Alignment of a Practical 3D Displacement and Frequency Noise Free Interferometry (DFI) Setup

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Introduction

Displacement and Frequency Noise Free Interferometry (DFI) is a new method of gravitational wave detection. It distinguishes itself from other forms of interferometry based gravitational wave detectors by using multiple interferometers for redundant sensing of the displacement of the test masses to cancel out all kinds of displacement noise. The belief previously held was that free falling test masses were necessary to detect signals as small as a gravitational wave, but DFI has shown that this stipulation is unnecessary. This results from the fact that mirror displacement and gravitational wave signals affect the propagation of the laser along the beam axis differently.

Background / Previous Work

Displacement and Frequency Noise Free Interferometry was first proposed by S. Kawamura and Y. Chen (2004 Phys. Rev. Lett. 93, 211103)[1]. In this letter, they make the case for DFI as a new method of gravitational wave detection free of many of the pitfalls of other methods. The following is a brief summary. A thought experiment is put forth in which three freely falling objects (A, B, and C) are placed on a line with B in the center of A and C. A, B, and C are assumed to each contain an initially synchronized clock, and there is a gravitational wave which is propagating along a path normal to the line connecting these three objects and has a period that is equivalent to distance between A and B divided by the speed of light (figure 1). Note that all the objects are subject to random fluctuations in position. A burst of light is emitted from object B toward both A and C at a time measured by B, and B_e is the displacement of B at the time of emission. The pulse will be reflected by A and C at times A_r and B_r measured by A and C, respectively. The pulses will then reach B again at times B_{rA} and B_{rC}, respectively, as measured by B. The difference between the

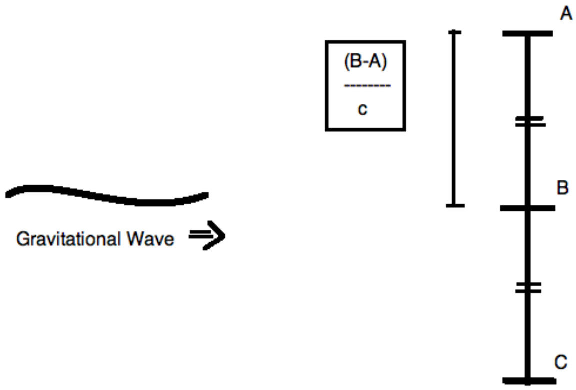


Figure 1

two times is of the order $O(\frac{hL}{c})$ for a gravitational wave with an amplitude of *h*. So the difference between $\frac{B_{rA}}{c}$ and $\frac{B_{rC}}{c}$ is of the order $(\frac{v}{c})(\frac{hL}{c})$ where v is the velocity of object B. Since v is much less than c, we can say that $B_{rA} \simeq B_{rC}$. Letting S_g represent the gravitational wave signal, the light travel times are then given as:

$$\tau_{BA} = +S_g + \frac{(B_e - A_r)}{c} \qquad (1)$$

$$\tau_{AB} = -S_g + \frac{(B_{rA} - A_r)}{c} \qquad (2)$$

$$\tau_{BC} = +S_g + \frac{(C_r - B_e)}{c} \qquad (3)$$

$$\tau_{CB} = -S_g + \frac{(C_r - B_{rC})}{c} \qquad (4)$$

These four quantities can be combined in such a way as to cancel out the displacements.

$$\tau_0 = \tau_{BA} - \tau_{AB} + \tau_{BC} - \tau_{CB} \qquad (5)$$

Working this out yields simply $\tau_0 = 4S_g$. This is significant because a signal that had been riddled with displacement noise is now purely comprised of the gravitational wave signal.

This is all made possible by the fact that laser interferometers are affected differently from mirror displacement than they are by gravitational waves. The effects of mirror displacement are compounded at the location of the mirror, whereas the effects of a gravitational wave are continuous along the propagation path of the laser. A previous experiment [2] that utilized a two dimensional version of the three dimensional instrument which this paper will go on to discuss demonstrates this principle. Their setup made use of a pair of counterpropagating Mach-Zehnder interferometers aligned on a plane, as opposed to the non-planar setup used in this paper's experiment, to demonstrate the cancellation of mirror displacement noise (figure 2). It made use of four folding mirrors, FM1 and FM2, for the inner Mach-Zehnder and FM3 and FM4 for the outer. By combining the two signals from the inner interferometer, the noise resulting from the displacement of FM1 and FM2 is canceled out. The second, outer interferometer is similarly insensitive to the motion of FM3 and FM4. Since these two bidirectional interferometers share beam splitters, combining all four signals allows us to cancel the noise of the displacement of the beam splitters, and thus allows the system to be completely insensitive to displacement noise. An electro-optical modulator (EOM) is placed at the location of FM1 and used to simulate the signal of mirror displacement. A second EOM, which is moved from one place along the laser's path to the next with the signals from each run being added together, is used to measure the interferometer's response to gravitational waves. When the displacement noise is simulated in FM1, this introduces a phase change to the laser beams which are reflected off of the mirror in each direction. Since this is at the center of the beam path, and the effect of displacement noise is concentrated in only the one location, the evidence of this phase change will reach output 1 and output 2 at the same time, and thus be canceled out. However, one can think of the effect of a gravitational wave as giving the medium of laser propagation a refractive index that varies with time and position. The simulated gravitational wave, which will affect the laser's phase at every point and not just the midpoint, should be preserved. The transfer function from the displacement of FM1 to the output sensor is

$$H_{D1(a,b)} = \pm\left(\frac{\omega}{c}\right)\exp\left(-il_{1B}\frac{\Omega}{c}\right) \qquad (6)$$

$$H_{D2(a,b)} = \pm\left(\frac{\omega}{c}\right)\exp\left(-il_{1\alpha}\frac{\Omega}{c}\right) \qquad (7)$$

and assuming that $l = l_{1\alpha} = l_{1B}$ since FM1 is at the midpoint of the beam's propagation, all signals will be of the form

$$H_{D(1,2)(a,b)} = \pm\left(\frac{\omega}{c}\right)\exp\left(-il\frac{\Omega}{c}\right) \qquad (8)$$

but with different signs. In contrast, if, for simplicity's sake, we substitute a phase modulation at some point other than the midpoint for an exact simulation of a gravitational wave and combine output 1 and output 2 of the inner Mach-Zehnder, we obtain

$$H_{sGW} = \pm\left(\frac{\omega}{c}\right)\left(\exp\left(-il_{1\alpha}\frac{\Omega}{c}\right) - \exp\left(-il_{1B}\frac{\Omega}{c}\right)\right)$$

and so all displacement noise has been canceled while the gravitational wave signal survives. Data taken using this setup is reproduced in Figures 3 and 4.

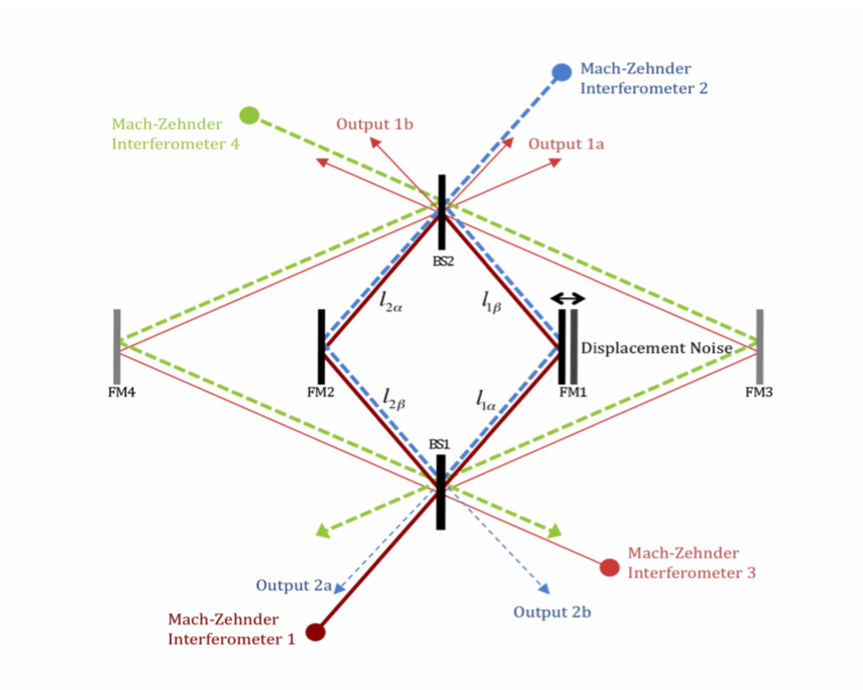


Figure 2 (S. Sato,*et al.*, 2007 *Phys. Rev. Lett.* 97, 141101)

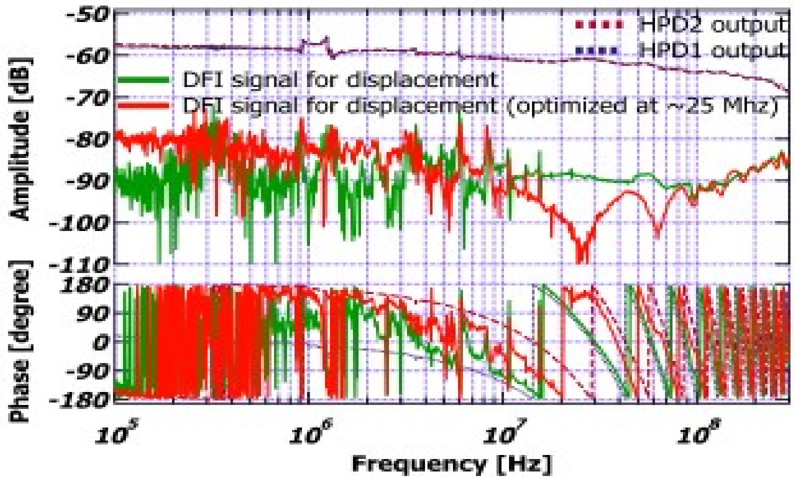


Figure 3 The red and blue lines near the top are individual photodetectors, and they almost overlap. Combining the signals yields the green curve, which shows about 30 dB of suppression between 10 kHz and 30 MHz. (S. Sato,*et al.,* 2007 *Phys. Rev. Lett.* 97, 141101)

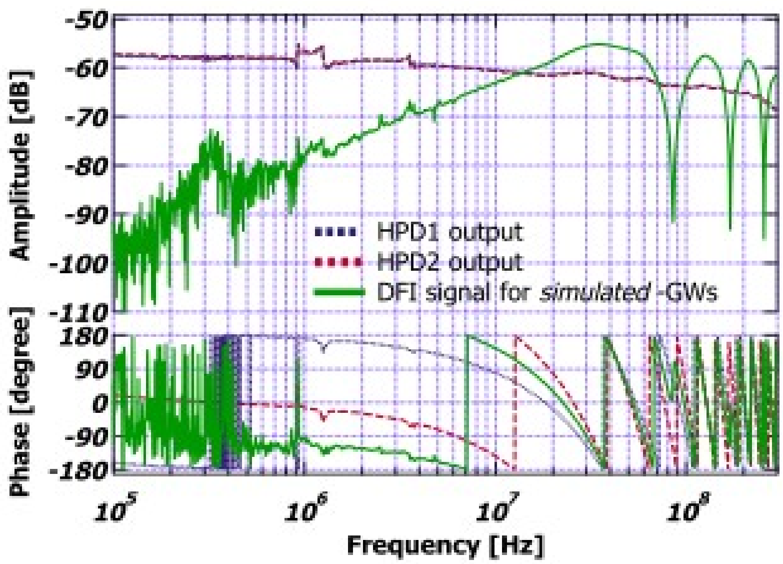


Figure 4 This is a graph of the response to a simulated gravitational wave. Again, the amplitude of the output of the of the photodetectors is similar, but the phase delay in the higher frequency region causes the signals to add instead of interfere. (S. Sato ,*et al.,* 2007 *Phys. Rev. Lett.* 97, 141101)

Purpose

This past summer, construction was begun on a new three dimensional DFI setup at the NAOJ campus in Mitaka. This three dimensional design was first suggested by Y. Chen *et al* in the 2006 Physics Review Letters 97 151103. It is considered to be a possible forerunner to both future ground and space-based gravitational wave detectors. It maintains all of the benefits of the previous two dimensional DFI setup, including the insensitivity to mirror and beam splitter displacement. In addition, for reasons beyond the scope of this paper, it has the advantage of less restrictive shot noise limitations at low frequencies, improving from ~ f^3 , the best possible from any two dimensional scheme compliant with our definition of DFI, to ~ f^2 , the best available from any possible DFI configuration. This is because, as was previously shown, at frequencies near zero, gravitational wave signals become indistinguishable from mirror displacements.

Experimental Setup

Ideally, the three dimensional DFI setup we were to build would have the shape of a regular octahedron, with each arm the same length as every other, as shown in figure 5. The left portion of the figure shows it tilted to emphasize its shape; while the right demonstrates how it would exist on the optical table. Much like the previous two dimensional incarnation, this setup employs two bidirectional Mach-Zehnder interferometers for a total of four interferometry signals. The four interferometers, MZ1-4, would be constructed such that MZ1 and MZ3, referred to as the “lower” interferometers, would overlap using mirrors C2 and D2, and the addition of these signals is insensitive to the displacement of these two mirrors. Similarly, MZ2 and MZ4, or the “upper” interferometers, would be insensitive to displacement in C1 and D1. Once the setup was aligned, the addition of all four signals would be insensitive to displacement of the beam splitters as well. This shape would become distorted during the building and alignment phases, however. This was due to a variety of reasons, some practical, such as keeping the upper interferometer below eye level, as well as some simply resulting from the many adjustments being made in order to align the interferometers. Fortunately, the strict symmetrical shape of the proposed design is not theoretically necessary for true DFI.

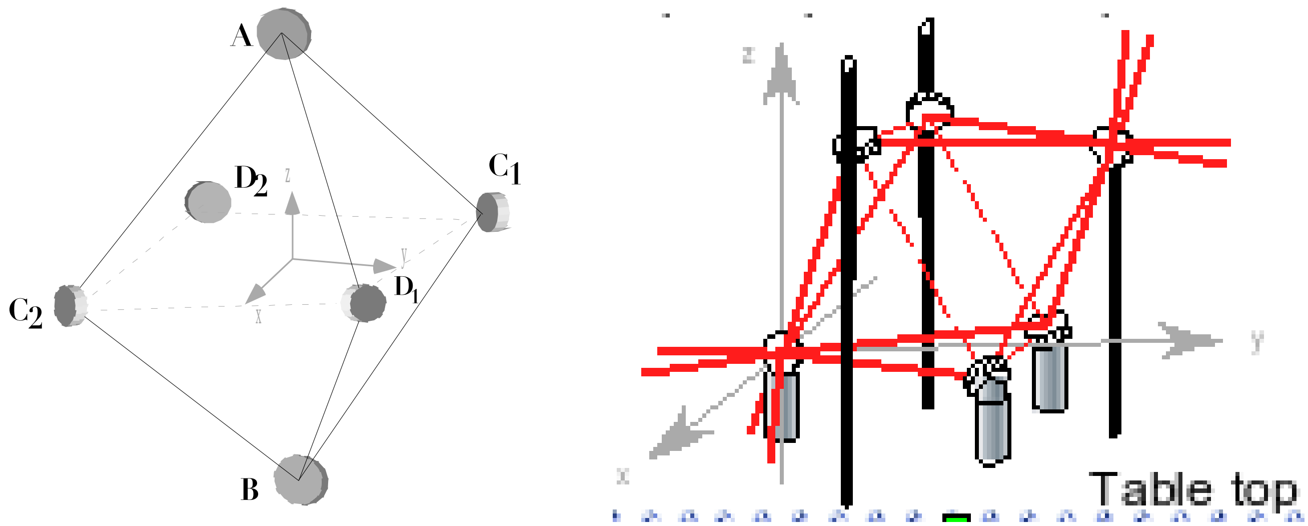


Figure 5: Left: Original concept of an octahedron formed by beam splitters A and B, mirrors C1 and D1 for the first bidirectional Mach-Zehnder, and C2 and D2 for the other.

Right: A concept drawing of how it should be built on the optical table. Can be thought of as tilting the octahedron on its side.

I made an early attempt to align the setup, but as expected, achieving alignment of both the upper and lower bidirectional interferometers simultaneously was not simple. Once the lower interferometer was aligned, attempting to align the higher without adjusting the beam splitter seemed futile. An attempt was made to simulate the situation using Mathematica to calculate how precisely the optics must be placed to determine if it would be realistic to expect to obtain suitable contrast by hand. To do this, a gaussian beam was defined with properties similar to the one used in the setup. The equations and properties F given below, where the wavelength is 1064nm, the beam waist is 200µm, the desired spot size is 5mm, and the separation distance of the beams is given by *d*.

$$u1=\frac{1}{q}\exp(-ik\frac{(x^2+y^2)}{2\mathrm{q}}) \qquad u2=\frac{1}{q}\exp(-ik\frac{((x-d)^2+(y-d)^2)}{2\mathrm{q}})$$

$$z=zR\sqrt{\frac{5\mathrm{mm}}{\omega_0}-1} \qquad q_0=i\pi\frac{\omega_0^2}{\lambda} \qquad k=\frac{(2\pi)}{\lambda} \qquad q=q_0+z \qquad (10)$$

The method used was to take a beam with the same beam waist as the laser, propagate it until reaching the typical spot size of 5mm, and then see how large of a separation distance would be acceptable. A plot of the contrast as a function of separation distance is given in figure 6. Note that contrast is defined by $C = \frac{(P_{max} - P_{min})}{(P_{max} + P_{min})}$. It was decided that a contrast ratio of 0.5 would be acceptable. This occurs at around *d* = 300µm, which is quite difficult to obtain by hand.

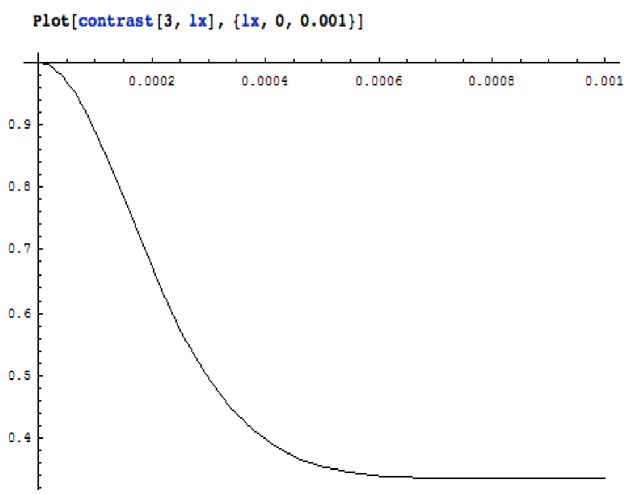


Figure 6

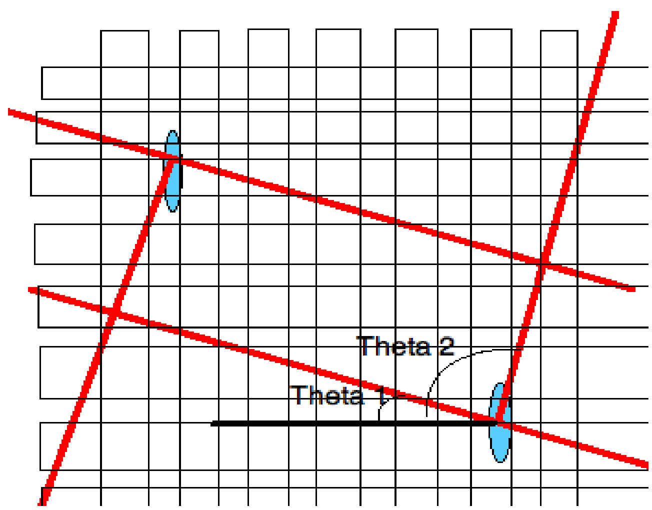


Figure 7

In light of this, a new method of aligning the interferometers was developed. A sliding stage was installed beneath the higher beam splitter. The grid marks in figure 7 represent the holes in the optical table. The angles Θ1 and Θ2 were measured for both beam splitters using the optical table as a reference and then these measurements were used to ensure that the beam splitter angles were identical. Next, sliding stages were installed beneath the higher and lower mirrors of one side of the interferometer (those on the rightmost side of figure 9 below) such that moving the stage would slide the mirrors along the beam axis coming from the higher beam splitter. An optical lever was constructed to sense the position and angle of the lower beam splitter (as shown in figure 8). This was done by using a beam splitter to input a laser beam to the lower beam splitter, then using a mirror to fold the beam reflected from the beam splitter on to piece of grid paper. The setup was then aligned for the lower interferometer and the position of the optical lever marked using an infrared sensor card. The position of the sliding stage for the lower mirror was then moved randomly a small amount. The interferometer was then realigned and the position remarked. This established a line on the grid paper which could be used to predict the optical lever's response to small changes in the position of the lower mirror. Next, this procedure was repeated for the higher mirror. The

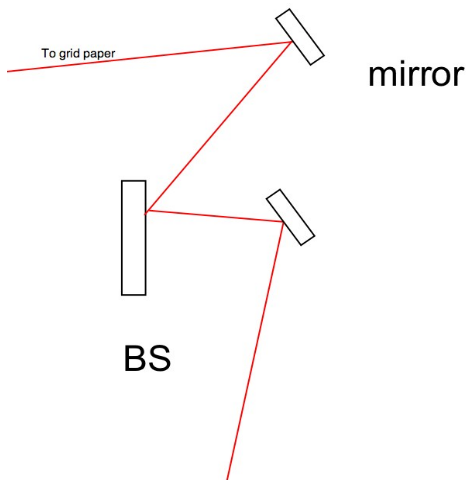


Figure 8

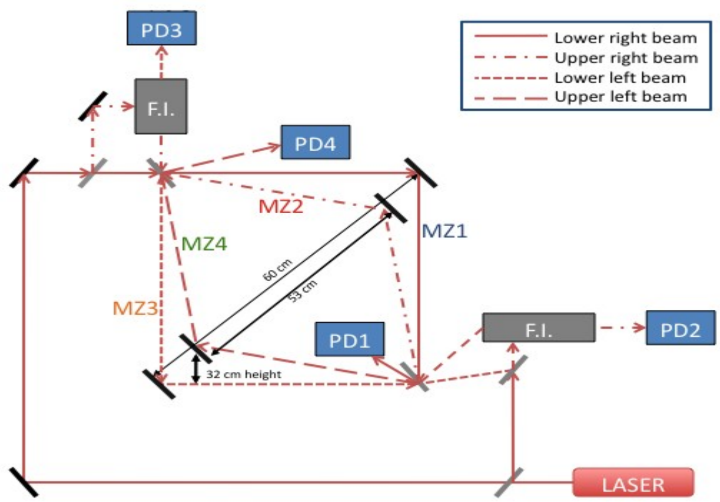
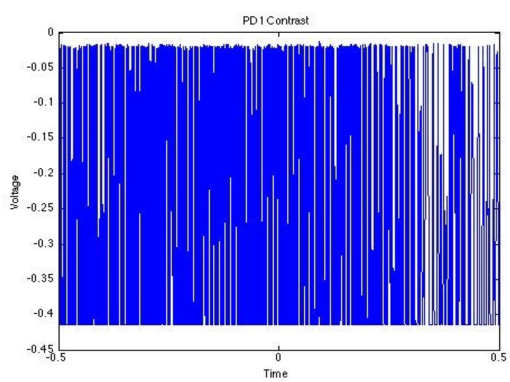


Figure 9

intersection point of the two lines was calculated and the mirrors moved along the sliding stages to the positions which should correspond to that point. The linear approximation of the optical lever's response to the mirror movement was accurate enough for small changes that this was successful after only a few iterations. Figure 9 shows a diagram of the finished interferometer with roughly measured dimensions included. Photo detectors and faraday isolators were installed in their permanent locations to read all four outputs. Filters were also put in place in front of all four photo detectors in order to ensure that each measured a similar signal to ensure the best cancellation of displacement noise. Figures 10-13 below showcase the contrasts obtained from each interferometer.



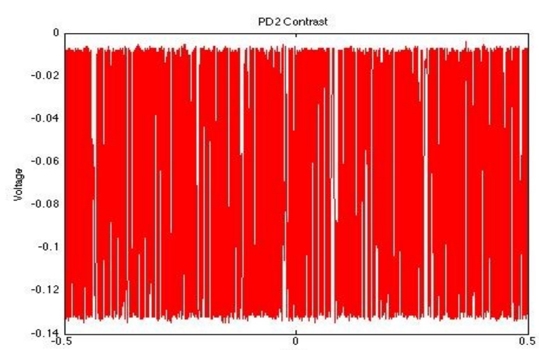
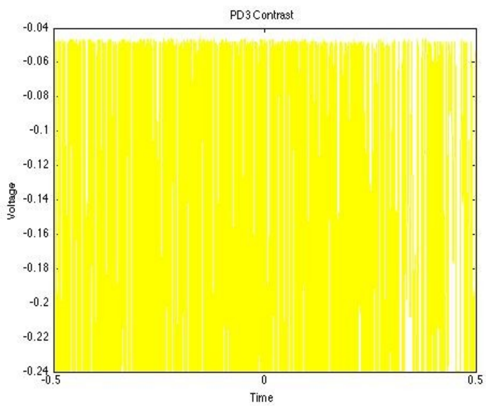


Figure 10

Figure 11



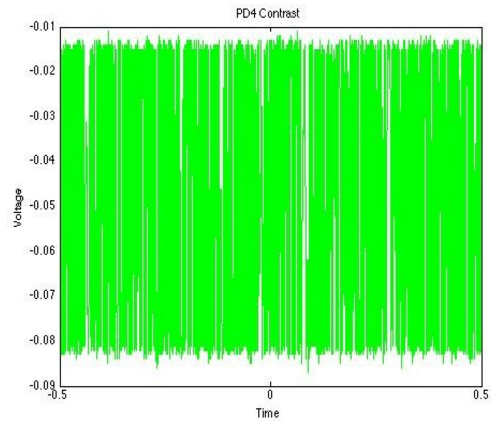


Figure 12

Figure 13

After the interferometers were all built and aligned, the process of controlling began. PZTs were in place on one higher and one lower mirror (the same mirrors that were placed on stages). The signal from PD1 was used to control the lower interferometers, MZ1 and MZ3 in figure 9 above, and PD4 was used to control the higher interferometers, MZ2 and MZ4. Figure 14 shows the control scheme. The signal from a photo detector is sent into the sum amplifier along with a signal of a DC generator. This DC signal is used to offset the PD signal to ensure a lock at the midfringe. Next the signal is sent through a low-pass filter. Initially the gain of the filter was too low (around 1), so it was altered to match the schematic shown in Figure 15. From the low-pass filter, the signal is sent to the PZT driver, which then controls the PZT. Figure 16 displays all four interferometer signals locked simultaneously.

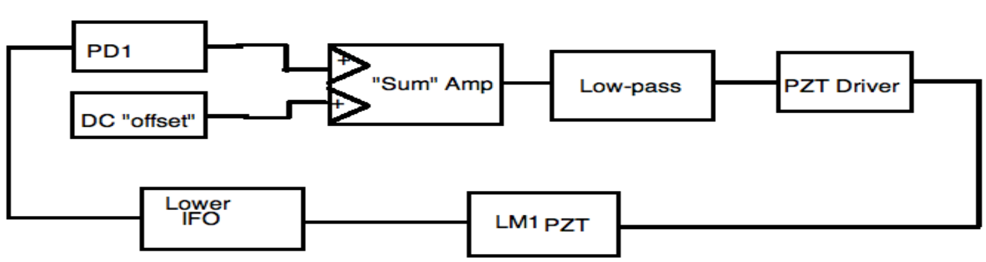
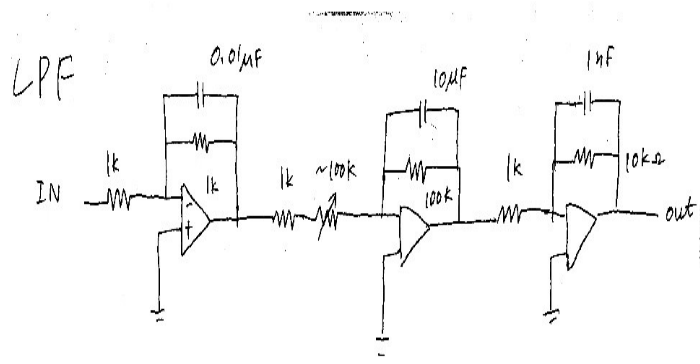


Figure 14



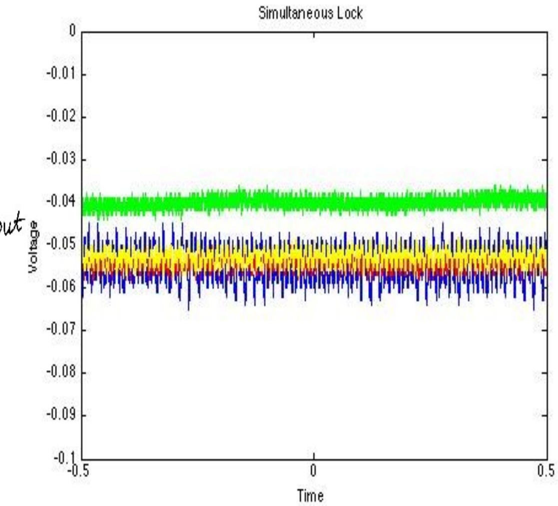


Figure 15

Figure 16

Conclusions / Future Work

The objective of this project was to build and align the first three dimensional DFI interferometry set up. As such, it was a success. However, proof of the 3D setup's superior sensitivity to gravitational waves, the purpose for building it, remains to be seen. Scientists and graduate students working at the NAOJ will expand upon the setup. An EOM will be added at the position of a mirror, at the midpoint of the laser's path, to phase modulate the laser and simulate mirror displacement. Another will be added away from the midpoint to map the setup's response to gravitational wave signals much like what was done for the two dimensional setup. This is likely to be finished in a matter of weeks as the most complicated, tedious, and unprecedented portion of the work, aligning the interferometers in three dimensions, has already been completed. The discipline of DFI as a whole will continue to be explored as a possibility for future large scale gravitational wave detectors.

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References:

1) (S. Kawamura and Y. Chen 2004 Phys. Rev. Lett. 93, 211103)

2) (S. Sato, *et al.,* 2007 *Phys. Rev. Lett.* 97, 141101)

3) (Y. Chen *et al* in the 2006 Physics Review Letters 97 151103)