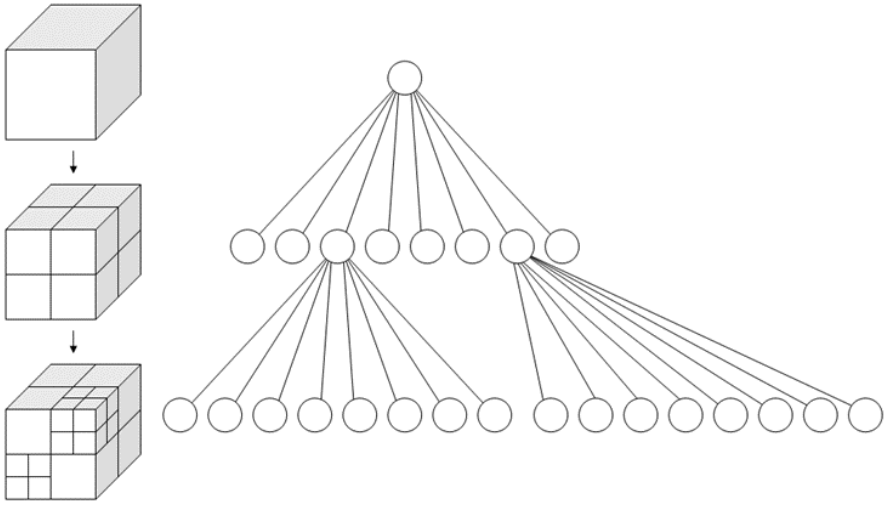
Most early simulation is based on this method. However it is difficult to implement periodic boundary conditions. The number of terms in the pairwise summation increases in proportion with N^2, where N is the number of particles. Which rapid increase in computational load with the number of particles. Most recent simulation adopt modified method to save computational load.

3.3: tree method

To avoid the huge computational spending, instead of calculate all the pairwise force, the force of a distant group of particles can

be approximated by the force due to a single pseudo particle located at the center of mass of the group, with mass equal to the total mass of the group of particles. This approximation changes the scaling of the number of calculations from N^2 to $NlogN$.

Tree method arranges particles into tree structure. The simulation volume is taken to be a cube and is divided into smaller cubes with 1/8 the volume each at every stage till the smallest cells have only one particle in them.



3.4: Particle-Meth method

In particle-Meth meth method, particles are used for representing the density and velocity field rather than the individual particles. We solve Poisson equation in Fourier space with the method of Fast Fourier Transforms, FFT requires sampling of functions at

uniformly spaced points, and a grid is used for this. Usually the simulation volume is taken to be a cube with equal number of grid points along each axis. Density contrast δ and the potential φ is defined on the grid for solving the Poisson equation.

Since the gird are fixed, as the simulation goed, gravity tends to clump things together, the particles flow from low density regions into high density regions. This leads to an excess of particles in certain cells whereas other cells are becoming more and more empty. We are left with the situation where we can not resolve structure formation on scales smaller than the cell spacing of the grid.

Improving force resolution in high density regions can improve the effectiveness of PM codes. Some other modified method are applied such as Adaptive Mesh Refinement, Particle-Particle + Particle-Mesh method, etc.