Discrepancies in the Measurement of Hubble Constant

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December 17, 2020

Abstract

Hubble's Law was the first observational evidence that explained the expansion of the universe, and the rate of this expansion is quantified by the Hubble Constant. Hubble's observations were made using data from 24 nearby galaxies. He used both the velocities and distances of these galaxies to make this historic observation. In the years after Hubble presented his result in 1929, there have been various independent methods to measure both the local and global value of this constant, H_0. In this project, we will study the main methods used currently to measure the Hubble Constant, and the discrepancies in this measurement which is famously referred to as the Hubble tension. We will cover some of the ways in which scientists have attempted to resolve this discrepancy. We will also go through some gravitational wave measurements in the future that would help in resolving the Hubble tension.

Contents

1 Introduction to Hubble Constant

The Hubble's law [5] gives us a relationship between between the recessional velocity, v with which galaxies move away from the Earth and their distance to the Earth, d.

$$v = H_0 \times d \tag{1}$$

H_0 gives us the rate at which the Universe expands. In his first ever calculation of the Hubble's constant, Hubble plotted the distance-velocity relationship of 24 extra-galactic nuclei (since distance measurements were only available for so many of them). Hubble's initial value for the Hubble Constant, was approximately 500 km/s/Mpc. This is very different from the measurements of H_0 in the years that followed because what Hubble thought were individual stars in his distant galaxies were actually star clusters, which cannot be thought of as 'standard candles'. Recent measurements of H_0 give us a value of approximately 70 km/s/Mpc. There are various experimental methods with different theoretical approaches used to measure H_0. Some of these methods are listed below:

1. *Planck* mission measurements of anisotropies in the cosmic microwave background. This method gives a measure of the global value of H_0.

2. Hubble Space Telescope in collaboration with SH0ES team using type 1a supernovae and Cepheid photometry to measure local value of H_0.

3. Using gravitational waves from binary neutron star merger along with the electromagnetic counterpart.

The major discrepancy has been in the local and global (Planck) measurements of the Hubble constant as seen in Figure. 1. There are two possible explanations for this. Either there are large systematic errors that are affecting our measurements of cosmological parameters, or we need new physics to understand our Universe and model it. In the following section we will discuss in some detail about each of these methods and their respective measurements of H_0.

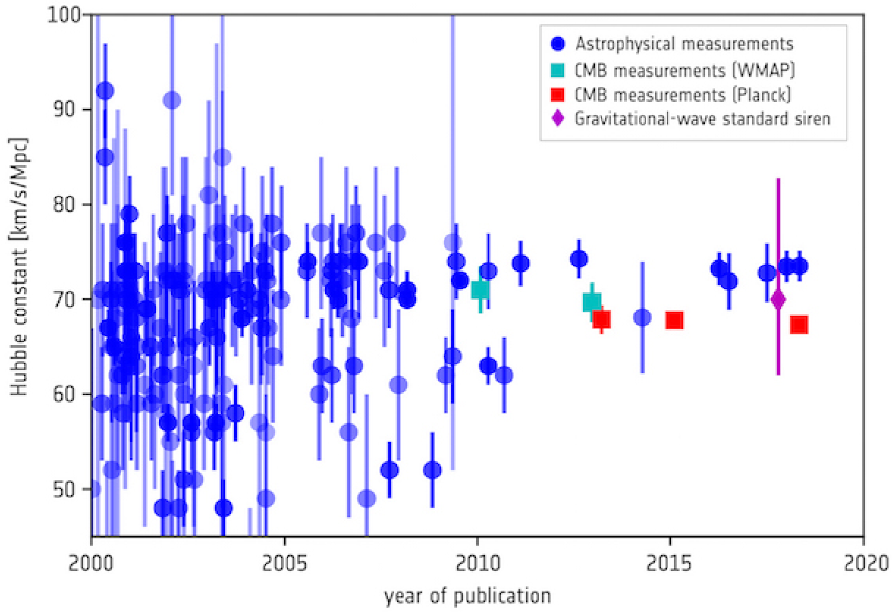

Figure 1: Measurements of the Hubble constant in the past two decades Ref: https://sci.esa.int/web/planck/-/60504-measurements-of-the-hubble-constant

2 Current methods to make measurements of H_0

2.1 *Planck* mission measurements

In this approach, the first assumption is a ΛCDM cosmological model [2]. Λ represents the cosmological constant. It is assumed that curvature, Ω_k is zero, and the equation of state of dark energy, $w = P/\rho = -1$. H_0 is indirectly measured by looking at anisotropies in the cosmic microwave background (CMB). An image obtained by the Planck satellite in their 2018 mission is shown in Figure. 3. Due to very high temperature in the early Universe, atoms could not be formed and the Universe was ionized. Around $z \approx 1000$, the temperature dropped low enough to allow formation of atoms. This is denoted as recombination. After recombination, photons could travel freely without being scattered by the ions. These redshifted photons formed the

CMB. Small density fluctuations in the early Universe got frozen into this photon field. These can be seen as peaks in the CMB spectrum. The angular separation gives us a way to quantify the expansion of the Universe. Since this angular diameter distance depends on various cosmological parameters, the positions of the acoustic peaks lets us place constraints on these cosmological parameters.
The angular power spectrum of the CMB as detected by the Planck satellite in 2013 is shown in Figure. 2. The x-axis shows us the length scale. Length is measured in the angular metric, and gives us the separation between points in space as they appear to us in the sky. To get an idea of what these distances mean, the diameter of the full Moon as seen from Earth measures about half a degree.
Let us understand the information about cosmological parameters that can be obtained from this plot as these cosmological parameters will be referred to later as well while discussing new physics models to alleviate H_0 measurement discrepancies.

1. **Curvature, k:** The first peak in the power spectrum comes from regions in space that were very dense in the beginning. The size of these regions is governed by the speed with which density fluctuations propagated in the primordial baryon-photon fluid. This means that the faster the speed of propagation, the larger the region in space. This is known as the sound horizon. This first peak in the power spectrum gives us information about the geometry of the universe. A flat Universe has the first peak located roughly where it is observed, around 0.8 degrees. What we can conclude from this is that the Universe is very nearly flat. It does not have positive or negative curvature.

2. **Ordinary matter density, Ω_m:** The odd-numbered peaks correspond to oscillations at maximum matter density and the even-numbered peaks to oscillations at minimum density. This implies that a higher overall density of ordinary matter enhances the odd-numbered peaks over the even-numbered ones.

3. **Dark matter density, Ω_{DM}:** It is important to note when minimum densities occur to understand how dark matter density can be inferred. Minimum density occurs when the interaction between ordinary matter and photons in an dense region causes pressure, which hurls ordinary matter away. However, dark matter does not interact with photons, its

density in a region is not affected by the pressure. Therefore, higher dark matter density implies a high third peak.

4. **Dark energy density, Ω_{DE}:** We know dark energy exists because of the following reason: Too low matter density implies a negatively curved Universe, too high matter density implies a positively curved Universe, and a critical density of matter that gives a flat universe. But the combination of dark matter and ordinary matter densities gives us nothing close to the density required to have a flat Universe. This cannot be directly detected and has only been indirectly seen through its gravitational effect on the expansion of the Universe, particularly it is the reason for the accelerating expansion. The acceleration is governed by Einstein's equations:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + 3P\right] \tag{2}$$

Therefore, something that drives acceleration of the Universe (positive $\ddot{a}$) must have negative pressure. In particular, its equation of state has to be $w = P/\rho < -1/3$

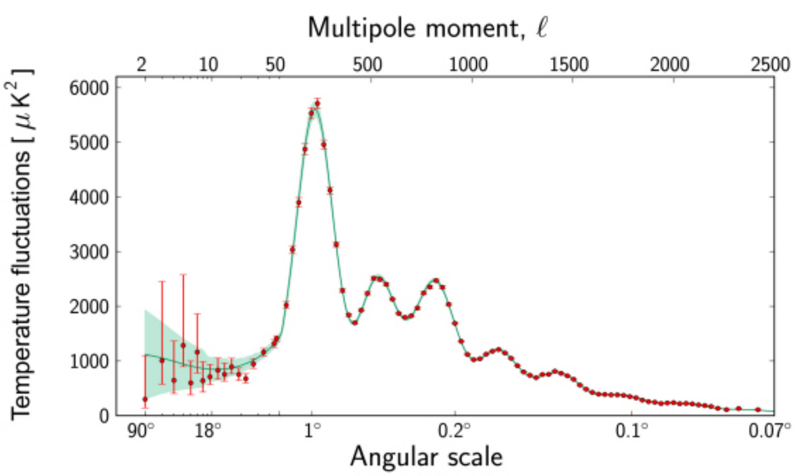

Figure 2: 2013 CMB Power Spectrum [March 2013] Ref: https://sci.esa.int/web/planck/-/51555-planck-power-spectrum-of-temperature-fluctuations-in-the-cosmic-microwave-background

Assuming the ΛCDM cosmology, Planck mission measurement gives a value of $H_0 = 67.3 \pm 1.2 kms^{-1}Mpc^{-1}$. It is important to note that Planck measurements are a good constraint on H_0 if we make the following assumptions: H_0 is independent of redshift, dark energy is a cosmological constant, and the curvature of the Universe is zero. Hence, the Planck measurement is highly dependent on the underlying cosmological model.

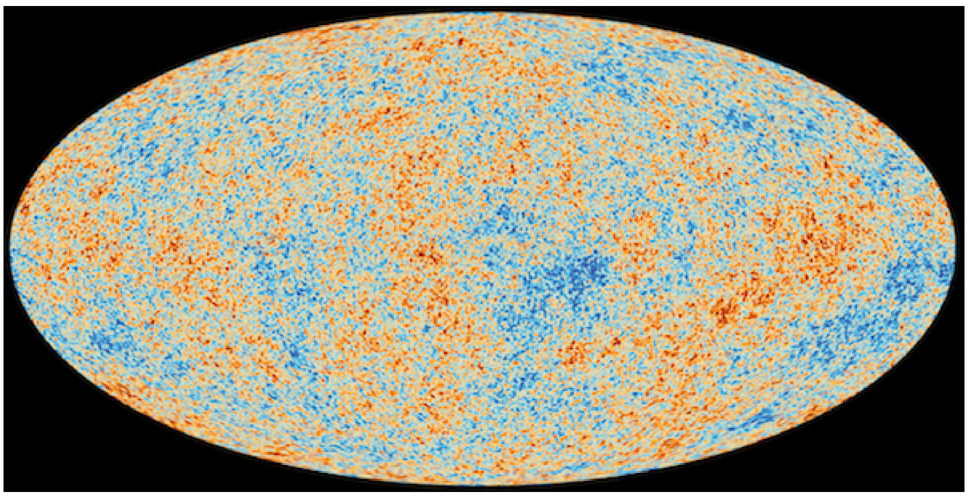

Figure 3: The cosmic microwave background [Planck 2018 mission] Ref: https://sci.esa.int/web/planck/-/60499-from-an-almost-perfect-universe-to-the-best-of-both-worlds

2.2 Hubble Space Telescope measurement in collaboration with SH0ES team

For calculation of Hubble's constant, we need to have the distance and velocity of the object in the Hubble flow. Recessional velocities are simpler to calculate by using an object with a spectral line. From this the wavelength emitted, we can get the redshift, z to the required object. To calculate the distance, in this method, Type 1a supernovae are used as standard candles. This is because we can only make precise direct distance measurements of objects that are close to us. To measure larger distances, we use successive measurements of shorter distances. This is known as the cosmic distance ladder method. Standard candles are objects such as Type 1a supernovae whose true brightness is presumed to be known. By making a measurement of their observed luminosity, we get an indirect estimate of their distance from us.

$$m - M + 5 = 5log_{10}(d) \tag{3}$$

where M is the absolute magnitude and m is the apparent magnitude. These are measures of the true luminosity and observed luminosity of the object respectively. d is the distance to the object in parsecs. Type 1a supernovae are used as standard candles because of the following reason. This category of supernovae are formed from slowly rotating white dwarfs composed of carbon and oxygen that are in binary systems. As the white dwarf accretes mass from the other star, there is a rise in temperature of its core that causes carbon fusion. This triggers a runaway process that causes a huge release in energy causing the star to explode. Since all type 1a supernovae occur from the same mechanism, and explode at a fixed mass, they have a near constant peak luminosity, which can be used to make indirect distance measurements. In the recent years [11], there has been an even better improvement in calculating the distance to these Type 1a supernovae. Scientists choose host galaxies that contain both Type 1a supernovae and Cepheid variable stars. These are stars that pulsates over a period of time. Studies indicate that the rate of pulsation and the true luminosity of such stars are very strongly correlated. Hence, by measuring the pulsation period, the absolute magnitude and hence the distance to this galaxy can be measured in a more precise fashion.

Using the apparent magnitude, the true magnitude and the redshift to the galaxies, Hubble's law can be re-written to obtain the Hubble constant [9]

$$log(H_0) = 0.2M_\lambda + 5 + a_\lambda \tag{4}$$

$$a_\lambda \equiv log(cz) - 0.2m_\lambda \tag{5}$$

The most recent and precise Hubble Telescope measurement reports a value of $H_0 = 73.24 \pm 1.74 kms^{-1}Mpc^{-1}$.

2.3 Gravitational wave measurements

The signal strength, specifically the signal to noise ratio of a gravitational wave is inversely proportional to the luminosity distance of the binary system merger that emitted the wave. This property lets us directly calculate

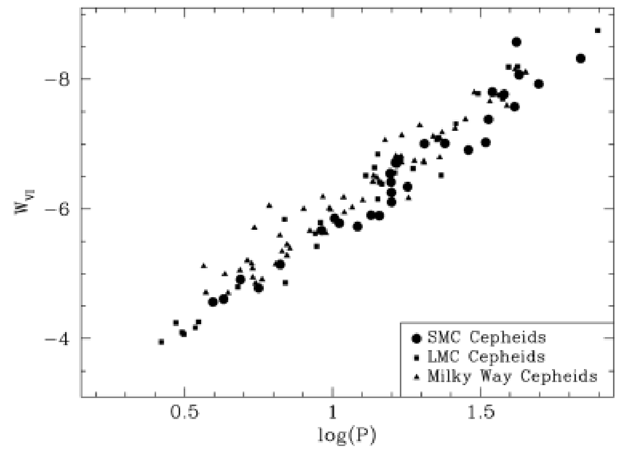

Figure 4: Luminosity period graph for cepheids in different galaxies. Ref: arXiv:1809.04073

the distance to the source without the use of any cosmic ladder method. In order to use this to calculate the Hubble constant, we require the recessional velocity of the source galaxy. By measuring the time delay between the arrival of the signals in the two detectors and detector response, the network of LIGO detectors can be used to establish the sky localization [12]. The first measurement of this kind was made with the detection of a binary neutron star merger [3]. This process results in the production of either gamma ray bursts or kilanovae, which can be detected by space telescopes to localize the source galaxy even further. The value of H_0 obtained from gravitational wave measurements is $70.0^{+12.0}_{-8.0} kms^{-1}Mpc^{-1}$. This measurement is consistent with both the HST and Planck measurements. It is important to note that for the Hubble velocity measurement of the host galaxy, the distance measurement of the source assumed $H_0 = 68.9kms^{-1}Mpc^{-1}$. Therefore, the posterior on H_0 is slightly biased and peaks around this value.
The local HST measurement and global Planck mission measurement of the Hubble constant differ by around 2.5σ. This raises the possibility that the tension is due to some non-standard physics. There have been various attempts to resolve this tension, some of which will be discussed in the next section.

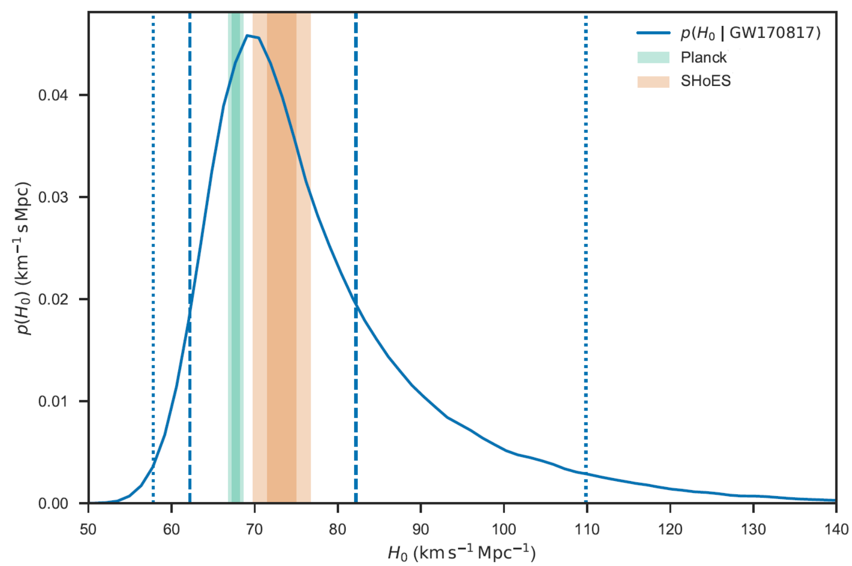

Figure 5: The dashed and dotted blue vertical lines show the 68.3% and 95.4% credible intervals respectively for H_0 calculation using GW170817. Ref: https://www.ligo.org/science/Publication-GW170817Hubble/

3 Attempts to resolve the tension

3.1 Scalar field Equation of State

Consider universe described by FRW metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) \tag{6}$$

where k =-1, 0 or +1 for open, flat or closed universe. In the presence of scalar field, we have the evolution equations:

$$3\frac{\dot{a}^2}{a^2} = 8\pi\rho + 8\pi\rho_f - 3\frac{k}{a^2} \tag{7}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(p+\rho) = -3H(p+\rho) \tag{8}$$

$$\dot{\rho}_f = -3\frac{\dot{a}}{a}(p_f+\rho_f) \tag{9}$$

since at any given, we have

$$H(t) = \frac{\dot{a}}{a} \tag{10}$$

ρ_f and p_f are the scalar field energy densities and pressure defined by:

$$8\pi\rho_f = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{11}$$

$$8\pi p_f = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{12}$$

The equation of state in cosmology is characterized by $w = \frac{p}{\rho}$.
For the scalar field, we would have the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \tag{13}$$

and the equation of state

$$w_f = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{14}$$

3.2 Early dark energy model

In this attempt to resolve the Hubble tension, the authors propose an early dark energy (EDE) model to resolve the Hubble tension [10]. This model proposes that EDE behaves as a cosmological constant at early times, upto $z \geq z_c$, and dilutes away at later times. Two physical models were proposed:

1. Oscillating scalar field model

2. Slowly rotating scalar field model

For the first model, we have a scalar field with potential $V(\phi) \propto [1 - cos(\frac{\phi}{f})]^n$. The field is constant for early times, $z > z_c = a_c^{-1} - 1$. Beyond the critical redshift, the field oscillates and behaves like a fluid with equation of state

$$w_n = \frac{n-1}{n+1} \tag{15}$$

For the second model, the potential is linear at early times and asymptotes to zero at later times. For the first model, the energy density and equation of state are given by

$$\Omega_\phi(a) = \frac{2\Omega_\phi(a_c)}{(a/a_c)^{3(1+w_n)}+1} \tag{16}$$

$$w_\phi(a) = \frac{1+w_n}{(a/a_c)^{3(1+w_n)}+1} - 1 \tag{17}$$

As $a \to 0$, $w_\phi(a) \to w_n$, and as $a \gg a_c$, $w_\phi \to -1$. This physically means that the field is frozen at early times and then dilutes away as $a^{3(1+w_n)}$. The authors [7] use a Markov Chain Monte Carlo code to solve the perturbation equations (will include these equations) for separate values of n. The posterior distributions indicate that the field must become dynamical at matter-radiation equality. The conclusions of this work are as follows: EDE that is constant at early times and dilutes away faster than radiation at $z_c \geq 3000$ can explain the tension between local and global measurements of H_0.
The next two approaches are methods to resolve the tension in the future with more gravitational wave measurements and the development of the space based telescope, LISA.

3.3 Sample of 50 binary neutron star observations along with EM counterpart could resolve tension

In this approach, the authors use the inverse cosmic distance ladder and gravitational wave standard sirens using the posterior predictive distribution (PPD) [4]. Inverse cosmic distance ladder is constructed from BAO oscillations and HST Type 1a supernovae measurements. If a model is assumed for H(z), the Hubble constant at redshift z, the BAO measurements can be extrapolated to redshift zero. This would be give us a way of calculating H_0. To constrain the exact model, additional distance measurement data is taken from modern supernovae data. The joint posterior of the inverse distance ladder parameters and the supernovae data parameters is constructed. The posterior predictive distribution is the sampled distribution for new data d' given known data d, model I and posterior of new data θ.

$$Pr(d'|d,I) = \int Pr(d'|\theta,I)Pr(\theta|d,I)d\theta \tag{18}$$

To see how many BNS were required to arbitrate the Hubble tension, the authors injected simulated binary neutron star data into detector data. The events are distributed uniformly in co-moving volume. 51 out of the injected BNS gravitational waves had a signal to noise ratio, SNR above a fixed threshold value. The sky position is then fixed assuming some known host galaxies. Each event's distance posterior is retained. Each source's peculiar velocity is generated from Gaussian measurement with standard deviation of $200kms^{-1}$. The H_0 posterior is computed by taking likelihood $Pr(H_0|H_0', I)$. Here, H_0' is the Plack H_0 assuming this is correct. The result is that the posterior for H_0 using cosmic distance ladder measurements gets shifted towards the left, alleviating the tension between Planck and HST measurements. This shows that around 50 BNS standard siren detections by the LIGO VIRGO detector network could in an independent way resolve the Hubble tension.

Following this method once we have enough gravitational wave detections with the assumptions made by the authors about the distribution of gravitational wave source parameters [4], we would obtain a value of H_0 close to the Planck measurement and is inconsistent with the local H_0 measurement.

3.4 Observation of stellar mass blackholes using low frequency detectors like LISA could resolve the Hubble tension

For the purpose of measurement of H_0, space detectors have the following advantages over ground based detectors: since ground noise is eliminated, the amplitude of the gravitational wave can be measured more accurately, making luminosity distance measurements even more precise. The annual motion of the LISA detector also helps in making more accurate sky localization measurements of the sources. In this paper [6], the authors propose using monochromatic stellar mass binary black holes like the first LIGO detection GW150914 with LISA to determine the local value of the Hubble constant. The error in measuring H_0 using this method stems from statistical errors in luminosity distance and the cosmological redshift due to peculiar velocities of the host (The peculiar velocity of a galaxy is its relative velocity to its motion due to the expansion of the Universe). This error is shown to be $\propto [D^{-1}]$ at small distances and $\propto [D]$ at large distances. This error reduces as we increase the number of standard sirens to make the calculation. The effects on H_0 and errors due to cosmic variance [8] are not discussed in this article. It is important to note that each of these sources must be associated with a unique host galaxy. In the scenario where all the standard sirens are associated with giant galaxies, we would then need around 60 GW standard sirens arising from sources within distances $\leq 100Mpc$ to suppress the error due to peculiar velocities. Another important feature of such a measurement is that because the redshifts are $z \geq 1$ for these sources, they would also contribute to bridge the current gap between local and global measurements of H_0. Hence, this method would help in understanding the point at which the H_0 measurement discrepancy arises.

3.5 A Simple Phenomenological Emergent Dark Energy Model (PEDE) can Resolve the Hubble Tension

This is a phenomenological dark energy model based on a hyperbolic tan function in which dark energy has no effective presence at early times and emerges are later times [7]. This model has a symmetric behavior around present time. The reason for doing so is because for the present time we have comparable dark energy and dark matter densities. Within the FLRW metric, dark energy density can be described as:

$$\widetilde{\Omega}_{DE}(z) = \Omega_{DE,0} \times exp\left[3\int \frac{1+w(z')}{1+z'}dz'\right] \tag{19}$$

$w(z)$ is the equation of state for dark energy where $w(z) = p_{DE}/\rho_{DE}$ as we had before. For the ΛCDM model, we have $w(z) = -1$ and $\widetilde{\Omega}_{DE}(z) = 1 - \Omega_m =$ constant. Here Ω_m is the matter density at present time. In this paper, the authors [7] introduce a phenomenological dark energy model given by

$$\widetilde{\Omega}_{DE}(z) = \Omega_{DE,0} \times [1 - tanh(\log_{10}(1+z))] \tag{20}$$

Here, $\Omega_{DE,0} = 1 - \Omega_{0m}$. The equation of state for this model is as follows:

$$w(z) = -\frac{1}{3ln10} \times (1 + tanh(\log_{10}(1+z))) - 1 \tag{21}$$

using

$$w(z) = \frac{1}{3}\frac{dln\widetilde{\Omega}_{DE}(z)}{dz}(1+z) - 1 \tag{22}$$

In early times, the equation of state would be $w(z) = -\frac{2}{3ln10} - 1$. For present time, we would have $w(z=0) = -\frac{1}{3ln10} - 1$. In the far future, it would evolve to $w(z) = -1$.
Constraints are then placed using Markov Chain Monte Carlo estimation. The authors use different combinations of available data to study how the constraints on H_0 change for different redshift values. These data sets are as follows:

1. Lower redshift data: Measurements from Type 1a supernovae, particularly the new "Pantheon" sample that contains data for redshift in the range $0.01 < z < 2.3$. This is used in combination with low redshift Baryon Acoustic Oscillations (BAO) data.

2. Higher redshift data: Higher redshift observations from $Ly\alpha$ BAO measurements are used. BAO are acoustic oscillations in the primordial plasma before decoupling of baryonic matter and radiation. These oscillations cause an oscillatory pattern in the correlation properties of dark matter.
   $Ly\alpha$ forest measurements are from absorption lines of distant quasars. These arise from the Lymann - α electronic transitions in the Hydrogen atom. Using this, the angular size and redshift extent of these quasars. The absorption lines spread out into a forest of lines due to each line being redshifted by an amount proportional to the quasar's distance from us. This "forest" gives us an idea of the correlation properties of dark matter that we can use to derive the value of cosmological parameters [13].
   This BAO + $Ly\alpha$ forest data is used in combination with CMB data from Planck measurements to apply constraints on higher redshift measurements of H_0 [1].

The authors perform analysis assuming no priors as well as H_0 priors from the local measurement of the constant by [11]. The results for these are as follows:

1. The PEDE model gives values of both H_0 and Ω_m a higher value for the low redshift data sets when no prior is considered.

    - H_0 from PEDE: $72.84^{+3.814}_{-3.530} kms^{-1}Mpc^{-1}$
    - H_0 from ΛCDM: $66.94^{+3.721}_{-3.256} kms^{-1}Mpc^{-1}$

2. However, upon adding CMB data and higher redshift BAO data, and applying an H_0 prior from the local measurements by [11] gives the following values:

    - H_0 from PEDE: $71.19^{+1.306}_{-0.001} kms^{-1}Mpc^{-1}$
    - H_0 from ΛCDM: $71.19^{+0.271}_{-0.000} kms^{-1}Mpc^{-1}$.

Therefore, the proposed model significantly alleviates the tension in estimation of H_0 using both low and high redshift data.

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