

Gravitational Physics International REU  
*Research at the Australian National University*

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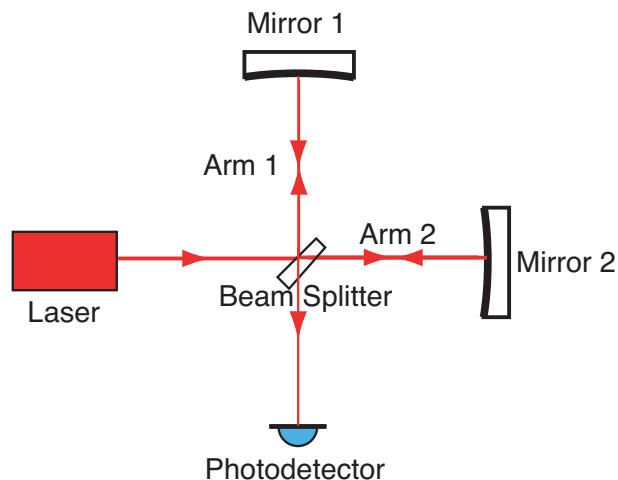
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## 1. Gravitational Waves & Interferometry Background

The classical Newtonian theory of gravity was reconciled with special relativity when Einstein published his General Theory of Relativity in 1916. Einstein proposed that gravity is not a force but rather a result of the geometry of space-time. Because massive objects curve space-time, an accelerating mass would cause the surrounding curvature to change. These perturbations in space-time can cause ripples to propagate away from curvature-changing events, and these ripples are called gravitational waves.

Until recently, the area of gravitational physics was dominated by theorists. Experiments to test theories of gravity and the existence of gravitational waves were rare due to a lack of technology. The area of experimental gravitational physics, however, has grown significantly with the development of new technology and sensitive instruments allowing for small-scale measurements.

One very promising setup intending to directly measure gravitational waves utilizes a Michelson interferometer configuration. A simple Michelson interferometer consists of a light source split into two beams by a beam splitter. Each beam travels a distance down the "arm" of the interferometer before being reflected back by a mirror at each arm's end. The two reflected beams recombine at the beam splitter and the resulting intensity is measured by an instrument called a photodetector. A schematic of a simple Michelson interferometer is given in Figure 1. Thus an interferometer compares the amount of time it



**Figure 1.** A simple Michelson interferometer. After being reflected by mirrors at the end of each arm, the light beams interfere at a beam splitter and are added according to the superposition principle.

takes for light beams in the two arms to complete their paths. When a gravitational wave passes by, it distorts the space-time around it, causing small changes in the lengths of an

interferometer's arms. The Laser Interferometer Gravitational-Wave Observatory (LIGO) operates three interferometers, two having 4-km arms and one having 2-km arms. An international collaboration between LIGO and other groups in gravitational physics has been established, and the area of gravitational wave detection continues to grow. By having a network of interferometers in different locations, the positions of gravitational wave sources can be determined by a method called triangulation.

Large amounts of energy carried in a gravitational wave only distort the surrounding space by a small amount, and thus the length changes in the interferometer arms are very minimal. Coalescing black holes, binary neutron stars, and other astronomical events are likely to be the most probable source of gravitational waves that current interferometers will be able to detect [2]. The strain of an interferometer is defined to be:

$$h = \frac{2 \Delta L}{L} \quad (1)$$

where  $h$  is the strain,  $L$  is the arm length of the interferometer, and  $\Delta L$  is the differential length change in the arms [5]. Gravitational waves are predicted to have strains on the order of  $h \approx 10^{-21}$  [2]. For an interferometer having 4-km arms, we can rearrange equation 1 to solve for the expected change in arm length  $\Delta L$ :

$$\Delta L = \frac{hL}{2} = \frac{(10^{-21})(4000 \text{ m})}{2} = 2 \times 10^{-18} \text{ m} \quad (2)$$

This means that to see the effect of gravitational waves, one must be able to measure extremely small changes in length. Notice that  $\Delta L$  could be increased by increasing the arm length  $L$  of the interferometer. For this reason Fabry-Perot cavities are often added in the arms to increase the effective length of the light traveled.

Lasers are the light sources used in gravitational wave interferometers because the emitted light is coherent and in phase [1]. When an atom is raised into an excited energy state, it can revert to a lower energy state through a process called photon emission. When the atom is stimulated by electromagnetic radiation to release the photon, the mechanism is referred to as stimulated emission. When this process is used, the emitted photon propagates in the same direction, has the same polarization, and is in phase with the electromagnetic radiation that triggered the process. When a large percentage of the atoms in a system are excited, the necessary population inversion is achieved. A pump source allows the laser to emit a continuous beam of coherent light.

For the setup in Figure 1, the electric field of the laser can be expressed as  $E_{laser} = E_0 e^{i\omega t}$ . Assuming none of the laser's energy is lost or absorbed, a 50/50 beam splitter will reflect half of the light into one arm and transmit half of the light down the other arm. When the reflected beams are recombined at the beamsplitter, the resulting electric field at the photodetector is:

$$E_{PD} = \frac{E_o}{\sqrt{2}}e^{i(\omega t + \phi_1)} + i\frac{E_o}{\sqrt{2}}e^{i(\omega t + \phi_2)} \quad (3)$$

where  $\phi_1$  and  $\phi_2$  are the phase shifts resulting from the roundtrip time in arm 1 and arm 2. Thus, denoting  $E_{PD}^*$  as the complex conjugate of  $E_{PD}$ , the power detected by the photodetector will be:

$$P_{PD} = E_{PD}^* E_{PD} = E_o^2 - E_o^2 \sin(\phi_2 - \phi_1). \quad (4)$$

Even with current technology and the most sensitive instruments, achieving the desired performance in gravitational wave detectors is very challenging due to various types of noise. Seismic noise, shot noise, thermal noise, radiation pressure noise, and gravity gradient noise limit the sensitivities of the interferometers [5]. Many feedback mechanisms have been developed to compensate for these types of noise. One common method used in measuring the length noise in a cavity is the Pound-Drever-Hall technique [3]. This technique can be used to stabilize a laser's frequency by locking it to a Fabry-Perot cavity (or vice versa). By setting up a feedback mechanism, one can measure the noise in the cavity based on the amount of compensation performed by the feedback circuit to keep the system in resonance. Because the photodetector only provides a measurement of the intensity at the output, the phase of the input light needs to be modulated so a proper feedback circuit can be implemented. Modulating the input light allows the system to know which side of resonance it is on and thus how to respond to re-achieve resonance.

## 2. Digitally Enhanced Heterodyne Interferometry

Obtaining precise measurements of the displacements between mirrors or other optical components would be very useful in providing accurate feedback to a system and maintaining lock. Digitally Enhanced Heterodyne Interferometry [5] is a technique that modulates a Pseudo-Random Noise (PRN) code on the laser to provide realtime information on the relative positions of multiple mirrors.

Heterodyne interferometry involves combining a high-frequency signal with another to produce a low-frequency signal. An Acousto-Optic Modulator uses an acoustic wave to shift the frequency of the light. When an Acousto-Optic Modulator is used in an interferometer arm to shift the beams by  $\omega_{AOM}$ , then the electric field of frequency-shifted beam [which we will call the Local Oscillator (LO)], can be expressed by:

$$E_{LO} = E_o e^{i[(\omega + \omega_{AOM})t + \phi_1]} \quad (5)$$

When the LO beam combines with the beam from the other arm, the resulting electric field at the photodetector is:

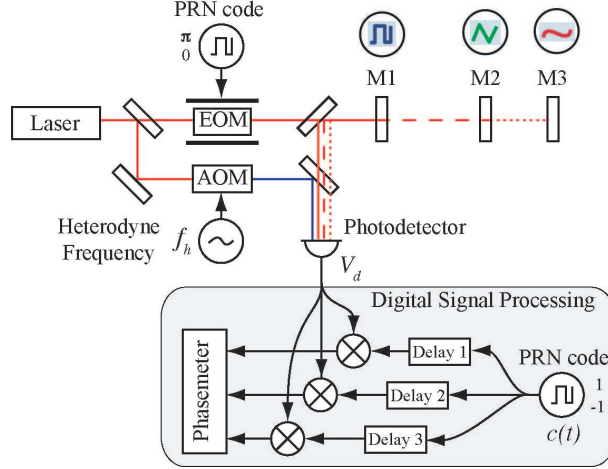
$$E_{PD} = \frac{E_{LO}}{\sqrt{2}} + i\frac{E_o}{\sqrt{2}}e^{i(\omega t + \phi_2)} \quad (6)$$

and the power detected at the photodetector is:

$$P_{PD} = E_{PD}^* E_{PD} = E_o^2 - E_o^2 \sin(\omega_{AOM}t + \phi_{LO} - \phi_2) \quad (7)$$

Note how this result differs from Equation 4.

In our experiment, we used a beam splitter to split the laser into two beams: a Local Oscillator (LO) and a probe beam. Please see figure 2 for a diagram of the experimental setup. An Electro-Optic Modulator (EOM) was used to modulate the phase of light. A



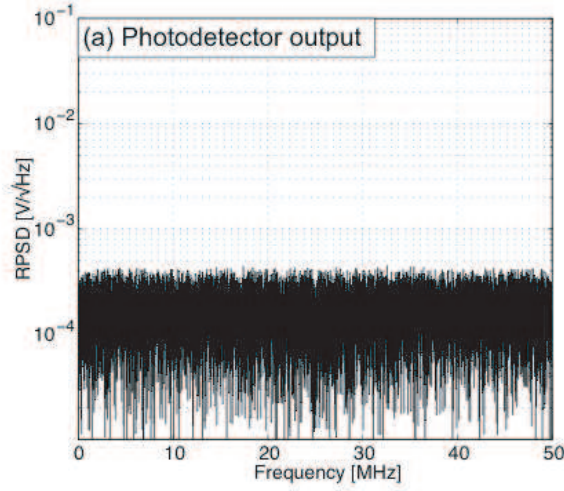
**Figure 2.** A Digitally Enhanced Interferometer. This was the setup used for the PRN experiments. Illustration with permission from reference Shaddock.

pseudo-random noise code generator sent a PRN code to the EOM, and the EOM modulated the phase of the light on the probe beam appropriately. The phase was either shifted by 0 or  $\pi$ . This modulated light was reflected off three mirrors, combined with the frequency-shifted LO, and was observed at the photodetector. Figure 3 shows a simulation of the output of the photodetector.

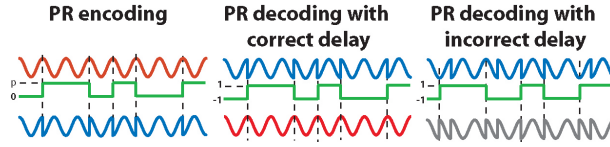
The modulated signal could be recovered by multiplying the signal by the original signal but with an appropriate time delay. By using appropriate time delays, reflections from individual optical components were isolated. Figure 4 provides an excellent demonstration of the technique.

To test the effectiveness of this technique, we modulated the first mirror in our experiment with a square wave, the second mirror with sawtooth wave, and the third mirror with a sine wave. Appropriate times delays were applied in LabView and we were able to track the movement of each mirror. See Figure 5.

Though the technology and the use of PRN is not new, this is the first time that this technique has been applied to gravitational wave detectors. This technique may be very useful in lock acquisition of Advanced LIGO and the Laser Interferometer Space Antenna



**Figure 3.** A simulation of the output of the photodetector before demodulation. Illustration with permission by Reference [4].



**Figure 4.** A PRN code modulates the phase of the light by 0 or  $\pi$ . When an appropriate time delay is applied to the original code, the time-shifted code can be multiplied to the photodetector signal to obtain the original signal. When an incorrect time delay is applied, demodulation will not result in the carrier signal.

(LISA). Because the signals from each optical component are isolated, the overall sensitivity of the system is increased. Spurious interference is suppressed by  $1/(2^N - 1)$ , where  $N$  is the length of the code. This technique still retains the sensitivity of interferometry, providing a full-range readout ( $0$  to  $2\pi$ ). Applying multiple delays to the same signal, one output signal can be decoded to isolate different reflections. The technique would be especially useful in LISA as it has the ability to track motion over multiple fringes. Further work and experiments will be performed at the ANU to optimize the code and improve suppression by using a longer averaging time.

**Figure 5.** Preliminary experimental demonstration of technique employing PRN to track displacement between mirrors.

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