Holographic theory, and emergent gravity as an entropic force

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Abstract

In this report, we are going to study the holographic principle, and the possibility of describing gravity as an entropic force, independent of the details of the microscopic dynamics. We will first study some of the deep connections between Gravity laws and thermodynamics, which motivate us to consider gravity as an entropic force. Lastly, we will investigate some theoretical criticisms and observational evidences on emergent gravity.

Contents

1 Introduction

The holographic principle was inspired by black hole thermodynamics, which conjectures that the maximal entropy in any region scales with the radius squared, and not cubed as might be expected for any extensive quantities. We will start with a brief introduction on black hole thermodynamics in Section 2 Black holes thermodynamics and the holographic principle where we address the Hawking temperature and the black hole entropy formula, also known as the Bekenstein–Hawking formula. We then investigate some deep connections between Gravity laws and thermodynamics in Section 3 Connection between gravity and thermodynamics. We first consider two examples from ideal gas and Einstein's field equations near the black hole event horizon as motivations [1, 2]. Then we derive the Einstein's law of gravity from the entropy formula and the law of thermodynamics in all its glory following [1]. Afterwards, in Section 4 Gravity as an entropic force we follow [3] and explain gravity as an entropic force generated by changes in the information associated with the positions of material bodies in both non-relativistic and relativistic settings. With this logic in mind we recover the Newton's law of gravity in a simple non-relativistic setting and Einstein's field equation when we generalize these results to the relativistic case. What's more, when space is emergent even Newton's law of inertia and and Einstein's equivalence principle need to be reexplained. So we will give new interpretations on Newton's law of inertia and the meaning of acceleration (both concepts in the original theories are related to spacetime-specific notions such as coordinates) in this new emergent space scenario where only the fundamental concepts such as entropy and temperature are essential. The equivalence principle can also be explained more naturally and unavoidably in a theory in which space is emergent through a holographic scenario since both gravity and acceleration are emergent phenomena. Last but not least, we summarize some of the theoretical criticism and observational evidences about emergent gravity theory of [3] in Section 5 Theoretical and experimental criticism. Despite all of the theoretical evidences presented in this report, there is a decreasing research interest into the emergent gravity theories within the community in recent years as far as I can tell. Maybe the last section can provide some clues.

We will use the mostly pluses signature $(-, +, +, +)$ throughout and units with $k_B = c = \hbar = 1$.[1]

2 Black holes thermodynamics and the holographic principle

2.1 Hawking temperature, entropy

2.1.1 Uniformly accelerated observer and the Unruh effect

We are going to show in explicit details that for a uniformly accelerated observer [4], the ground state of an inertial observer is seen as a mixed state in thermodynamic equilibrium

[1] for the most part, since sometimes having $\hbar$ in our result explicitly for example can emphasize the its quantum nature.

with a non-zero temperature bath, and the observed frequency spectrum is given by familiar Planck's law with the temperature $T = 1/\beta = a/(2\pi)$, where a is the constant acceleration.

In the reference of a stationary observer, the four-acceleration of our uniformly accelerated observer is $a^\alpha = \ddot{x}^\alpha = (0, \boldsymbol{a})$, where $|\boldsymbol{a}| = a$ is constant. We can convert this condition into a covariant form

$$\eta_{\alpha\beta}\ddot{x}^\alpha\ddot{x}^\beta = a^2 \tag{1}$$

we also have the normalization condition

$$\eta_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = -1 \tag{2}$$

Without lose generality, let's assume that the trajectory is contained in the t–x plane, i.e. $\boldsymbol{a} = (a, 0, 0)$. Switching to the light-cone coordinates

$$u = t - x \quad \text{ and } \quad v = t + x \tag{3}$$

the line elements can be written as (suppress the transverse coordinates y and z)

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 = -\mathrm{d}u\mathrm{d}v \tag{4}$$

The acceleration equation 1 and the normalization equation 2 can be written as

$$\ddot{u}\ddot{v} = -a^2 \,\text{and}\, \dot{u}\dot{v} = 1 \tag{5}$$

The two equations in 5 can be combined to eliminate one of the variable say u, we obtain

$$\frac{\ddot{v}}{\dot{v}} = \pm a \tag{6}$$

This is integrated to be

$$v(\tau) = \frac{A}{a}\exp(a\tau) + C \tag{7}$$

and, using $\dot{u} = 1/\dot{v}$ we got

$$u(\tau) = -\frac{1}{Aa}\exp(-a\tau) + D \tag{8}$$

go back to the original Cartesian coordinates, we obtain[2]

$$t(\tau) = \frac{1}{a}\sinh(a\tau) \text{ and } x(\tau) = \frac{1}{a}\cosh(a\tau) \tag{9}$$

where we have chosen the integration constants such that the initial conditions $t(0) = 0$ and $x(0) = 1/a$ are satisfied.

Now consider a monochromatic wave for the inertial observer

$$\phi(k) \propto \exp(-\mathrm{i}\omega(t - x)) \tag{10}$$

[2]This result is also in one of our homework questions.

An accelerated observer does not see a monochromatic wave however, but a superposition of plane waves with varying frequencies. We can see this by inserting the trajectory 9 into the monochromatic wave

$$\phi(\tau) \propto \exp\left[-\frac{\mathrm{i}\omega}{a}[\sinh(a\tau) - \cosh(a\tau)]\right] = \exp\left[\frac{\mathrm{i}\omega}{a}\exp(-a\tau)\right] \tag{11}$$

As the next step, we want to determine its power spectrum $P(\nu) = |\phi(\nu)|^2$ and relate it to the Plank's formula. First let us calculate the wave 11 in the Fourier space $\phi(\nu)$. According to the Fourier transform, we obtain

$$\phi(\nu) = \int_{-\infty}^{\infty} \mathrm{d}\tau \phi(\tau) \mathrm{e}^{-\mathrm{i}\nu\tau} = \int_{-\infty}^{\infty} \mathrm{d}\tau \exp\left(\frac{\mathrm{i}\omega}{a}\exp(-a\tau)\right)\mathrm{e}^{-\mathrm{i}\nu\tau} \tag{12}$$

change variable to $y = \exp(-a\tau)$, we obtain

$$\phi(\nu) = \frac{1}{a}\int_0^{\infty} \mathrm{d}y y^{\mathrm{i}\nu/a-1}\mathrm{e}^{\mathrm{i}(\omega/a)y} \tag{13}$$

we notice the above integral is similar to the Gamma function integral

$$\int_0^{\infty} \mathrm{d}t t^{z-1}\mathrm{e}^{-bt} = b^{-z}\Gamma(z) = \exp(-z\ln b)\Gamma(z) \tag{14}$$

except our integral 13 is not convergent. Here we added an infinitesimal positive real quantity ε to ensure the convergence. Combine equations 13 and 14, we obtain

$$\phi(\nu) = \frac{1}{a}\cdot\exp\left(-i\nu/a\ln(-i\omega/a + \epsilon)\right)\cdot\Gamma(i\nu/a) \tag{15}$$

where we also have $\lim_{\varepsilon\to 0}\ln\left(-\frac{\mathrm{i}\omega}{a} + \varepsilon\right) = \ln\left|\frac{\omega}{a}\right| - \frac{\mathrm{i}\pi}{2}\,\mathrm{sign}(\omega/a)$. Use this we obtain

$$\phi(\nu) = \frac{1}{a}\left(\frac{\omega}{a}\right)^{-\mathrm{i}\nu/a}\Gamma(\mathrm{i}\nu/a)\mathrm{e}^{-\pi\nu/(2a)} \tag{16}$$

for positive ω/a. Thus, the power spectrum is

$$P(\nu) = |\phi(\nu)|^2 = \frac{1}{a^2}\mathrm{e}^{-\pi\nu/a}\Gamma(\mathrm{i}\nu/a)\Gamma(-\mathrm{i}\nu/a) \tag{17}$$

use Euler's reflection formula

$$\Gamma(\mathrm{i}\nu/a)\Gamma(-\mathrm{i}\nu/a) = \frac{-\pi}{\frac{\mathrm{i}\nu}{a}\cdot\sin(\mathrm{i}\pi\frac{\nu}{a})} = \frac{\pi a}{\nu\sinh\frac{\pi\nu}{a}} \tag{18}$$

equation 17 now becomes

$$P(\nu) = \frac{\pi}{a^2}\frac{\mathrm{e}^{-\pi\nu/a}}{(\nu/a)\sinh(\pi\nu/a)} = \frac{\beta}{\nu}\frac{1}{\mathrm{e}^{\beta\nu}-1} \tag{19}$$

with $\beta = 2\pi/a$. This is the Planck's formula. Therefore, classically, a uniformly accelerated detector will measure a thermal Planck spectrum with temperature $T = 1/\beta = a/(2\pi)$. This phenomenon is called Unruh effect [5].

This result can also be easily generalized to quantum field theory. Quantum mechanically, in the canonical quantization formalism[3], we can follow the same steps and consider instead any quantum fields with harmonic expansions similar to the classical monochromatic wave we considered above, but with integration over all available frequency modes. As a result, the vacuum expectation value of the number density operator of scalar particles detected by a uniformly accelerated observer for example is [4]

$$\langle \tilde{n}_\Omega \rangle = \frac{1}{\exp(2\pi\Omega/a) - 1} \tag{20}$$

which is again, the black body radiation with the temperature given by $T = 1/\beta = a/(2\pi)$.

In special relativity, an observer moving with uniform proper acceleration through Minkowski spacetime is conveniently described with Rindler coordinates, which are related to the standard (Cartesian) Minkowski coordinates by

$$\begin{aligned} x &= \rho \cosh(\sigma) \\ t &= \rho \sinh(\sigma) \end{aligned} \tag{21}$$

The line element in Rindler coordinates is

$$\mathrm{d}s^2 = -\rho^2 \mathrm{d}\sigma^2 + \mathrm{d}\rho^2 \tag{22}$$

where $\rho = \frac{1}{a}$ and $\sigma = a\tau$ for an uniformly accelerated observer, see equation 9.

In the next section where we derive the black hole entropy, we are going to massage the Schwarzschild metric outside a black hole into the above 22 form and interpret the resulting Unruh temperature of stationary observer at the spatial infinity as the well-known Hawking temperature. Combine the Hawking temperature and the second law of thermodynamics we can finally derive the black hole entropy formula.

2.1.2 Hawking temperature and black hole entropy

As briefly outlined above, in order to calculate the black hole entropy, let's start with the Schwarzschild metric [2, 7]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{23}$$

where $f(r) \equiv 1 - 2GM/r$. Let's consider the region near (but outside) the horizon $r = r_s = 2GM$, $f(r) \overset{r \to r_s}{=} f'(r_s)(r - r_s)$. Introducing the proper distance ρ

$$d\rho = \frac{dr}{\sqrt{f}} \tag{24}$$

we can write

$$f(r) = K^2\rho^2 + \cdots \tag{25}$$

[3]In the path integral formalism, we can arrive the result by utilizing the formalism similarity between quantum statistical mechanics and quantum field theory, i.e. the generating function in quantum field theory v.s. the partition function in statistical mechanics [2, 6].

where $K \equiv \frac{1}{2} f'(r_s) = \frac{1}{4G_N M}$ is the surface gravity. So near the horizon, the metric 23 can be rewritten as

$$ds^2 = -K^2 \rho^2 dt^2 + d\rho^2 + r_s^2 d\Omega_2^2 = -\rho^2 d\eta^2 + d\rho^2 + r_s^2 d\Omega_2^2 \tag{26}$$

with $\eta = Kt = \frac{t}{2r_s}$. This metric is similat to the Rindler metric 22 except for the last term that is not relevant for our purposes. An observer at $r = \text{const}$ is equivalent to an observer with $\rho = \text{const}$. Such an observer has a constant proper acceleration $a = \frac{1}{\rho}$, and the acceleration seen by O_∞ is redshifted to be $a_\infty = a(r) f^{1/2}(r) = \frac{1}{\rho} \cdot K\rho = K$.

The local temperature at ∞ (T_H) can be calculated by the acceleration and Unruh effect

$$T_H = \frac{a}{2\pi} = \frac{1}{8\pi GM} = \frac{\hbar c^3}{8\pi GM} \tag{27}$$

which is the Hawking temperature of the black hole!

With the black hole temperature, it is straightforward to calculate the black hole entropy. The change in entropy when a quantity of heat dQ is added is

$$\mathrm{d}S = \frac{\mathrm{d}Q}{T} = \frac{8\pi GM}{\hbar c^3} \mathrm{d}Q = \frac{8\pi GM dM}{\hbar c} = \frac{c^3}{4\hbar G} 4\pi d \left(\frac{2GM}{c^2}\right)^2 = \frac{dA}{4 \cdot \ell_\mathrm{P}^2} \tag{28}$$

where $A = 4\pi r_s^2$ is the event horizon area, and $\ell_\mathrm{P} = \sqrt{G\hbar/c^3}$ is the Planck length. This means

$$\mathrm{S} = \frac{A}{4 \cdot \ell_\mathrm{P}^2} \tag{29}$$

This is the Bekenstein–Hawking formula!

2.2 Holographic principle

The entropy of a thermodynamics system is normally extensive and proportional to its volume. However, as shown in equation 28, the entropy of a black hole is proportional to its area. It is as if the entropy/information of a black hole were to reside completely on its surface. The holographic principle was exactly inspired by this remarkable result [8, 9], which conjectures that the maximal entropy in any region scales with the radius squared, and not cubed as might be expected. We are going to show this result in the following with a thought experiment.

Consider [7,10] an isolated system mass E and entropy S_0 in an asymptotic flat spacetime. Let A be the area of the smallest sphere hat encompasses the system, and M_A to be the mass of a black hole with the same horizon area. We must have $E < M_A$, otherwise the system would be already a black hole. Now add $M_A - E$ energy to the system while keeping the area fixed, we shall obtain a black hole with mass M_A. Since the second law of thermodynamics, we have

$$S_{BH} \leq S_0 + S' \tag{30}$$

where S' is the entropy of the added entropy. We then conclude

$$S_0 \leq S_{BH} = \frac{A}{4\ell_\mathrm{P}^2} \tag{31}$$

That is to say the maximum entropy inside a region bounded by area A is $\frac{A}{4\ell_{\mathrm{P}}^2}$.

This completes our derivation of the black hole entropy formula and the holographic principle. It's the aim of this report to re-interpret Newton and Einstein's laws of gravity in a non-relativistic and relativistic setting respectively with some basic assumptions such as the holographic principle and emergent spacetime.

However before we jump in right away, in order to motivate our study on the emergent gravity scenario, let us investigate some deep connections between Gravity laws and the laws of thermodynamics first. In particular, we are going to re-derive the Einstein's law of gravity as an *equation of state* assuming the holographic principle and thermodynamics in the next section.

3 Connection between gravity and thermodynamics

3.1 Motivations (from ideal gas and GR near event horizon)

There is a deep connection between Gravity laws and the laws of thermodynamics. It has been shown in [1, 2] that one can derive Einstein's field equation as an equation of state by combining the Bekenstein–Hawking formula 28 and the fundamental relation $\delta Q = TdS$ relating heat, entropy, and temperature in thermodynamics.

To see logically how this is possible, let us consider a similar example in the context of ideal gas [1]. Our goal is to derive the "field equation" of any thermodynamic systems given its entropy formula $S(E, V)$ and the thermodynamics relation $dE = TdS - PdV$. In general we have

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V \text{ and } \frac{P}{T} = \left(\frac{\partial S}{\partial V}\right) \tag{32}$$

and in the case of ideal gas, the entropy is given by the logarithm of the number of possible states, i.e. it goes like $S = N \log V + f(E)$ with some function $f(E)$. The above thermodynamics relation 32 then yield $PV = NT$, which is the well-known equation of state of an ideal gas.

Viewed in this way, if we can derive Einstein's field equation from the entropy formula (holographic principle) and general rules of thermodynamics, the Einstein equation is demoted to the status of an equation of state just like the ideal gas example. This perspective suggests that it may be no more appropriate to canonically quantize the Einstein equation than it would be to quantize the wave equation for sound in air [1], and one of the apparent difficulties in quantizing gravity than quantizing the ideal gas for example, I guess, is the non-linearly in the Einstein's equation.

In deriving the Einstein's equation as an equation of state, the key idea is to demand the thermodynamic relation $\delta Q = TdS$ holds for all local Rindler causal horizons through each spacetime point with δQ and T interpreted as the energy flux and Unruh temperature seen by an accelerated observer just inside the horizon. This requires that gravitational lensing by matter energy distorts the causal structure of spacetime in just such a way that the Einstein equation holds. Let's see how this is done intuitively.

Let's consider an infinitesimal amount of heat δQ going through a local Rindler causal horizon through an arbitrary spacetime point. δQ is given by integrating the energy flux, which depends on the energy-momentum tensor $T_{\mu\nu}$. On the other hand, the right hand side of the relation $\delta Q = TdS$ is related to the area change δA through the Bekenstein–Hawking formula, which itself is determined by the curvature of spacetime $R_{\mu\nu}$ through the Raychaudhuri equation. So as a result of this procedure, we have a equation relating the energy momentum tensor and the Ricci tensor. This relation turns out to be Einstein's field equation, as we will show in the following.

Before we show the derivation explicitly, let's look at yet another interesting case where you can clearly see the deep intricate relation between Gravity and Thermodynamics provided by Prof. Padmanabhan [2].

Let's consider a static, spherically symmetric horizon, in a spacetime described by a metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{33}$$

notice unlike equation 23, this doesn't have to be Schwarzschild metric. We can follow the same steps as section 2.1.2 and show that the metric reduces to the Rindler metric near the horizon in the $r-t$ plane with the surface gravity $K = \frac{1}{2}f'(a)$ and Unruh temperature

$$T_H = \frac{a}{2\pi} = \frac{f'(a)}{4\pi} \tag{34}$$

where $f(a) = 0$ and $f'(a) \neq 0$.

Given the metric 33, we can write down the Einstein equation following the normal procedure. We obtain

$$(1-f) - rf'(r) = -(8\pi G)\, Pr^2 \tag{35}$$

with $P = T^r_r$ is the radial pressure (spherically symmetric). Evaluate on the horizon $r = a$ and use the fact that on the horizon $f(a) = 0$, we get the result

$$\left[\frac{1}{2}f'(a)a - \frac{1}{2}\right]/G = 4\pi Pa^2 \tag{36}$$

multiply the above equation by da on both sides and integrate we get

$$\frac{\hbar c f'(a)}{4\pi}\frac{c^3}{G\hbar}d\left(\frac{1}{4}4\pi a^2\right) - \frac{1}{2}\frac{c^4 da}{G} = Pd\left(\frac{4\pi}{3}a^3\right) \tag{37}$$

where we have restored all the $\hbar$ and c. This is exactly the thermodynamics relation $TdS - dE = PdV$ if we recognize

$$S = \frac{c^3}{4G\hbar}\left(4\pi a^2\right) = \frac{1}{4}\frac{A_H}{\ell_{\mathrm{P}}^2}; \quad E = Mc^2 = \frac{ac^2}{2G}c^2 = \frac{c^4}{2G}a \tag{38}$$

where A_H is the horizon area, and $\ell_{\mathrm{P}} = \sqrt{G\hbar/c^3}$ is the Planck length as shown in the Bekenstein–Hawking formula above 28. The result shows that the Einstein's field equations evaluated on the horizon reduces to a thermodynamic identity. Some people believe this is not a simple reversing of the arguments or a coincidence, but due to a much deeper connection between Gravity and Thermodynamics.

3.2 Derive Einstein's law of gravity from the entropy formula and thermodynamics

Okay now let's shut up and calculate. We are going to re-derive the Einstein's field equation relating the energy momentum tensor to the Ricci tensor from the Bekenstein–Hawking's entropy formula and the law of thermodynamics. As shown in the intuitive discussion above, we are going to need the Raychaudhuri equation relating the amount by which light rays converge or diverge with the spacetime curvature. So first of all, let's derive the Raychaudhuri equation from the geodesic equation.[4]

3.2.1 the Raychaudhuri equation

Let's give a derivation of the Raychaudhuri equation we are going to use. Consider [6] a bundle of time-like geodesics $x^\mu\left(\tau,\sigma^1,\sigma^2,\sigma^3\right)$ labeled by three real value $\sigma^{1,2,3}$. Pick a point P on one specific geodesic and the tangent vector is $V^\mu=\frac{dx^\mu}{d\tau}$. We also have the geodesic equation $V^\mu D_\mu V^\nu=0$. Now consider the three deviation vectors $W^\mu=\frac{dx^\mu}{d\sigma}$, then

$$\frac{DW^\mu}{D\tau}=V^\nu D_\nu W^\mu=W^\nu D_\nu V^\mu\equiv B^\mu_\nu W^\nu \tag{39}$$

where the last equality serve only as the definition of B^μ_ν, which tell us the rate of change of W. Also, the second equality comes from the fact that the Lie derivative of the deviation vectors W^μ vanishes

$$\mathcal{L}_V W^\mu=V^v\partial_\nu W^\mu-W^\nu\partial_\nu V^\mu=V^v D_\nu W^\mu-W^\nu D_\nu V^\mu=\frac{dW^\mu}{d\tau}-\frac{dV^\mu}{d\sigma}=\frac{d^2x^\mu}{d\sigma d\tau}-\frac{d^2x^\mu}{d\tau d\sigma}=0 \tag{40}$$

it follows that $V^v D_\nu W^\mu=W^\nu D_\nu V^\mu$, which is the second equality in equation 39.

The idea is to compute the rate of change for $B_{\mu\nu}$. First of all, we can decomposed a generic 2-index tensor into three irreducible parts

$$B_{\mu\nu}=\sigma_{\mu\nu}+\frac{1}{3}\theta P_{\mu\nu}+\omega_{\mu\nu} \tag{41}$$

where $P_{\mu\nu}\equiv g^{\mu\nu}+V^\mu V^\nu$ is the projection operator in the three dimensional subspace spanned by $\sigma^{1,2,3}$. To see this, we can easily check $P^{\mu\nu}V_\nu=0, P^{\mu\nu}P^\lambda_\nu=P^{\mu\lambda}$. The traceless symmetric part $\sigma_{\mu\nu}\equiv\frac{1}{2}\left(B_{\mu\nu}+B_{\nu\mu}\right)-\frac{1}{3}\theta P_{\mu\nu}$ describes shear, the antisymmetric part $\omega_{\mu\nu}=\frac{1}{2}\left(B_{\mu\nu}-B_{\nu\mu}\right)$ describes rotation, and the trace part $\theta=P^{\mu\nu}B_{\mu\nu}=D_\mu V^\mu$ describes expansion.

Now let's differentiate $B_{\mu\nu}$, we get

$$\begin{aligned}\frac{DB_{\mu\nu}}{D\tau}&=V^\lambda D_\lambda B_{\mu\nu}=V^\lambda D_\lambda D_\nu V_\mu=V^\lambda D_\nu D_\lambda V_\mu+V^\lambda\left[D_\lambda,D_\nu\right]V_\mu\\&=D_\nu\left(V^\lambda D_\lambda V_\mu\right)-\left(D_\nu V^\lambda\right)\left(D_\lambda V_\mu\right)-V^\lambda R^\sigma_{\mu\lambda\nu}V_\sigma\\&=-B_{\mu\lambda}B^\lambda_\nu-R_{\sigma\mu\lambda\nu}V^\sigma V^\lambda\end{aligned} \tag{42}$$

[4] I did not know this material will be discussed in the lectures while I was working on this draft. The following derivation found in A. Zee's book [6] is quite similar to the lecture note.

where we have used the product rule in the second to last equality, and in the last equality the first term is zero because of the geodesic equation. So we can see, indeed, in the equation governing $B_{\mu\nu}$, the Riemann curvature tensor appears. This equation, known as the Raychaudhuri equation, tells us how $B_{\mu\nu}$ varies as we move along a geodesic.

In the next section where we derive the Einstein's equation from the entropy formula and the law of thermodynamics, we are interested in how the expansion parameter θ changes, i.e. how a bundle of geodesics converges (or diverges). Since $\frac{D\theta}{D\tau} = g^{\mu\nu}\frac{DB_{\mu\nu}}{D\tau}$, contrast equation 42 with $g^{\mu\nu}$ we can get an equation for $\frac{D\theta}{D\tau}$. The first term on the right hand side becomes

$$\begin{aligned} g^{\mu\nu}B_{\mu\lambda}B^{\lambda}_{\nu} = B_{\mu\nu}B^{\nu\mu} &= \left(\sigma_{\mu\nu} + \frac{1}{3}\theta P_{\mu\nu} + \omega_{\mu\nu}\right)\left(\sigma^{\mu\nu} + \frac{1}{3}\theta P^{\mu\nu} - \omega^{\mu\nu}\right) \\ &= \sigma_{\mu\nu}\sigma^{\mu\nu} + \frac{1}{3}\theta^2 - \omega_{\mu\nu}\omega^{\mu\nu} \end{aligned} \tag{43}$$

We thus obtain the desired result relating the expansion parameter θ and the Ricci tensor

$$\frac{D\theta}{D\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}V^{\mu}V^{\nu} \tag{44}$$

We are interested in non-rotational geodesics in what follows, so we have $\omega^{\mu\nu} = 0$. We finally obtain the equation for the expansion for time-like geodesics

$$\frac{d\theta}{d\lambda} = -\frac{1}{3}\theta^2 - \sigma^2 - R_{\mu\nu}V^{\mu}V^{\nu} \tag{45}$$

For null geodesics [11], however, the expansion equation is slightly different. First of all, we need to define the null tangent vector with an affine parameter λ with $dx^a = k^a d\lambda$ with $k^a k_a = 0$ and $k^a\xi_a = 0$ (with ξ being the deviation vectors). We need to change the projection operator since now $P^{\mu\nu}k_\nu \neq 0$ is no longer true. Instead, we need to introduce an auxiliary null vector N^ν with $k_\nu N^\nu = -1$. Then the projection operator now can be chosen to be $P_{\mu\nu} = g_{\mu\nu} + k_\mu N_\nu + k_\nu N_\mu$. This satisfies $k^\mu P_{\mu\nu} = 0$ and $N^\mu h_{\mu\nu} = 0$. This means that $P_{\mu\nu} = 0$ construsted this way will be two dimensional instead of three dimensional as we have in the time-like geodesics case above. The expansion equation for the case of null geodesics is

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b \tag{46}$$

And this is the equation we are going to utilize in the next section.

3.2.2 Einstein's equation from entropy formula and thermodynamics

First of all, let's specify our system [1]. The equivalence principle is invoked to view a small neighborhood of each spacetime point p as a piece of flat spacetime. It is always possible to choose a small spacelike 2-surface element P through p so that the expansion and shear vanish in a first order neighborhood of p. Therefore the θ^2 and σ^2 terms in the expansion equation 46 are higher order contributions that thus can be neglected compared with the last term when integrating to find the expansion θ near P. This integration yields

$$\theta = -\lambda R_{ab}k^a k^b \tag{47}$$

for small enough λ.

As illustrate above, for the small spacelike 2-surface element P one has an approximately flat region of spacetime with the usual Poincare's symmetries according to the equivalence principle. Also, there is an approximate Killing vector field χ^a generating boosts orthogonal to P and vanishing at P. What's more, according to the Unruh effect 27, the boost observer will experience a thermal state with temperature $T = \frac{\kappa}{2\pi}$, where κ is the acceleration of the Killing orbit on which the norm of χ^a is unity. The heat flow then is defined by the boost-energy current of the matter, $T_{a,b}\chi^a$, where $T_{a,b}$ is the energy momentum tensor.

Now consider any local Rindler horizon through a spacetime point p with Killing vector χ^a generating this horizon. The heat flux is given by the following integral

$$\delta Q = \int_{\mathcal{H}} T_{ab}\chi^a d\Sigma^b \tag{48}$$

In terms of the tangent vector k_a, the Killing vector and the area element can be written as

$$\chi^a = -\kappa\lambda k^a \text{ and } d\Sigma^a = k^a d\lambda d\mathcal{A} \tag{49}$$

where κ is the acceleration.

Assuming the Bekenstein–Hawking relation 28, the entropy change the heat exchange can be written as $dS = \delta\mathcal{A}/4\ell_{\mathrm{P}}^2$, where $\delta\mathcal{A}$ is the area variation. The area variation $\delta\mathcal{A}$ can be further written in terms of the expansion parameter θ as $\delta\mathcal{A} = \int_{\mathcal{H}} \theta d\lambda d\mathcal{A}$. The entropy change then is

$$dS = \frac{\int_{\mathcal{H}} \theta d\lambda d\mathcal{A}}{4\ell_{\mathrm{P}}^2} = -\frac{\int_{\mathcal{H}} \lambda R_{ab}k^a k^b d\lambda d\mathcal{A}}{4\ell_{\mathrm{P}}^2} \tag{50}$$

where we have used equation 47 in the last equality.

Now we have equations for the heat flux δQ and entropy change dS in equations 48 and 50. We can simply relate them through the law of thermodynamics $\delta Q = TdS$. Notice that $\ell_{\mathrm{P}}^2 = G$ in natural unit, we obtain

$$T_{ab}k^a k^b = \frac{1}{8\pi G} R_{ab}k^a k^b \tag{51}$$

for any null-like k_a. This implies $T_{ab} = \frac{1}{8\pi G}R_{ab} + f g_{ab}$ for some f. Local conservation of the original energy momentum tensor implies that T_{ab} is divergence free and therefore this posts a constraint on the additional function f. Using the contracted Bianchi identity we obtain $f = -R/2 + \Lambda$ for some constant Λ. We thus obtain the Einstein equation with the mysterious cosmological constant term

$$R_{ab} - \frac{1}{2}R g_{ab} + \Lambda g_{ab} = 8\pi G T_{ab} \tag{52}$$

We thus see if we assume the equilibrium thermodynamic relation $\delta Q = TdS$ holds, where δQ is interpreted as the energy flux through the local Rindler horizons and dS is propotional to the area change, the Einstein equation holds. It seems like the physics of gravity has to be arranged in precisely such a way as the form of Einstein's equation so that the formula $S \propto A$ (instead of say $S \propto V$) holds.

4 Gravity as an entropic force

4.1 General philosophy

As we have seen in the previous sections, there exists a deep and mysterious connection between the field equations of gravity and the horizon thermodynamics [1,2]. In the following we will argue that the central notion needed to derive gravity is information. More precisely, it is the amount of information associated with matter and its location, in whatever form the microscopic theory likes to have it, measured in terms of entropy. Changes in this entropy when matter is displaced leads to an entropic force [3], which as we will show takes the form of gravity. In this way, gravity should be understood from general principles that are independent of the specific details of the underlying microscopic theory, which coincides with the universality of gravity. In the following, we are going to start from first principles, using only spacetime-independent concepts like energy, entropy and temperature to show that Newton and Einstein's laws of gravity appear naturally and practically unavoidably.

In order to interpret gravity as an entropic force caused by a change in the amount of information associated with the positions of bodies of matter, we are going to use the holographic principle and assume that the amount of information associated with a given spatial volume is proportional to the area. On the other hand, the energy, that is equivalent to the matter, is distributed evenly over the degrees of freedom, and thus leads to a temperature. We then derive the entropic force associated with this process, which turns out to be the Newton's law of gravity. We are also going to extent this reasoning in a relativistic setting to derive the Einstein's equation.

4.2 Entropic force in general

Before we derive Newton and Einstein's Gravity laws as entropic forces, let's first look at entropic force in general and see how it arises in thermodynamics.

An entropic force acting in a system is an emergent phenomenon resulting from the entire system's statistical tendency to increase its entropy, rather than from a particular underlying force on the atomic scale. In particular, unlike the fundamental interactions such as electromagnetic and weak forces, there is no fundamental field associated with an entropic force. For definiteness [3], let's consider a system with entropy equals to $S(E,x) = k_B \log \Omega(E,x)$, where $\Omega(E,x)$ is the number of microscopic states. In the canonical ensemble the force F is introduced in the partition function

$$Z(T,F) = \int dEdx\Omega(E,x)e^{-(E+Fx)/k_BT} \tag{53}$$

the force F required to keep the system at equilibrium at state x, E is

$$F = T\frac{\partial S}{\partial x} \tag{54}$$

We can see from the above equation that entropic forces result from entropy gradient and always point in the direction of increasing entropy, and it is also proportional to the temperature of the system.

4.3 Newton's law of gravity as an entropic force

We now derive the Newton's gravity law as an entropic force. First of all, from the holographic principle, the description of a volume of space can be thought of as N bits of binary information, encoded on a boundary to that region, a closed surface of area A. Each bit of information requires an area equals to the Planck area $\ell_{\rm P}^2$, and the information is evenly distributed on the surface. Thus we get

$$N = \frac{A}{\ell_{\rm P}^2} = \frac{Ac^3}{\hbar G} \tag{55}$$

In order to derive Newton's law of gravity, lets look at a flat non-relativistic space. Let's consider a small piece of an holographic screen at the boundary, and a particle of mass m that approaches it from the side at which spacetime has already emerged. Let's follow Bekenstein's original thought experiments and postulate when a particle is one Compton wavelength from the holographic screen, it is considered to be part of the area. Let's postulate that the change of entropy associated with the information on the boundary equals

$$\Delta S = 2\pi k_B \text{ when } \Delta x = \frac{\hbar}{mc} \tag{56}$$

or we can write it in a more general form and assume

$$\Delta S = 2\pi k_B \Delta x \frac{mc}{\hbar} \tag{57}$$

This will be our basic assumption about the entropy change that is associated with the displacement of matter. Before we move on, let's first give an motivation about the above postulate. First of all, the entropic force satisfies

$$F\Delta x = T\Delta S \tag{58}$$

combine this with our postulate 57 and use the Unruh temperature 27 here as T, we get $F = ma$, where a denotes the acceleration. So we have shown that starting from the postulate 57, we recover the Newton's second law. Keep in mind that in the entropic gravity scenario we are considering, we will discard all the quantities related to spacetime (as it is emergent and not fundamental), such as coordinates and acceleration, while keeping quantities like entropy and temperature as more fundamental. As a result, we will use the Newton-equivalent entropy postulate 57 as our starting point from now on.

Now we are going to derive the Newton's gravity law from the postulate 57. First of all, combine with $F\Delta x = T\Delta S$, we get

$$F = \frac{2\pi k_B mc}{\hbar} \cdot T \tag{59}$$

Let's find what T is. Since we are interested in thermal equilibrium, from the statistical equipartition theorem, the temperature of a system with N degrees of freedom and energy E (which in our case is equal to the mass of the material) is

$$T = \frac{2E}{k_B N} = \frac{2Mc^2}{k_B N} = \frac{2Mc^2 \ell_{\rm P}^2}{k_B A} \tag{60}$$

where we have used the formula for the number of degrees of freedom 55. Substitute 60 into 59 and use the fact that $A = 4\pi r^2$, we get

$$F = \frac{2\pi k_B mc}{\hbar} \cdot \frac{2Mc^2 \frac{\hbar G}{c^3}}{k_B 4\pi r^2} = \frac{GMm}{r^2} \tag{61}$$

We have recovered Newton's law of gravitation! You may argue, as I said above, that the reason why we can recover the Newton's law is simply that we reversed all of the usual arguments. This might be true, but the more important message here is the underlying logic, which states the origin of gravity: it is an entropic force. If true, this should have profound consequences, as we will discuss in the following sections.

4.4 Einstein's law of gravity as an entropic force

In order to derive Einstein's law of gravity as an entropic force, let's consider a static background with a global time-like Killing vector ξ_a. We need a factor e^ϕ representing the redshift factor that relates the local time coordinate to that at a reference point with $\phi = 0$, where ϕ is the gravitational field. A natural generation of the Newton's potential in general relativity is [12]

$$\phi = \frac{1}{2} \log\left(-\xi^a \xi_a\right) \tag{62}$$

The four velocity and acceleration can be written as [5]

$$u^b = e^{-\phi}\xi^b, \quad a^b \equiv u^a \nabla_a u^b = e^{-2\phi}\xi^a \nabla_a \xi^b \tag{63}$$

we can rewrite the acceleration by using the Killing equation $\nabla_a \xi_b + \nabla_b \xi_a = 0$ and the expression for ϕ. We get

$$a^b = e^{-2\phi}\xi^a \nabla^b \xi_a = \frac{1}{2} e^{-2\phi} \nabla^b \left(\xi_a \xi^a\right) = -\frac{1}{2} e^{-2\phi} \nabla^b e^{2\phi} = -\nabla^b \phi \tag{64}$$

Similar to the non-relativistic case 57 in the previous section, we are going to follow Bekenstein and postulate that the change of entropy at the screen is $2\pi k_B$ for a displacement by one Compton wavelength normal to the screen

$$\nabla_a S = -2\pi k_B \frac{mc}{\hbar} N_a \tag{65}$$

where N_a is a unit outward pointing vector perpendicular to the screen S and to ξ_b. The extra minus sign compared to 57 comes from the fact that the entropy gradient is in the opposite direction as the outward vector (i.e. entropy increases when we cross from the outside to the inside). Same logic as the non-relativistic case, in order to give a motivation to postulate 65, we can look at the Unruh temperature and see that the resultant entropic force leads to Newton's $F = ma$. More specifically, a natural generation of the Unruh temperature

$$T = \frac{1}{2\pi} e^\phi N^b \nabla_b \phi \tag{66}$$

[5] I do not not quite understand why we are ignoring the term proportional to $\nabla_a \phi$ in the acceleration. The original source I found is Equ. 11.2.3 in Wald's book [12] where he talked about the concept of "staying in place".

where we have added the extra redshift factor e^{ϕ} and the unit vector N^b. Then the entropic force in this system is

$$F_a = T\nabla_a S = -me^{\phi}\nabla_a\phi \tag{67}$$

which is exactly $F = ma$ 64 with an additional factor e^{ϕ} due to the redshift.

Now let's derive Einstein's field equation as an entropic force. First of all, we immediately see that we cannot apply the same logic as the non-relativistic case since now we want to find a field equation relating the energy momentum content and the curvature of spacetime instead of a law for the 'force'. We have seen above the local temperature 66 and our postulate 65 are equivalent. Now let's start with the holographic principle. We assume that holographic screen is enclosing a certain static mass configuration with total mass M. The bit density on the screen is then given by

$$dN = \frac{dAc^3}{G\hbar} \tag{68}$$

Again by equipartition we have

$$Mc^2 = \frac{1}{2}k_B\int_{\mathcal{S}} TdN \tag{69}$$

substitute in equations 66 and 68 we get

$$M = \frac{1}{4\pi G}\int_{\mathcal{S}} e^{\phi}\nabla\phi\cdot dA \tag{70}$$

which is precisely Komar's definition of the mass contained inside an arbitrary volume inside any static curved space time. It can be derived by assuming the Einstein's field equation. In our reasoning, however, we are coming from the other side. In the following I will follow chapter 11 in book [12] to derive Einstein's field equation from Komar's mass formula 70.

Start with equation 70. Notice the integral is taken over any topological 2-sphere, which encloses all of the sources. Let's use the equality 64 to write the right hand side integral as an integration over the Killing vector

$$M = -\frac{1}{4\pi G}\int_{\mathcal{S}} e^{-\phi}\xi^a\nabla_b\xi_a N^b dA \tag{71}$$

Use Killing's equation $\nabla_a\xi_b = \nabla_{[a}\xi_{b]}$, we can write the above as

$$M = -\frac{k_B}{8\pi G}\int_{\mathcal{S}} N^{a,b}\nabla_{[a}\xi_{b]}dA \tag{72}$$

where $N^{a,b} = 2e^{-\phi}\xi^{[a}N^{b]}$ is the normal 'bi-vector' to S. We can continue to write the above equation to as an integration over a two-form

$$M = -\frac{1}{8\pi G}\int_{\mathcal{S}} dx^a\wedge dx^b\epsilon_{abcd}\nabla^c\xi^d \tag{73}$$

where ϵ_{abcd} is the volumn element on spacetime associated with the spacetime metric. We can convert the surface integral 73 to a volumn integral using Stokes's theorem

$$M = -\frac{3}{8\pi G}\int_{\Sigma} \nabla_{[e}\left\{\epsilon_{ab]cd}\nabla^c\xi^d\right\} \tag{74}$$

In order to see what the integrand is, let's calculate[6]

$$\begin{aligned}
\epsilon^{efab}\nabla_f\left[\epsilon_{abcd}\nabla^c\xi^d\right] &= \epsilon^{efab}\epsilon_{abcd}\nabla_f\nabla^c\xi^d \\
&= -4\nabla_f\nabla^{[e}\xi^{f]} \\
&= 4\nabla_f\nabla^f\xi^e \\
&= -4R^e_f\xi^f
\end{aligned} \tag{75}$$

And then, multiply the above equation with ϵ_{elmn} and contract over e we get

$$\nabla_{[l}\left\{\epsilon_{mn]cd}\nabla^c\xi^d\right\} = \frac{2}{3}R^e_f\xi^f\epsilon_{elmn} \tag{76}$$

Substitute this into equation 74 we get

$$\begin{aligned}
M &= -\frac{3}{8\pi G}\int_\Sigma \nabla_{[e}\left\{\epsilon_{ab]cd}\nabla^c\xi^d\right\} \\
&= -\frac{1}{4\pi G}\int_\Sigma \nabla_{[e}\left\{\epsilon_{ab]cd}\nabla^c\xi^d\right\} \\
&= -\frac{1}{4\pi G}\int_\Sigma R^d_f\xi^f\epsilon_{deab} \\
&= \frac{1}{4\pi G}\int_\Sigma R_{ab}n^a\xi^b dV
\end{aligned} \tag{77}$$

where n^a is the normal vector to Σ, so that $\varepsilon_{\mathrm{abc}} = n^d\epsilon_{dabc}$ is the natural volume element represented by dV.

On the other hand, we can see that the energy momentum tensor emerges naturally on the left hand side of the original equality 70 to rewrite the total mass. So we end up with

$$2\int_\Sigma\left(T_{ab}-\frac{1}{2}Tg_{ab}\right)n^a\xi^b dV = \frac{1}{4\pi G}\int_\Sigma R_{ab}n^a\xi^b dV \tag{78}$$

where the particular combination of the stress energy tensor on the left hand side can be fixed by combining the conservation laws of the tensors that occur in the integrals [7]. Notice that equation 78 only gives us only a certain component of the Einstein's equation. To get to the full Einstein we now use a similar reasoning as Jacobson [1] for time-like screens instead of null-like screens. We will end up with the full Einstein equations, same as what we had in equation 52

$$R_{ab} - \frac{1}{2}Rg_{ab} + \Lambda g_{ab} = 8\pi G T_{ab} \tag{79}$$

4.5 New interpretations on inertia, acceleration and the equivalence principle

The acceleration due to the entropic force (gravity), can be calculated with

$$a = \frac{F}{m} = T\cdot\frac{\frac{\Delta S}{\Delta x}}{m} = \frac{2c^2}{k_B n}\frac{\Delta S}{\Delta x} \tag{80}$$

[6] We used one of the results of the homework problems.

[7] This particular choice of the left hand side, in my opinion, is similar to what we did in Equ.[51] where we applied additional constraints/arguments such as the contracted Bianchi identity that we assume holds

where n denotes the amount of new bits of information created by m when m finally merges with the microscopic degrees of freedom on the screen. Equation 80 shows that the acceleration is proportional to the entropy gradient [8]. This will be one of our main principles: inertia is a consequence of the fact that a particle in rest will stay in rest because there are no entropy gradients.

This new way of thinking can also shed light on the equivalence principle. The equivalence principle tells us that redshifts can be interpreted in the emergent space time as either due to a gravitational field or due to the fact that one considers an accelerated frame. However, as shown in the last subsection, both views are equivalent in the relativistic setting, but neither view is microscopic or fundamental in the sense that they are both emergent phenomena.

5 Theoretical and experimental criticism

5.1 Theoretical criticism

In its current form, entropic gravity has been severely challenged on formal grounds. Let's look at two examples.

Matt Visser points out [13] that if we wish to retain key parts of Verlinde's proposal (an Unruh-like temperature, and entropy related to "holographic screens") and to model conservative forces in the general Newtonian case (i.e. for arbitrary potentials and an unlimited number of discrete masses), the resulting model with a single heat bath is at best orthogonal to Verlinde's proposal. So that leads to unphysical requirements for the required entropy and involves an unnatural number of temperature baths of differing temperatures. Visser concludes that based on the work of Jacobson [1], Padmanabhan [2], and others, there are good reasons to suspect a thermodynamic interpretation of the fully relativistic Einstein equations might be possible. Whether the specific proposals of Verlinde [3] are anywhere near as fundamental is yet to be seen — the rather baroque construction needed to accurately reproduce n-body Newtonian gravity in a Verlinde-like setting certainly gives one pause.

Tower Wang also considered [14] a wide class of modified entropic gravity models of the form

$$E = \frac{aAc^2}{4\pi G} f(a, A) \tag{81}$$

where $f(a, A)$ is a model-dependent function of acceleration and area of the holographic surface [9]. Notice in the case of Verlinde [3], $f = 1$.

Wang has shown that the inclusion of the covariant conservation law of energy-momentum tensor severely constrains viable modifications of entropic gravity. In addition, he has shown that assuming the FLRW metric (i.e. assume cosmological homogeneity and isotropy), there

[8] We are considering a process where a particle of mass m is approaching the holographic screen and we are trying to re-interpret its acceleration. So I guess even though the background (M at the center for example) is in thermal equilibrium there is still entropy gradient caused by the displacement Δx of m relative to M.

[9] My best guess for how to choose the area A of mass m for this generalized entropic gravity model would be the horizon area of the corresponding black hole with the same mass m, which is bounded by the minimum encompassing area of the particle.

is a discrepancy in deriving cosmological equations unless a condition for $f(a, A)$ is satisfied [10]

$$-\partial_t^2 f + \frac{\ddot{R}r}{\dot{R}} \partial_t \partial_r f = 0 \tag{82}$$

where $R(t)$ is the cosmological scale factor. For a model with a specific function of f, the condition is not always satisfied. So Wang concluded that the modified entropic gravity models of form 81, if not killed[11], should live in a very narrow room to assure the energy-momentum conservation and to accommodate a homogeneous isotropic universe.

5.2 Cosmological observations

Verlinde proposed [15] that the observed excess gravity in galaxies and clusters that is normally accounted for by the mysterious 'dark matter' is a consequence of entropic gravity. He shows that the emergent laws of gravity contain an additional 'dark' gravitational force describing the 'elastic' response due to the entropy displacement. Verlinde gives an estimate of the excess gravity in terms of the baryonic mass, Newton's constant and the Hubble acceleration scale $a_0 = cH_0$, and argues that this additional 'dark gravity force' explains the observed phenomena in galaxies and clusters currently attributed to dark matter. A team from Leiden Observatory [16] observed statistically the lensing effect of gravitational fields at large distances from the centers of 33,613 galaxies, found that those gravitational fields were in good agreement with the prediction from entropic gravity. Whereas using conventional gravitational theory, the lensing effect of gravitational fields implied by these observations (as well as from measured galaxy rotation curves for example) could only be ascribed to a particular distribution of dark matter.

However, there are also other cosmological observations that do not agree with the entropic gravity predictions. For example, Nieuwenhuizen [17] focuses on the good lensing and X-ray data for A1689, and finds out the need for dark matter starts near the cluster centre, where Newton's law is still supposed to be valid for entropic gravity. Nieuwenhuizen concludes that entropic gravity is strongly ruled out unless additional (dark) matter like eV neutrinos is added.

Also, Kris Pardo [18] developed equations for entropic gravity's velocity curve predictions given a realistic, extended mass distribution, and applied it to isolated dwarf galaxies. He found that entropic gravity severely underpredicts the maximum velocities for those galaxies with measured velocities $v > 165$km/s. Kris concluded that rotation curves of these isolated, HI gas-rich, nearly spherical dwarf galaxies would provide the definitive test of entropic gravity.

[10] This means that the generalized entropic gravity model 81, according to Wang [14], is severely limited if not completely killed.

[11] Verlinde's idea with f being a constant 1 agrees with Equ. 82 however. So I guess Wang's objection is against a even more generic (but similar) emergent gravity model than Verlinde's.

6 Final thoughts on this[12]

Despite all the theoretical and observational evidences against the emergent gravity scenario, there are still some aesthetically pleasing aspects in its formalism. Just to name a few: the deep connections between Gravity and Thermodynamics in general and the derivation of Newton's and Einstein's laws of gravity with spacetime-independent quantities such as energy, entropy and temperature by assuming *linearity*[13] between the entropy change associated with the position change of the material bodies and the matter displacement are all quite shocking to me the first I saw it. Some people might argue these are simply due to a reserve of arguments, and practically speaking both directions of the arguments are equivalent since they obtain the same equations of motion. This might be true but in my opinion, this does not necessarily mean the other side of arguments is unnecessary or simply redundant. Let me briefly illustrate what I mean here.

For any fundamental physical (or even mathematical for that matter) theories, we start with some basic assumptions as our starting points and then use mathematical logic to derive and present its consequences as mathematical equations of motion mostly. In my opinion, however, a complete understanding of a physical theory is not just about the equations, but more about the fundamental assumptions that lead to the equations and reflect our understanding of the physical process. In this sense, even though we arrived at the same equations in the emergent gravity scenario as in the traditional approaches, they still provide different insights about our understanding of gravity, i.e. it is an emergent phenomenon with no fundamental field associated with it. The only way to distinguish them (the fundamental assumptions or the way we understand the gravity) from each other is of course by observations. All of the observational evidences against emergent gravity that I can find however, including the ones I listed above, are all suggesting emergent gravity is not enough *by its own* in explaining the observed excess gravity in galaxies and clusters. But that does not completely rule out emergent gravity as a useful interpretation of gravity. I am happy to see more decisive evidences in the future, if any.

[12] This section summarizes some thoughts I had while preparing this draft. It is not mature in anyway and shouldn't considered as part of the review of this topic.

[13] See assumptions 57 and 65. It would be better if these assumptions can arise from a deeper argument instead of the simple agreements with the known physical laws, as provided in this report. But again, we can treat the linearly as the first principle of our emergent gravity theory.

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