Mode Mismatch Detection using an Electro-optic Lens Device

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Since optical losses limit the sensitivity gains from squeezed light technology, new generational gravitational wave detectors require the precise detection and reduction of mode mismatch — a significant source of laser power loss. Characterized by the amount of coupling to the higher-order spatial modes of an optical cavity, mode mismatch is dependent on the waist size and position of the incident laser beam. The Electro-optic Lens (EOL) device, composed of lithium niobate crystals, can alter the laser waist by applying varying electric fields to the crystals. To produce the mismatch error signal, the device generates one resonant and one non-resonant sideband around the carrier signal in the first-order Laguerre-Gaussian mode. By demodulating the beat signal between the non-resonant sideband and the carrier component that couples to higher order spatial modes, the EOL device obtains an error signal on the mismatch between the waist of the beam and the waist required by the cavity.

I. INTRODUCTION

Since the first observance of gravitational waves in 2015, scientists of the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo Scientific Collaboration have been exploring new techniques to enhance the sensitivity of gravitational wave detectors. One technique that promises impressive results is the use of squeezed light technology to reduce photon shot noise. The benefits of this method, however, are significantly reduced by optical losses from detecting elements. Thus, new generational detectors require precise sensing technology to identify and reduce sources of power loss. Since mode mismatch — the imperfect matching of the spatial characteristics of the laser beam and those required by an optical cavity— is a key contributor to optical loss, researchers of the Virgo Padova-Trento group are developing an electro-optic lens (EOL) device for mode mismatch detection. The development and testing of the EOL device were conducted at the Instituto Nazionale di Fisica Nucleare, in association with the Universita di Padova.

A. Gravitational Wave Detection

Gravitational waves are disturbances in the curvature of space-time that result from massive accelerating objects. Radiating outward at the speed of light, these waves can be detected billions of light years away from their source. The strongest waves are generated from catastrophic events such as colliding black holes, coalescing neutron stars, and supernovae. Although gravitational waves were first observed in 2015, their existence was predicted by Albert Einstein in 1916 using the general theory of relativity. Therefore, detection of gravitational waves not only provides insight on the origins of violent cosmic events but also on the geometric nature of gravitation established by Einstein's theory [1].

Currently, the most effective method for detecting gravitational waves is laser interferometry. The basic optical layout of the Virgo Interferometer, a ground-based detector located in western Italy, is shown in Figure 1. As a gravitational wave passes through the detector, the distortion of space-time changes the length of one detector arm to be longer than the other. When the laser beams travelling through each arm recombine, the difference in path lengths results in a detectable interference pattern. To ensure sensitivity to gravitational waves, each interferometer arm has a Fabry-Perot optical cavity that increases the effective length of the laser path. The mirrors of the cavity allow the laser light to reflect back and forth multiple times before merging together to interfere. For the Virgo interferometer, the addition of cavities increases the effective length of its interferometer arms from its physical size of 3 km to more than 200 km [2].

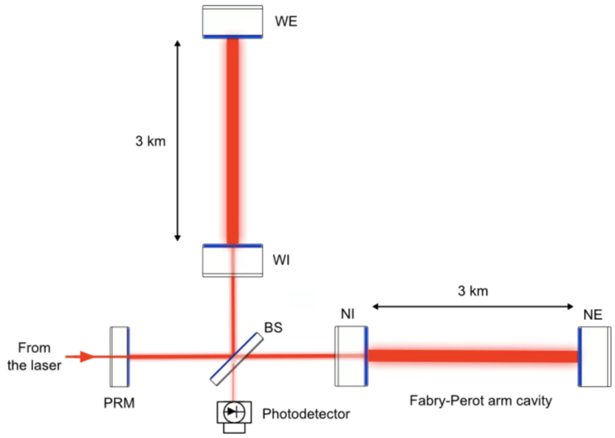

FIG. 1. The basic optical layout of a gravitational wave interferometer, with the labeled dimensions of the Virgo Interferometer. As the laser light enters the detector, it is split into two arms by a beam-splitter. Fabry-Perot cavities allow the laser light to propagate back and forth in each arm before they merge and interfere. When a gravitational wave changes the length of each arm, the recombined light produces a distinct interference pattern that is detected by a photodiode. A recycling mirror (PRM) is used to increase the light power in the interferometer [2].

B. Mode Mismatch

While it increases sensitivity, optical cavities also introduce power losses in gravitational wave detectors when they are not stable or properly aligned with the the laser beam. Stable optical cavities satisfy the following two conditions: (1) the laser frequency matches the cavity resonance frequency and (2) the laser beam wavefront matches the curvature of the cavity reflection mirror. Failure in meeting the second condition, the geometric condition, is mode mismatch and the process to recover the condition is mode matching.

The transverse mode of a laser beam refers to the radiation pattern that forms in the plane perpendicular to the beam's propagation. Laser beams with rectangular symmetry can be described by Hermite-Gaussian modes, as shown in Figure 2, and beams with cylindrical symmetry can be described by Laguerre-Gaussian Modes, as shown in Figure 3 [3]. For both families of modes, the gaussian beam — a beam with a transverse magnetic and electric field profile described by the Gaussian function — is the fundamental transverse electromagnetic (TEM_{00}) mode. When a cavity and a laser beam are properly matched to minimize optical losses, the beam only couples to one mode of the cavity, typically the TEM_{00} mode [4]. Mode mismatch is thus characterized by the amount of coupling to the higher-order spatial modes of the cavity.

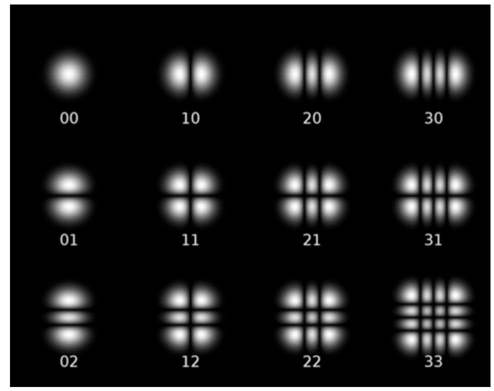

FIG. 2. The intensity profiles of Hermite-Gaussian modes, which describe laser beams with rectangular symmetry [3].

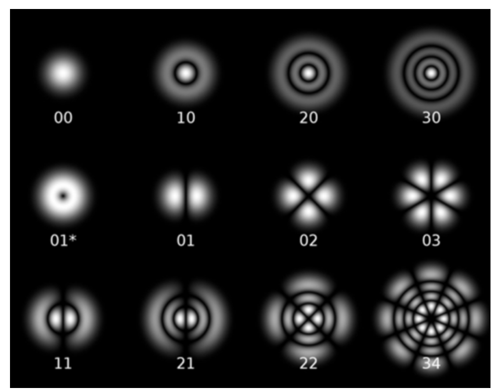

FIG. 3. The intensity profiles of Laguerre-Gaussian modes, which describe laser beams with cylindrical symmetry [3].

The level of mode mismatch in an optical system is dependent on the waist size and position of the laser beam with respect to the cavity mirrors. The beam waist, or focus, is the location along the laser's propagation direction at which the beam radius is at a minimum. As shown in Figure 4, the size and positon of the waist w determine how well the laser beam matches the curvature of the cavity mirrors. An arrangement of lenses can added to the system to obtain the desired waist size and position. In the optical setup of Figure 4, lens 3 (L_3) changes the waist size from w_0 to w_1 and the combination of lens 4 (L_4) and 5 (L_5) changes the waist position without changing the size of the laser beam [5]. This concept of using lenses to adjust the beam waist is the basis of the electro-optic lens (EOL) device, which produces small changes in mode mismatch to obtain an error signal on the mismatch between the waist properties of the laser beam and those required by the cavity.

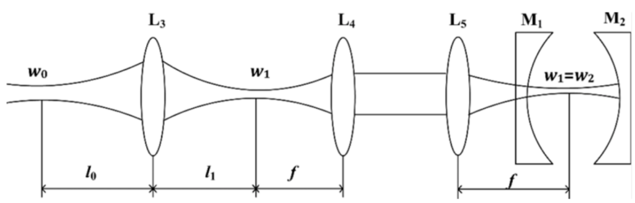

FIG. 4. An optical arrangement that adjusts the beam waist to match the laser beam wavefront with the curvature of the optical cavity mirrors. Lens 3 (L_3) changes the waist size from w_0 to the desired w_1 and the combination of lens 4 (L_4) and 5 (L_5) changes the waist position without changing the size [5].

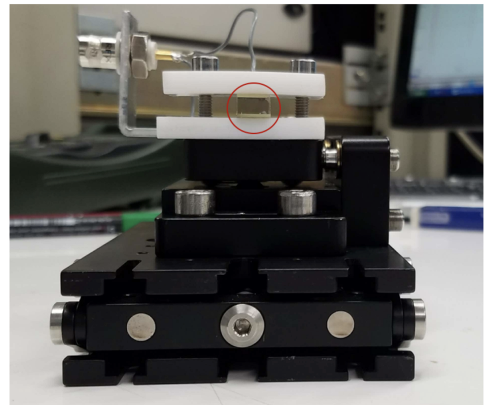

FIG. 5. The electro-optic lens (EOL) device for mode matching. The gold piece, circled in red, is composed of lithium niobate ($LiNbO_3$) crystals that change in refractive index when an electric field is applied. This enables the device to act as a lens with various focal lengths, resulting in different waist sizes and positions of the incident laser beam.

II. THEORY

Unlike an arrangement of lenses, the EOL device, pictured in Figure 5, is able to vary the waist of the laser light without physically moving different components in a system. This ability stems from the electro-optic effect: the device's composition of lithium niobate ($LiNbO_3$) crystals allows an applied electric field to alter its index of refraction. Since the refractive index of a lens determines its focal length, applying different electric fields to the EOL device allows it to act as a lens with different focal lengths, thus obtaining different waist sizes and positions of the laser beam. By continuously varying the beam waist in such a manner, the device can modulate a laser beam at radio frequencies.

EOL modulation can produce a mismatch error signal using a technique similar to the Pound-Drever-Hall locking scheme for frequency stabilization. The EOL device mdoulates the laser light with an angular frequency Ω and generates sidebands around the carrier in the first-order Laguerre-Gaussian (LG_{01}) mode. The modulated light can be described by a superposition of a LG_{00} carrier with frequency ω and two LG_{01} sidebands with frequencies $\omega + \Omega$ and ω - Ω. If mode mismatch is present, then the carrier will have a component that couples to the higher-order modes of the cavity and will be reflected off from the cavity. Demodulating the beat between this component and the non-resonant sideband provides the error signal for mode mismatch. Due to the symmetry of the sidebands, the detected beat will not produce amplitude modulation if both sidebands are reflected off from the cavity. Thus, the light is modulated at a frequency Ω such that the LG_{01} sideband with frequency $\omega + \Omega$ is resonant and completely transmits into the cavity. The detected beat signal between the non-resonant sideband and the carrier component is then demodulated in the I and Q quadratures. By changing the phase of the quadratures, we can obtain two isolated error signals on the mismatch in waist size and position.

The mismatch between the waist size of the incident laser beam $w_{0,IN}$ and the size required by the cavity $w_{0,CAV}$ can be described with the parameter

$$\beta = \frac{\delta w_0}{w_{0,CAV}} \tag{1}$$

where $\delta w_0 = w_{0,IN}$ - $w_{0,CAV}$. Similarly, the mismatch between the waist position of the incident laser beam $z_{0,IN}$ and the position required by the cavity $z_{0,CAV}$ can be described with the parameter

$$\gamma = \frac{\delta z_0}{2 z_{R,CAV}} \tag{2}$$

where $\delta z_0 = z_{0,IN}$ - $z_{0,CAV}$ and $z_{R,CAV}$ is the Rayleigh range of the laser beam as required by the cavity. These two parameters can be used as correction factors to describe the waist mismatch in different cavity modes.

The difference in waist size, if described in the fundamental mode of the cavity, can be described in the first order cavity mode with the equation

$$(\delta V_{00})_{w,size} = -\beta V_{01} \tag{3}$$

where V_{00} and V_{00} are the cavity LG_{00} and the LG_{01} mode transverse functions, respectively. Inversely, the waist size difference described in the first order mode can be described with the fundamental mode as:

$$(\delta V_{01})_{w,size} = \beta V_{00}. \tag{4}$$

The difference in waist position can be similarly related to the two mode descriptions with the equations:

$$(\delta V_{00})_{w,position} = i\gamma(V_{00} + V_{01}) \tag{5}$$

$$(\delta V_{01})_{w,position} = -2i\gamma[V_{02} - \frac{1}{2}(V_{01} - V_{00})] \tag{6}$$

where V_{02} is the LG_{02} mode transverse function of the cavity.

By combining the above equations, we can describe the mode transverse functions of the laser beam in the cavity basis as:

$$U_{00} = (1 + i\gamma)V_{00} + (i\gamma - \beta)V_{01} \tag{7}$$

$$U_{01} = (\beta - i\gamma)V_{00} + (1 + i\gamma)V_{01} - 2i\gamma V_{02} \tag{8}$$

where U_{00} and U_{01} are the LG_{00} and the LG_{01} mode transverse functions of the laser beam.

After the EOL device has modulated the incident laser beam with angular frequency Ω, the differences in waist size and position between the modulated (MOD) beam and the incident (IN) beam can be described with parameters:

$$B = \frac{\delta w_0}{w_{0,IN}} = \frac{m_B}{2}(e^{i\Omega t} + e^{-i\Omega t}) \tag{9}$$

$$G = \frac{\delta z_0}{z_{R,IN}} = \frac{m_G}{2}(e^{i\Omega t} + e^{-i\Omega t}) \tag{10}$$

where $\delta w_0 = w_{0,MOD}$ - $w_{0,IN}$, $\delta z_0 = z_{0,MOD}$ - $z_{0,IN}$, and $z_{R,IN}$ is the Rayleigh range of the incident laser beam. m_B and m_G are constants specific to the device that describe the modulation amplitudes of the waist size and position.

Using the parameters B and G, we can describe the beam immediately after EOL modulation in the laser basis as:

$$U_{MOD} = (1 + iG)U_{00} + (iG - B)U_{01}. \tag{11}$$

If the EOL device is located at a distance z away from the optical cavity, then the beam entering the cavity can be described by:

$$U_{CAV} = (1 + iG)U_{00} + (iG - B)U_{01}e^{i\Delta\Psi} \tag{12}$$

where the additional term with $\Delta\Psi = \arctan(z/z_{r,IN})$ accounts for the difference in Guoy phase between U_{00} and U_{01}. Using Eq. 7 and Eq. 8, we can transform the previous equation into the cavity basis:

$$\begin{aligned} U_{CAV} = (1 + iG)[(1 + i\gamma)V_{00} + (i\gamma - \beta)V_{01}] + \\ (iG - B)[(\beta - i\gamma)V_{00} + (1 + i\gamma)V_{01} - 2i\gamma V_{02}]e^{i\Delta\Psi}. \end{aligned} \tag{13}$$

If there is no mismatch between the waist of the laser and the waist required by the cavity, then the V_{02} term will disappear in Eq. 13. Thus, mode mismatch can be measured by the strength of the 02 mode in the incident laser beam [6].

III. EXPERIMENTAL SETUP

The testing and characterization of the EOL device were completed using an optical arrangement in the Instituto Nazionale di Fisica Nucleare - Laboratori Nazionali di Legnaro (INFN-LNL). The portion of the setup that includes the device and the optical cavity is shown in Figure 6. A triangular optical cavity was used in place of a linear cavity due to limitations in space; its small angles, however, allow it to be treated as two plano-concave cavities that are linearly aligned. As shown by Figure 6, the laser light reflected off from the triangular cavity is detected by a photodiode. The light transmitted to the cavity is divided by a polarizing beam splitter between a camera, which captures the laser modes, and a second photodiode. Before the laser light enters the cavity, it passes through an arrangement of lens 3 and 4 which were used to directly change the level of mode mismatch and verify if the recovered error signals were accurate. The optical setup also includes an electro-optic modulator (EOM), positioned before the EOL device, which uses the Pound-Drever-Hall locking scheme to match the laser frequency to the cavity resonance frequency. With accurate error detection, the combination of the EOM and EOL devices ensures that the two conditions for stable optical cavities are met.

IV. CHARACTERIZATION

The EOL device was characterized by scanning the triangular cavity at varying frequencies of the laser light. Since each mode resonates at a different frequency, a laser frequency scan provides the mode content of the

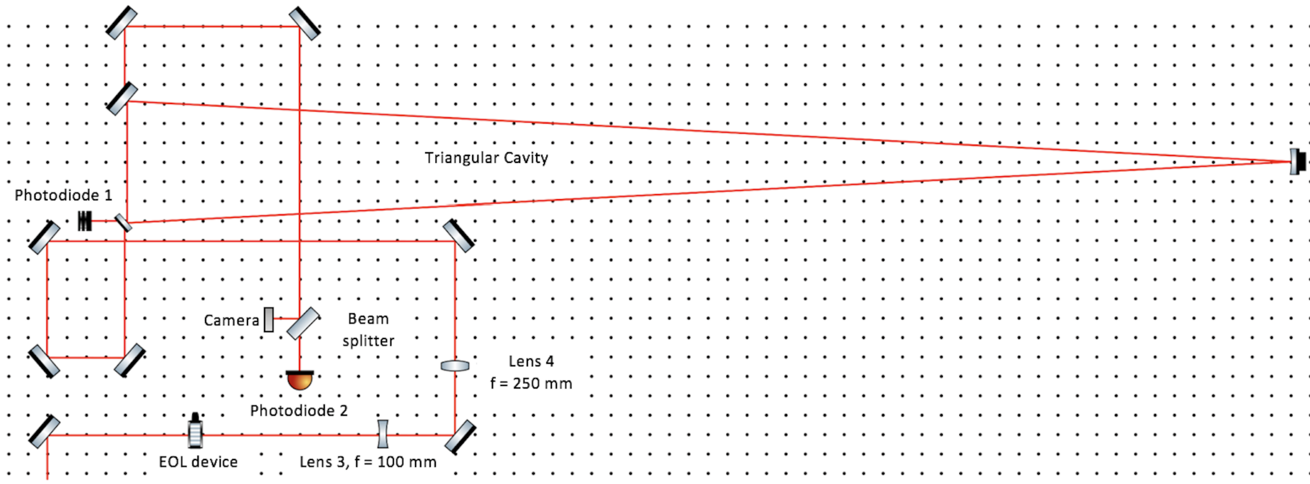

FIG. 6. The schematics for the portion of the optical setup with the EOL device. The distance between the black points corresponds to 25 mm in real scale. The light reflected off from the triangular optical cavity is detected by photodiode 1. The transmitted light from the cavity is split by a polarizing beam splitter to a camera and photodiode 2. Lens 3, with a focal length of 250 mm, and lens 4, with a focal length of 100 mm, were used to adjust the mode mismatch and test the effect of the EOL device.

beam: the power of each resonant peak is a measure of the strength of the corresponding mode within the beam. Figure 7 shows a sample laser frequency scan and identifies the carrier modes corresponding to each peak. The 10 and 01 mode peaks are due to mirror misalignment in the horizontal and vertical direction, respectively. The power of the 02/20 mode, as a percentage of the 00 mode peak power, was used as a measure of the mode mismatch percentage in the optical setup.

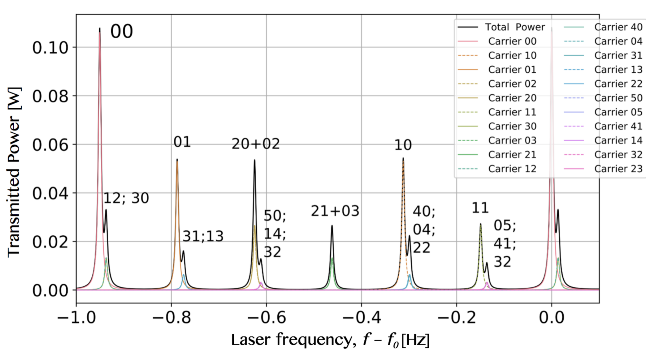

FIG. 7. A frequency scan of the triangular optical cavity, showing one free spectral range and identifying the modes corresponding to the resonant peaks. The 01 and 10 peaks are due to vertical and horizontal misalignments, respectively. The power of the 02/20 peak is a measure of the mode mismatch in the optical system.

To verify the effect of the lens arrangement on mode mismatch, the cavity was scanned to record the mode mismatch percentage for varying translation positions of lens 3 and 4. The program *Just Another Mode Matching Tool (JamMT)* was used to simulate the optical arrangement and provide expected values for the mode mismatch percentage, given by the value of 100 - V_x in the program. Table 1 provides these measured and expected values for each translation stage position of lens 3 and 4. As shown by the agreement between the measured and simulated values, the experimental setup can be well represented by a numerical simulation. Since it can vary the mismatch percentage by more than 10%, the results also confirm that the lens arrangement is an effective means of manually adjusting the mode mismatch to test the recovered error signals.

The effect of the EOL device on mode mismatch was then verified by comparing the power of the 02/20 mode peaks when the device was switched on and off. These measurements were completed by sending a square wave to the device that oscillates between 0 V and 400 V. The 02/20 mode peak distribution for a sample of approximately 200 peaks is given in Figure 8. As shown by the shift in peak distribution when the device is switched on, the EOL device produces a visible decrease in mode mismatch. To understand how the effect changes with EOL position, the EOL ON/OFF measurements were taken against translation and tilt movements, in both the horizontal and vertical direction. As expected, the difference in mode mismatch percentage was maximized when the

TABLE I. The experimental and expected values of the mode mismatch percentage against the position of lens 3 and 4. The expected values were given by the paramater parameter 100 - V_x from the program *Just Another Mode Matching Tool.*

Translation Stage Position (mm)		Mode Mismatch (%)	
Lens 3	Lens 4	Experimental	*JamMT* Simulation
6.50	7.50	8.197	6.510
6.50	12.50	4.144	3.607
6.50	17.50	2.344	1.475
6.50	22.50	0.424	0.289
11.50	22.50	0.258	1.495
16.50	22.50	1.759	3.668
21.50	22.50	5.691	6.686
26.50	22.50	11.220	10.362

device was centered on the laser beam. At large tilt and translation distances away from the center of the beam, the EOL device produced little to no effect. A negative effect was also observed at certain positions, as measured by the increase of 02/20 peak power when the EOL was switched on.

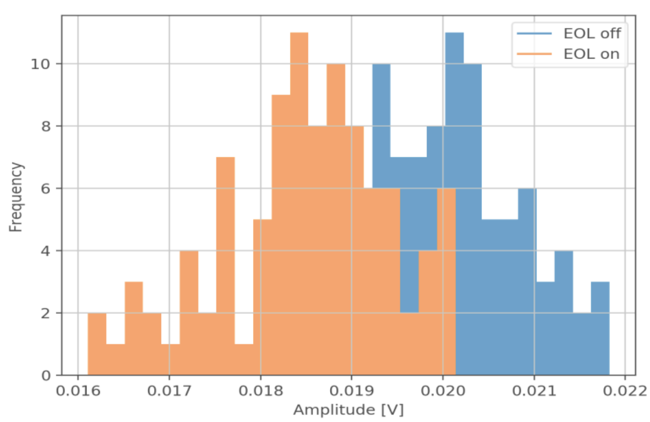

FIG. 8. The peak power distribution of approximately 200 peaks corresponding to the 02/20 mode. As shown by the left shift in distribution when the EOL is on, the EOL device produces a visible reduction in the power of the 02/20 mode peak and thus a visible decrease in mode mismatch.

Since changes in mirror alignment can prevent cavity stability, it is also critical to ensure that the EOL device does not produce an effect on misalignment with the same magnitude as that on mode mismatch. Thus, the 01 and 10 mode peaks, as percentages of the 00 mode peak, were measured to check the effect on vertical and horizontal misalignment, respectively. The misalignment peak modulation against EOL translation showed a sinusoidal pattern, similar to mode mismatch modulation. Thus, there were certain translation positions at which the misalignment modulation was maximized and those at which little to no effect was measured. By comparing the mode mismatch modulation and the misalignment modulation for a given position, we were able to confirm that there are positions at which the changes in the misalignment are minimized but the effect on mode mismatch is still visible. As an example, Figure 9 shows the effect of EOL horizontal translation on mode mismatch, horizontal misalignment, and vertical misalignment, respectively. At the 0.15 mm position, the EOL produces a relatively highly mode mismatch decrease without compromising horizontal and vertical alignment. The effect on mode mismatch is more than 50 times that on mirror alignment.

Lastly, the effect of the EOL device on mode mismatch was measured against its driving voltage, ranging from 0 V to 400 V. Contrary to expectations that the EOL effect will linearly increase with voltage, there was a range of voltages between which the EOL effect decreased with voltage. This trend was observed at different positions of the EOL device. Figure 10 shows the mode mismatch modulation against EOL driving voltage for two different positions of the EOL device in the optical arrangement. There is a clear zero crossing for both positions at which the EOL effect on mode mismatch reduction changes from negative to positive. The difference in voltage for the two zero crossings suggests an interference effect: the device's crystals may not be perfectly symmetric to each other. Since the device prototype used for characterization did not have anti-reflection coatings, the incident and reflected laser light may destructively or constructively interfere, depending on the path length difference between the two waves. The asymmetry of the devices crystals causes the path length difference to change with the position of the incident laser beam.

V. CONCLUSION

By measuring the strength of the 02/20 mode peaks in a laser frequency scan, we were able to observe a visible effect of the EOL device on mode mismatch reduction. The effect of the device decreased significantly as its center was tilted and translated away from the incident laser beam. The device characterization also showed an effect on horizontal and vertical misalignment that compara-

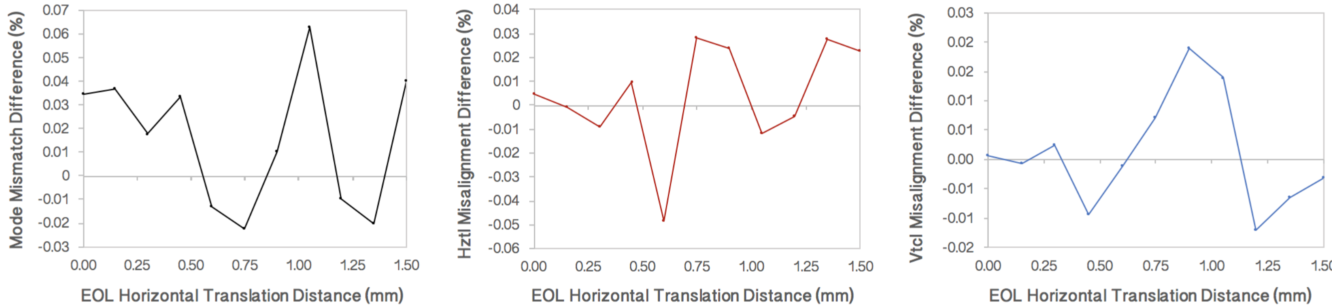

FIG. 9. The effect of EOL horizontal translation on mode mismatch percentage, horizontal misalignment, and vertical misalignment. Both mode mismatch and misalignment modulation show an oscillating trend but positions can be identified where misalignment modulation in both directions is minimal but the effect on mode mismatch is high.

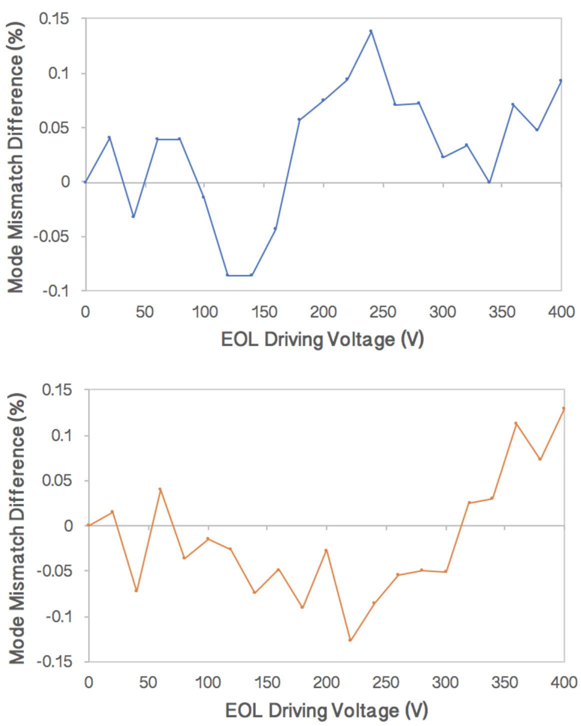

FIG. 10. Mode mismatch percentage modulation against EOL driving voltage for two EOL translation positions. Both positions show a range of voltages where the EOL effect becomes more negative with voltage. The difference in the y-intercept for the two plots suggest that the oscillating trend may be due to an interference effect from the device's asymmetric structure.

bly varied with position. Despite the similar sinusoidal trend for both mode mismatch and misalignment modulation, we were able to identify certain positons of the EOL device that produced a visible effect on mode mismatch without compromising alignment. From measuring the mode mismatch reduction against EOL driving voltage, we then observed a range of voltages that produced a decreasing effect from the EOL device as the voltage increased. Since the range of voltages changed with EOL position, the behavior may be due to an interference effect from the asymmetry of the device's crystals. To verify the cause of this effect and reduce its magnitude, more measurements must be made after the anti-reflection coatings have been added to the device.

Regarding future tasks of the EOL experiment, the Virgo Padova-Trento research group is currently automating the entire process of error detection and cavity stabilization with a PXI system using LabVIEW. While demonstration of cavity stabilization with the Pound-Drever Hall technique has been already completed, the demonstration of the mode mismatch error detection technique is still in process. For physical improvements to the device, anti-reflection coatings will be tested and added to reduce the interference effect observed in the device characterization. Furthermore, shielding material will be attached around the device to reduce the electronic pickup, detected in the cavity reflection signal, from sending the EOL voltage signal. With the promise of these physical improvements and the measured visible effect of mode mismatch, the EOL device seems to offer an effective means to detect and reduce optical loss in gravitational wave detectors.

VI. ACKNOWLEDGEMENTS

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