Methods for the Control of Coupled Fabry-Perot Cavities

Zachary Metzler

Issue:
Date: August 1, 2019

School of Physics and Astronomy
University of Birmingham
Birmingham, B15 2TT

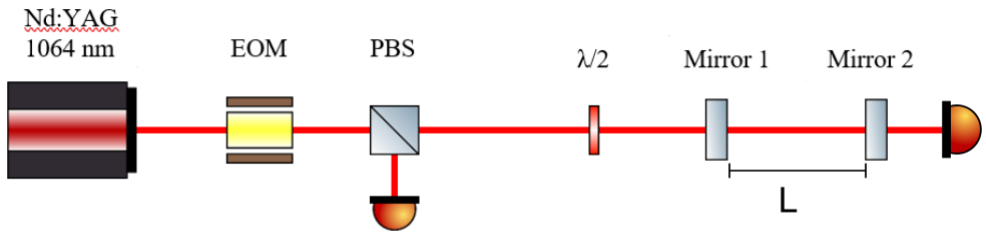

Figure 1: A schematic of a simple Fabry-Perot cavity along with two photodiodes to detect resonance.

1 Introduction and Motivation

1.1 Simple Fabry-Perot Cavities and LIGO/aLIGO

Optical cavities are the vital to the operation of modern gravitational wave detectors and many laser optics experiments.[3] A single Fabry-Perot Cavity is formed by two parallel mirrors at some distance apart. As seen in Figure 1, there is a macroscopic length L between the two mirrors, which is always a multiple of the lasers half-wavelength; however, there is also some tuning on the order of less than one wavelength. This tuning is conventionally given in degrees, where 360 degrees is equal to one wavelength. At 0 or 180 degrees, there is a resonant peak, which transmits the most light to the end photodiode, and reflects the least light to the photodiode connected to the beam splitter.

LIGO consists of a Michelson interferometer with a Fabry-Perot cavitiy in each arm. This is used to resonate the carrier frequency of the laser. In aLIGO, two Fabry-Perot cavities are also used to recycle power and the GW signal. Each of these raise the power of the pump or signal sidebands by a factor of 50 within the Michelson interferometer.[4] This improves upon LIGO, which used a marginally stable power recycling cavity between a power recycling mirror and one of the arms input mirrors. LIGO also did not use signal recycling.

1.2 Motivation

Coupled Fabry-Perot cavities expand on simple ones by splitting their resonant peaks. Where a simple cavity would have a resonant peak, a coupled cavity has two peaks. These peaks separation, respective heights, and width are dependent upon the mirrors parameters, the lengths of the sub-cavities, and beam parameters. Coupled cavity systems, however, are much less intuitive to align and control. The third mirror brings the number of sub-cavities to three and the number of degrees of freedom to two. Furthermore, positioning the mirrors to minimize losses to higher-order modes becomes more difficult as these higher-order modes may resonate throughout the coupled cavity system. This experiment aims to create a method for designing and controlling coupled cavities in order to manage these challenges.

1.3 Finesse

Finesse is the main tool used to model this coupled cavity system. It is a frequency domain simulation software with a python wrapper, PyKat. Currently, it is actively used to commission and design gravitational wave detectors and their subsystems. It is most often used in Jupyter or IPython notebooks.[2]

2 The Coupled Cavity System

2.1 Definitions

As shown in Figure 3, there are three mirrors named 1,2, and 3. The cavity formed by mirrors 1 and 2 is named Cavity A. Between 2 and 3 is Cavity B. Between 1 and 3 is Cavity C. In this experiment, the lengths of cavities A and B will be the same, in order to avoid having high power in either of the sub-cavities. With regards

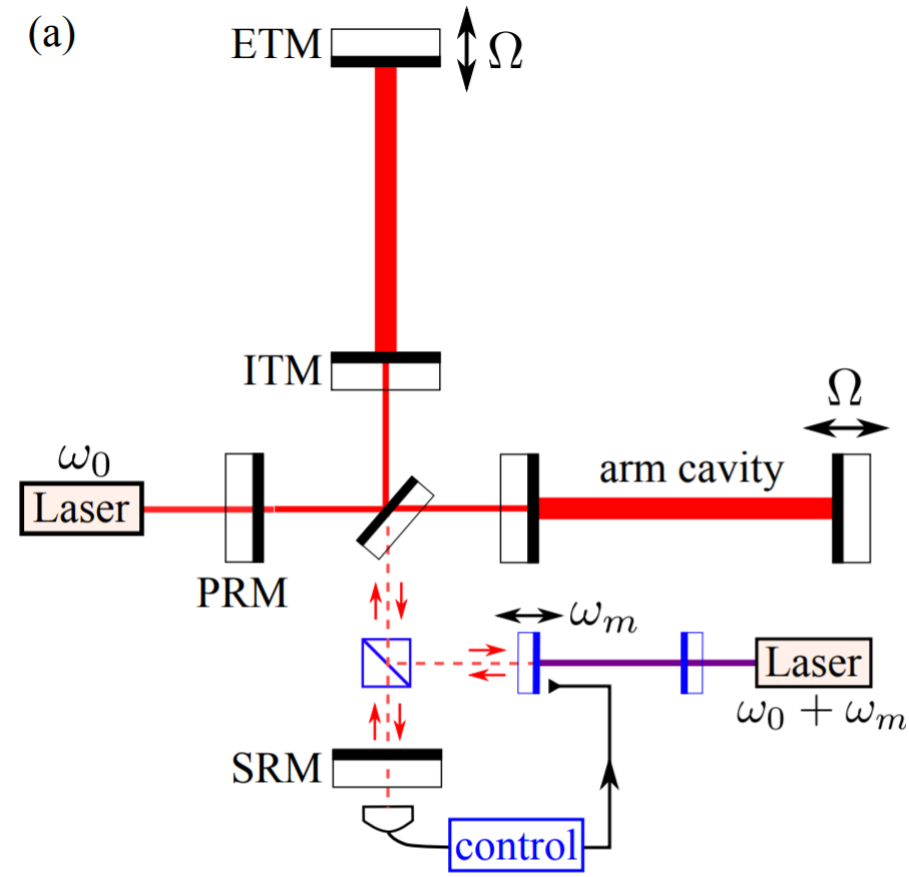

Figure 2: A schematic of aLIGO demonstrating where a coupled Fabry-Perot cavity would be used to compensate for the dispersion of the detectors arm cavities.

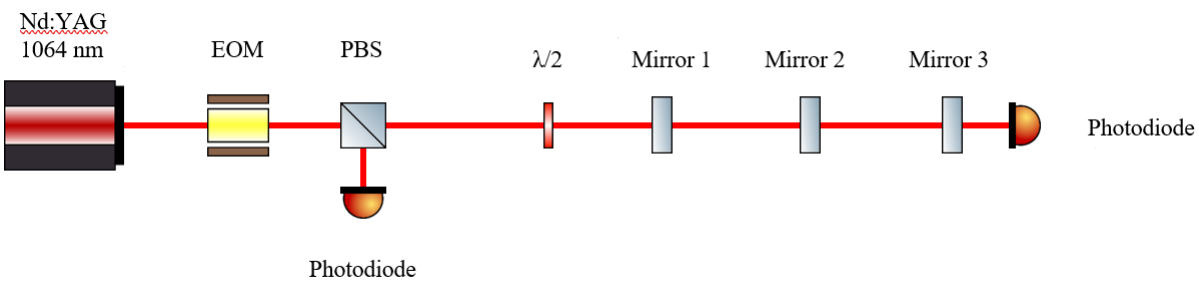

Figure 3: This is stripped-down model of the coupled cavity system, defines labels used later in this paper.

to the locking and control of the cavity, the lasers frequency and the position of Mirror 2 will be modulated over. Locking and control will be discussed more in Sections 4 and 5. The system is defined by the lengths of each sub-cavity, the transmission rates through each mirror and the radii of curvature of each mirror. For each sub-cavity there is a g-factor, which assesses the geometric stability of a potential cavity. Additionally, the frequency splitting, sharpness of the peaks at resonance, free spectral range, and finesse further define the coupled cavity system.

The following constraints are given for each of the above parameters.

- The g-factor for each cavity should be between 0.05 and 0.95, since a cavity becomes harder to control as its g-factor approaches 0 and 1. The cavities are unstable with a g-factor less than 0 or greater than 1.
- The frequency splitting should be greater than the width of the resonance peaks.
- The length of Cavity C should be less than 30 cm due to spatial constraints.
- Each cavity is over-coupled so that the light being reflected is dominated by the light from the cavity, not light simply bouncing off the input mirror.

2.2 The Chosen Parameters

This systems parameters are:

- Mirror 1 has a radius of curvature of 1m with transmission of 0.7%. The coated, curved side of the mirror faces the laser.
- Mirror 2 has a radius of curvature of 1m with transmission of 0.13%. The coated, curved side of the mirror faces the laser.
- Mirror 3 has a radius of curvature of -1m with transmission of ¡8ppm. The coated, curved side of the mirror faces away from the laser.
- The length of Cavities 1 and 2 are 0.287m.
- The frequency splitting should be 6 MHz.

2.3 The Beam Parameters

Once the cavities g-factors and lengths are defined, it is necessary to determine which beam parameters minimize the losses due to scattering into higher order gaussian modes. This is done by selecting two of the three mirrors in the coupled cavity system and setting the beam radius of curvature at each mirror to be identical to that of the selected mirrors. Figures 4-6 show the resonance peaks when mode-matching to each cavity according to a Finesse model.

3 Cavity Alignment

3.1 Methods

The alignment is easiest when done in multiple phases. Figure 7 shows the optical bench set-up for this experiment. The modulators and four surrounding lenses were already in place before this experiment began.

The first phase was to align the steering mirrors and two lenses between the modulators and the coupled cavity system. This was done by arbitrarily picking a path for the coupled cavity and aligning the steering mirrors to the path. Then, the lenses were included and were constrained to the path as well. The lenses are positioned such that the beam waist is approximately 372 um and located beyond the expected position of Mirror 3. This configuration is determined by calculating the ideal beam parameters such that the radii of curvature of Mirrors 1 and 3 match the beams radii of curvature. This produces resonance peaks as shown in Figure 6. Furthermore, it is more important that the beam waist be positioned properly than the beam waists size be perfect in terms of preserving the symmetry of the frequency splitting as shown in Figures 8,9. Next, the

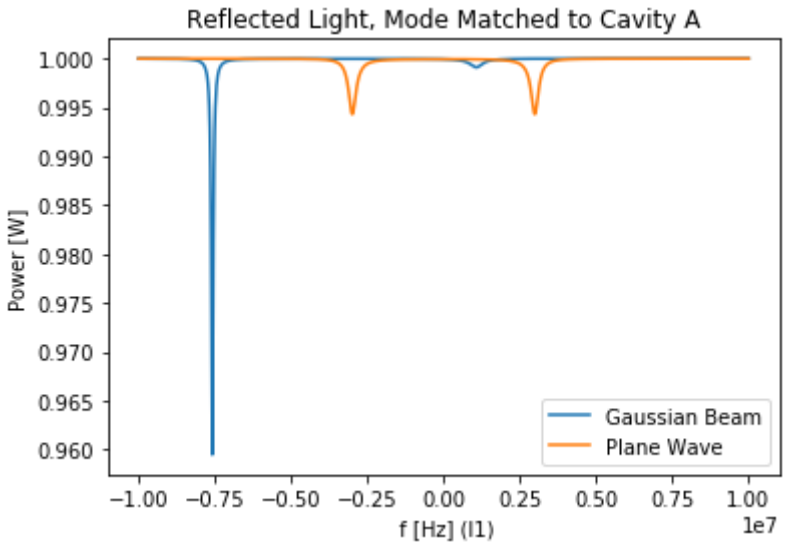

Figure 4: The reflected light when mode-matched to Cavity A over a range of laser modulation frequencies is shown here. In orange, there is a plane wave, which acts as though all the mirrors are flat. In blue, there is a gaussian beam, which has been mode-matched to Cavity A.

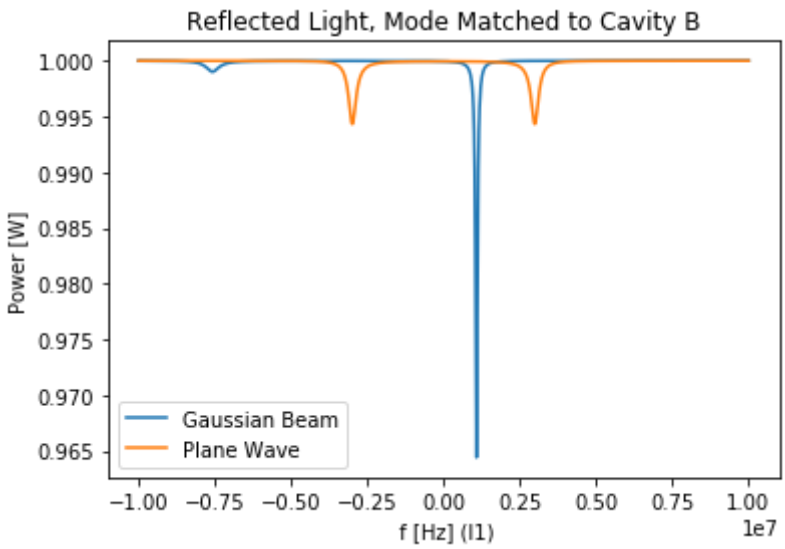

Figure 5: The reflected light when mode-matched to Cavity B over a range of laser modulation frequencies is shown here. In orange, there is a plane wave, which acts as though all the mirrors are flat. In blue, there is a gaussian beam, which has been mode-matched to Cavity B.

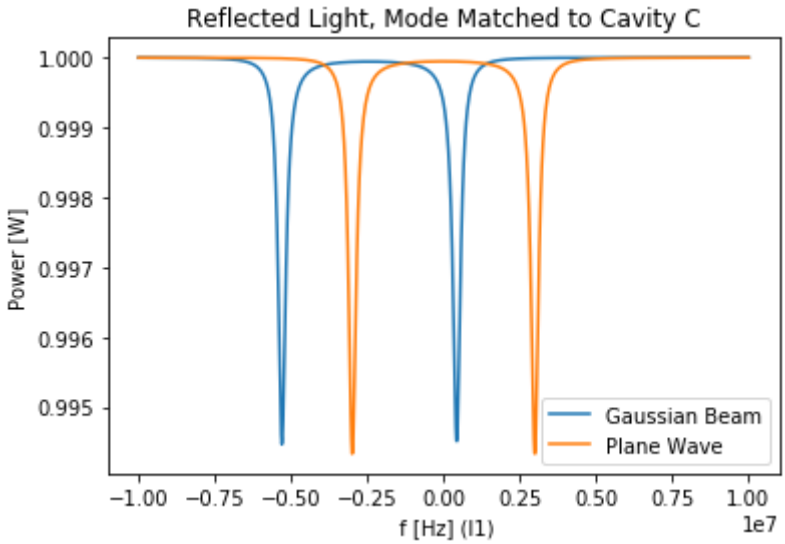

Figure 6: The reflected light when mode-matched to Cavity C over a range of laser modulation frequencies is shown here. In orange, there is a plane wave, which acts as though all the mirrors are flat. In blue, there is a gaussian beam, which has been mode-matched to Cavity C.

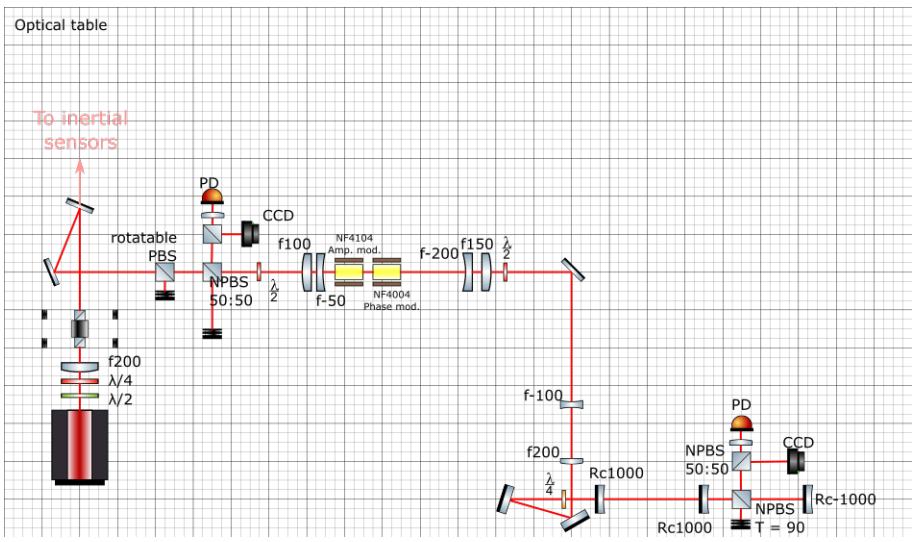

Figure 7: This shows the entire bench setup. The beam splitter in the bottom right-hand corner is to be used for alignment. The beam splitter in the top left-hand corner is to be used for both alignment and control.

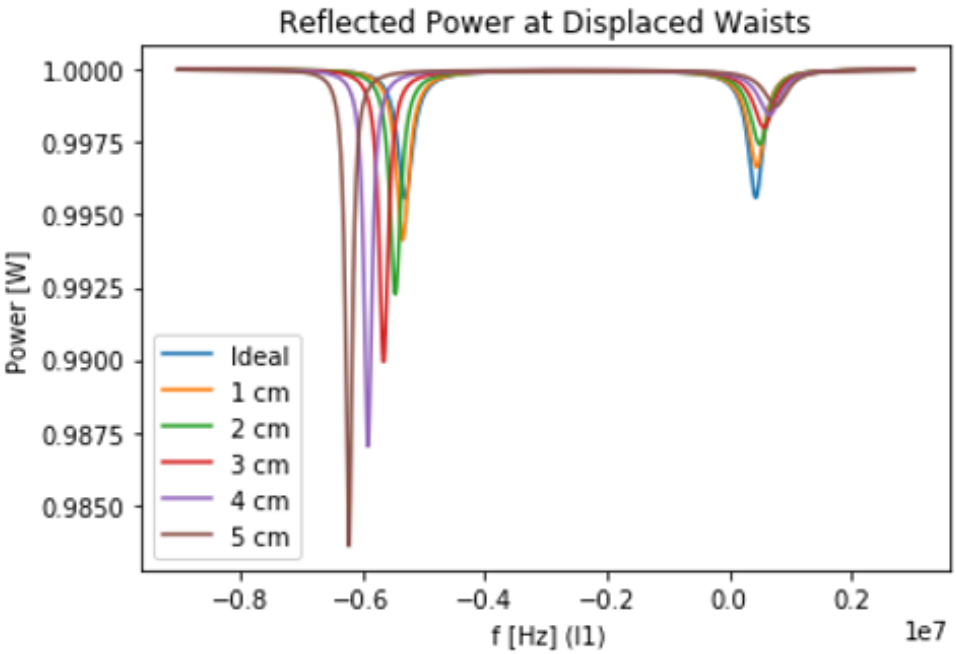

Figure 8: This demonstrates the effect of misplacing the beam waist by different values on the order of a few cm. This creates mode-mismatch and alters the symmetry of the frequency splitting.

two beam splitters mentioned above are positioned, and the alignment beam splitter is positioned such that its surface is aligned in reflection. Using the information regarding the location of the waist, Mirror 3 is positioned and aligned using reflection with the first beam splitter.

The following step is to align Mirror 2. This is accomplished in two parts: alignment using reflection and the first beam splitter, and alignment using the camera and photodiode at the second beam splitter.

3.2 Using a Beam Splitter Replaces Reflection, Transmission for Fine Alignment

Conventionally, Fabry-Perot cavities are aligned using reflection or transmission; however, the high reflectivity of the cavity mirrors make each of these options unfeasible. Transmission is ineffective due to the cavity being highly over coupled, so there is not enough transmitted light to be useful. Furthermore, reflection is not as effective as using a beam splitter inside the cavity. This is due to the large percentage of light reflected towards the photodiode even if the cavity is aligned.

As seen in Figures 10,11, any of the three techniques could be used to align Mirror 2 if Mirror 3 has a transmission rate of 10-5; however, the transmission is lower than this. Therefore, transmission can still be ruled out. With regards to reflection, the high finesse of the cavity makes the peaks in reflection very narrow. This makes difficult detecting the peaks and thus aligning Mirror 2. Meanwhile, using a beam splitter for alignment will allow for the alignment of both mirrors. Using a beam splitter is still not ideal. The high loss introduced widens the resonance peaks, but also lowers the peaks far below that in transmission or reflection. Additionally, once the cavity is aligned, the beam splitter will be removed. This creates risk of misaligning the cavity. Despite this risk, a slight misalignment can be accounted for once the beam splitter is removed to restore alignment when controlling the system.

4 Controlling the Coupled Cavity System

In this control scheme, one modulation frequency at two phases will be used to control the coupled Fabry-Perot cavities. The schematic for the control system is shown in Figure 12.

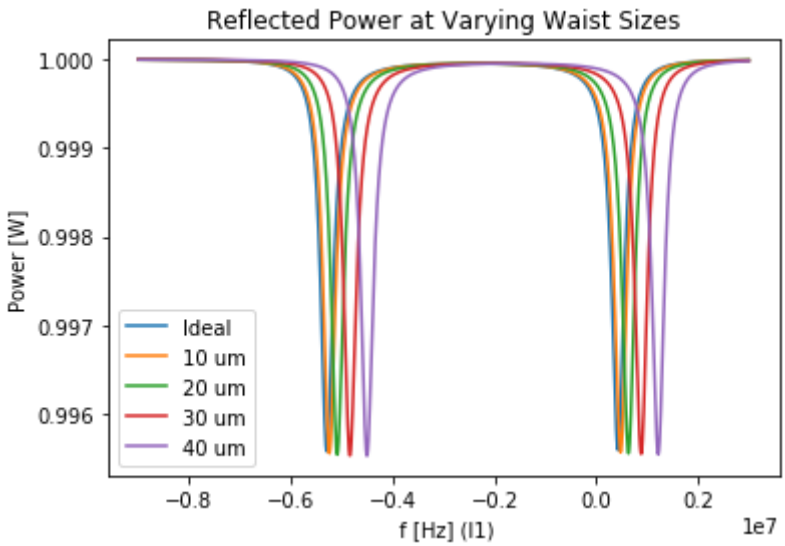

Figure 9: This demonstrates the effect of altering the beam size by different values on the order of 10s of um. This creates a slight mode-mismatch, but does not change the symmetry of the frequency splitting. However, it does shift the position of the frequency splitting towards the plane wave case (Fig. 6).

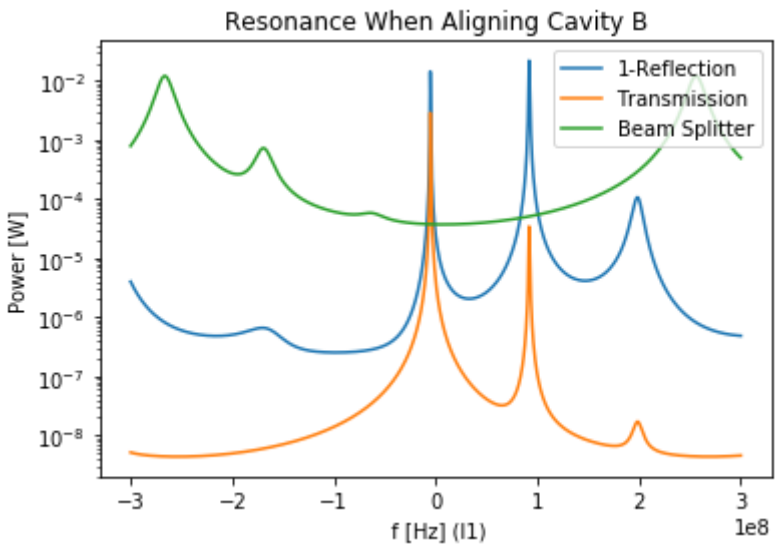

Figure 10: The resonance peaks are shown for three methods of aligning Mirror 2. The transmission rate for Mirror 3 is 10-5. The beam splitters transmission rate is 90%.

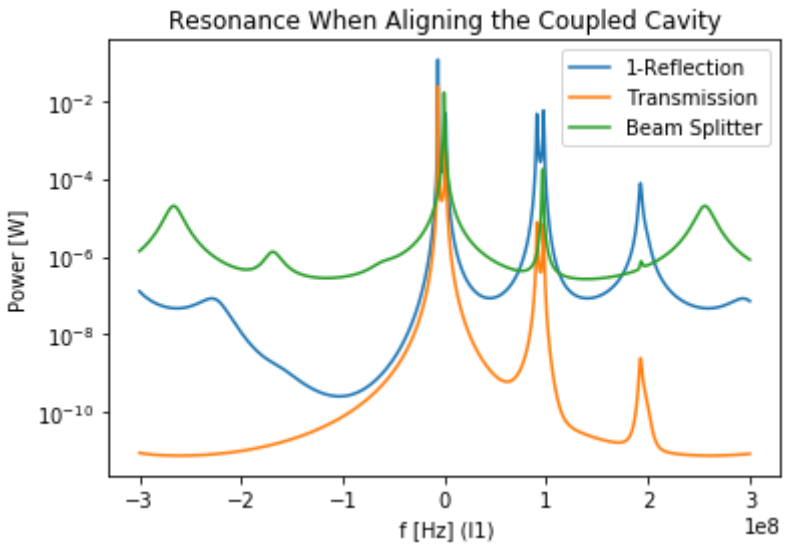

Figure 11: The resonance peaks are shown for three methods of aligning Mirror 3. The transmission rate for Mirror 3 is 10-5. The beam splitters transmission rate is 90%.

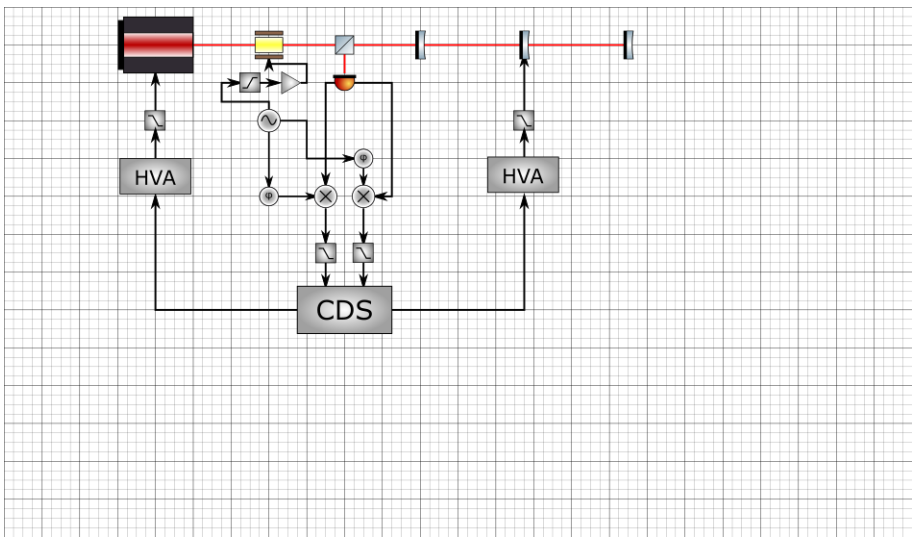

Figure 12: The schematic for the control system of the coupled Fabry-Perot cavities. This includes modulating on an amplitude modulator and a readout in reflection. The modulation frequency is then offset by two frequencies and mixed with the readout from the photodiode. This enters the control system, which will lock the laser first and Mirror 2 second.

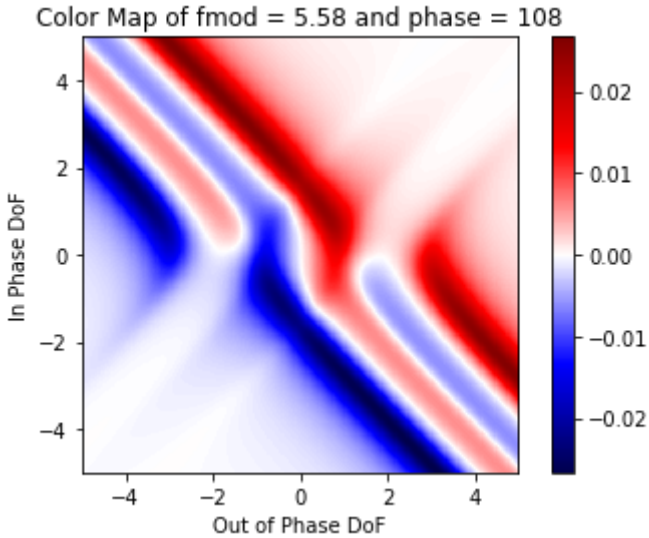

Figure 13: This heatmap shows where a lock could occur for any pair of degree of freedoms at 108-degree demodulation. Lock occurs along the stark white line along any of the stark white lines.

The control process begins by applying a modulation signal to the amplitude modulator at 5.58 MHz. This creates changes in the cavity, which are detected at the photodiode and mixed with the original 5.58MHz modulation frequency at offsets of 50 and 108 degrees. The demodulated signals are then sent to the control and data acquisition system (CDS). Within CDS, each signal is used to determine how to alter each degree of freedom to achieve lock. The first degree of freedom is the effective length of Cavity C. This is done by modulating the laser frequency using the 108-degree signal, as described in the next section. Second, CDS will operate on the position of Mirror 2 using information from the 50-degree signal. This controls the second degree of freedom, the difference between the lengths of Cavities A and B.

Furthermore, as we lock each degree of freedom separately, we can model in Finesse the allowable uncertainty of the other degree of freedom. This is demonstrated in Figures 13-14. As an example, when locking the out of phase motion, which corresponds to the lasers frequency, Figure 13 is used. As CDS locks to 0 degrees for the laser frequency, there is a window of about 2 degrees for Mirror 2 to move. If Mirror 2 moves outside of this window, CDS will lock to the wrong position. A similar process occurs for using Figure 14 to lock Mirror 2s position.

5 Error Signals

5.1 Definitions and Standards

Once the cavity is aligned, the next step is to control it so that it remains resonant. In order to accomplish this, an error signal is used to inform CDS where to lock the coupled cavity system. There are two degrees of freedom, which need to be locked. The first is the length of Cavity C, which equivalently would be the frequency of the input laser. The second degree of freedom is the differential motion between the lengths of Cavities A and B, which equivalently would be motion of Mirror 2. Since there are two degrees of freedom, two error signals are needed to control the system. A pair of error signals, one for each degree of freedom, is defined by the modulation frequency and the phase offset. Ideally, only one pair of error signals would be completely diagonalized. This situation means that as you modulate your signal, the ideal position of the sensitive degree of freedom is well defined, while having zero effect on the other degree of freedom. Unfortunately, there is no such pair of error signals for this coupled-cavity system, so the goal is to have two pairs of error signals at the same modulation frequency and as large of a phase difference as possible. It would be easiest if, in each pair, one degree of freedom was sensitive and the other was insensitive. However, it is possible to use one pair of error

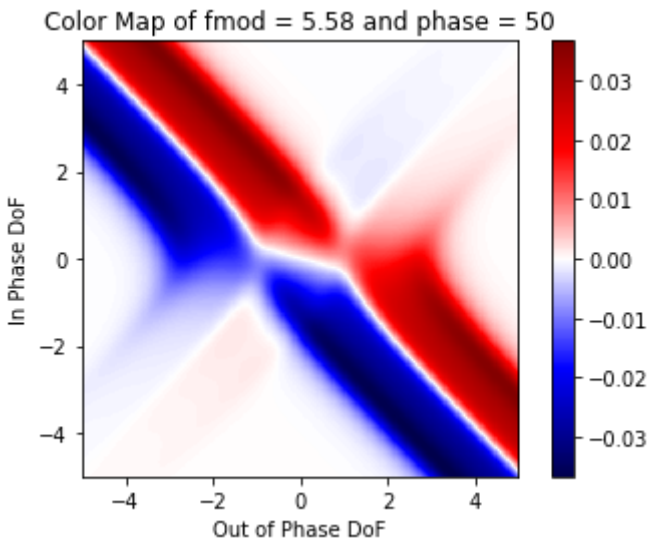

Figure 14: This heatmap shows where a lock could occur for any pair of degree of freedoms at 50-degree demodulation. Lock occurs along the stark white line along the diagonal.

signals as described above and one pair with two steep signals at the origin, since we can programmatically compensate for the signal that should be flat in the second pair. The error signals used in this experiment are shown in Figures 15,16.

5.2 Error Signal Selection Process

In order to filter through the set of all possible pairs of modulation frequencies and demodulation phases, several supplemental plots are used. First, the error signals are replaced by a transfer function, which displays the intensity of the transmitted light over a signal frequency. The intensity as the signal frequency vanishes is the slope of the error signal at the origin. Transfer functions are unique for some pair of modulation frequency and demodulation phase. The transfer functions for this experiment are given in Figures 17,18.

The next plot takes the point on a set of transfer functions where s=0 and shows the intensity at this point for a range of demodulation phases. This can also be thought of as plotting the slopes of the error signals at the origin for a range of demodulation phases.

In order to determine sets of valid error functions, pairs of modulation frequency and demodulation phase had to meet several criteria. For both the primary and secondary locks, the error signal could not cross the horizontal more than once. If either crossed the axis multiple times, this would run the risk of locking to the wrong position. Additionally, for the primary signal, one of the degrees of freedoms slope must be near zero. Once each of the pairs of modulation frequency and demodulation phase were found that met the primary signal conditions, all usable secondary signals at the same modulation frequencies as the primary signals were collected. This was done without picking which degree of freedom would be locked first. Finally, the triplets, consisting of modulation frequency and two demodulation phases, were sorted through manually in order to find the best option. See Figures 15,16 for the error signals chosen in this experiment.

6 Applications

Following the design, alignment, and control process outlined here, coupled Fabry-Perot cavities can be applied to many areas of modern physics research. One such application is extending sensistivity towards higher frequencies in gravitational wave detectors. An unstable optomechanical filter with a single cavity has been proposed to compensate for the dispersion of the detectors arms. (See Figure 2) This would be done by inducing a beat between radiation from the detector at 0 and radiation inside the filter at 0 + m, where 0 is the resonant

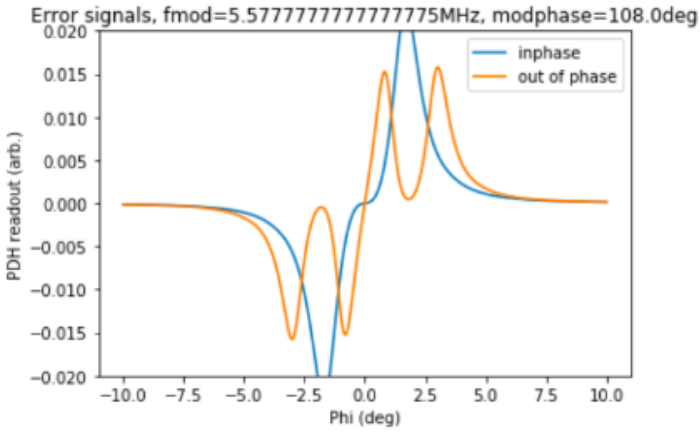

Figure 15: The first pair of error signals, which will be used to control the out of phase or differential degree of freedom. The inphase or common degree of freedom is flat at the origin, so this is unaffected within the flat area by the modulation.

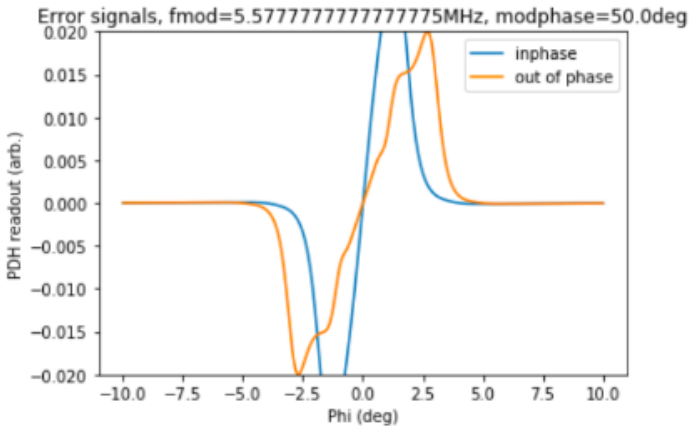

Figure 16: The second pair of error signals to control the inphase or common degree of freedom, while using the other pair of signals to eliminate the changes in the out of phase or differential degree of freedom.

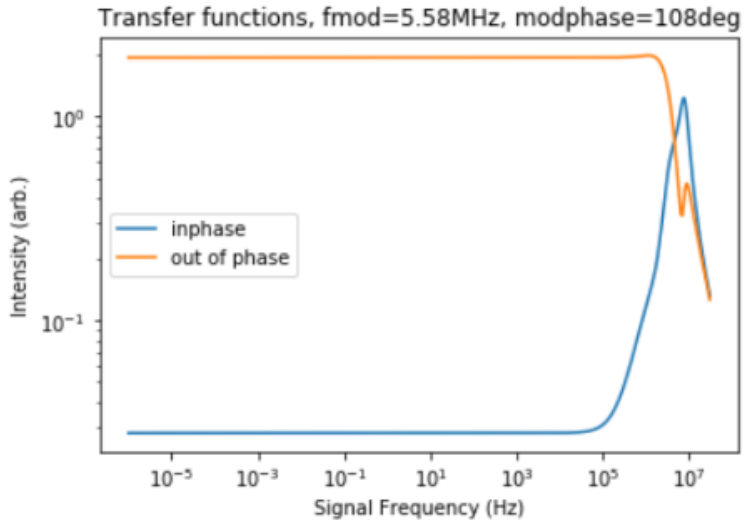

Figure 17: The first pair of transfer functions plot the slope at the origin of the corresponding error signals over a set of signal frequencies at one demodulation phase. The plateau to the left of the plot is the slope at the origin of Figure 15.

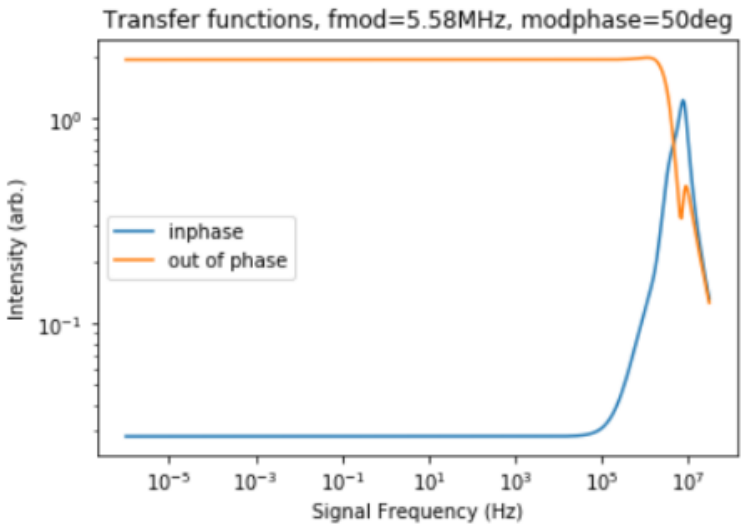

Figure 18: The second pair of transfer functions plot the slope at the origin of the corresponding error signals over a set of signal frequencies at one demodulation phase. The plateau to the left of the plot is the slope at the origin of Figure 16.

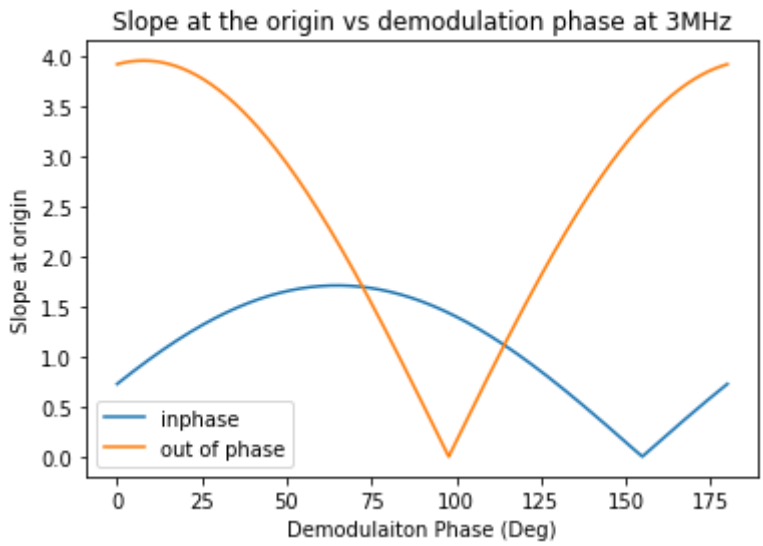

Figure 19: This is an example of the type of plot used to visualize the steepness and flatness of an error function. Each point is the slope at some demodulation phase, while the entire graph is at a given modulation frequency. This plot is at 3MHz modulation.

frequency of the detector and the filter cavity and m is the mechanical resonant frequency of the optomechanical element.[1]

However, the advantage to using a coupled cavity system is the split resonance. Resonating at both 0 and 0+ m would compensate for the arm lengths dispersion by a factor at the same order as the finesse of the cavity system. A similar mechanism can also be used as a cooling scheme. If instead there was a sideband at 0 - m, the resonant frequency removes mechanical energy from the system. Since 0 resonates in the cavity, and the pump frequency is less, the beat frequency at m causes the mirror to cool to quantum levels.

Finally, the method presented in this paper can be extended to any order of cavity. With four mirrors, there would be three resonant peaks, which could increase the sensitivity in LIGO by having sidebands at $\omega_0, \omega_0 + \omega_m$, and $\omega_0 + 2\omega_m$. This raises the sensitivity floor, retains symmetry about the pump frequency, and more accurately measures the movement of the arm cavities.

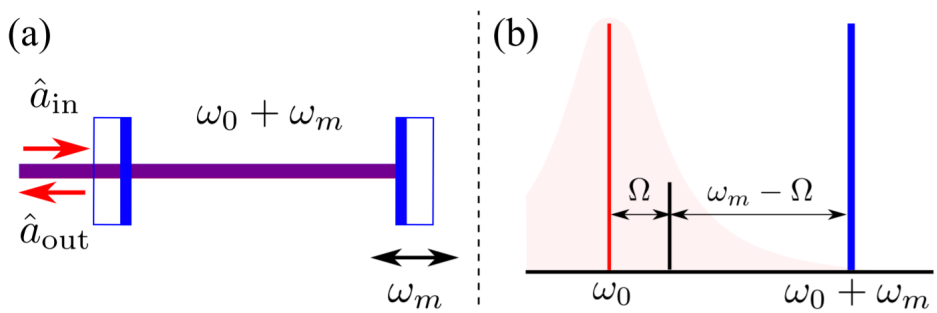

Figure 20: (a) A close-up schematic of the optomechanical filter (b) frequencies of interest: 0 is the resonant frequency of the cavity, m is the mechanical resonance of the filter, and 0+m is the frequency of the input laser, the other sideband at 0+2m is not shown.

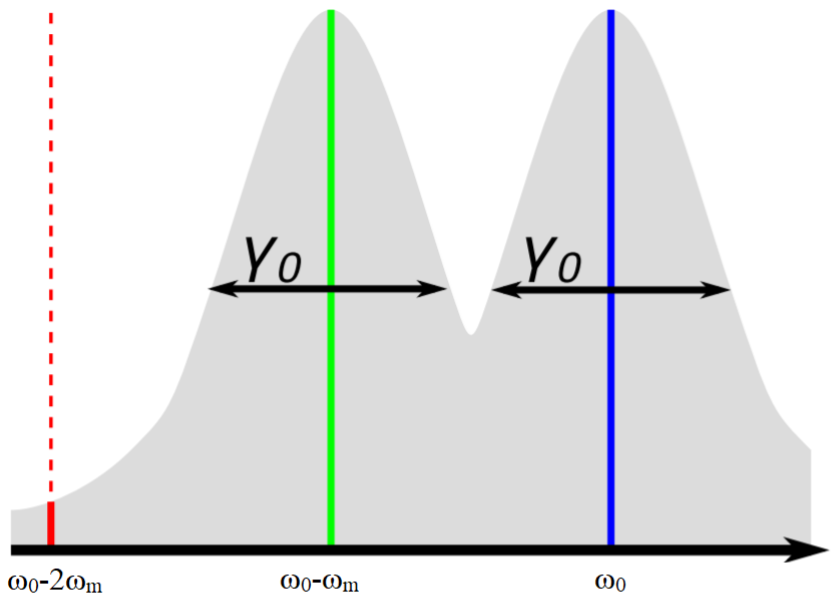

Figure 21: Frequencies of interest: frequencies of interest: ω_0 is the resonant frequency of the cavity, ω_m is the mechanical resonance of the filter, and $\omega_0 - \omega_m$ is the frequency of the input laser, $\omega_0 - 2\omega_m$ is the other sideband.

References

[1] Yanbei Chen. "Enhancing the bandwidth of gravitational-wave detectors with unstable optomechanical filters". In: *Physical Review Letters* (2015).

[2] Artemiy Dmitriev. *Gravitational Wave Advanced Detector Workshop*. https://agenda.infn.it/event/15928/contributions/89744/attachments/63368/76286/GWADW19_talk.pdf. Accessed on 2019-07-28. 2019.

[3] Andreas Freise. *gwopotics*. http://www.gwoptics.org/learn/. Accessed on 2019-07-28. 2009.

[4] Guido Mueller. "Design of the Advanced LIGO recycling cavities". In: *Optical Society of America* (2008).