

# Ranging Implementation: Signal Processing Development

Kendall Ackley  
University of Florida  
Max-Planck-Institut für Gravitationsphysik  
(Albert-Einstein-Institut)  
IREU

August 11, 2009

## Abstract

The objective of my project was to develop a Simulink model to track an incoming PRN code affected by Doppler-shifts to a local copy of the same code. For each independent satellite: using a model of the phase measurement system (PMS) and manipulating different discrete filters and gains of a phase-lock loop and a delay-lock loop, the two PRN codes are able to be correlated and their correlation to be maximized. Through the process of filtering, correlating, delaying and shifting, the two signals are able to be manipulated in such a way that the difference subtracted out between the two sources, the beat signal, will contain information that should ensure the first mathematical evidence for a gravitational wave.

## Introduction of LISA

The Laser Interferometer Space Antenna (LISA) is a joint project of the ESA and NASA to detect through observation what Albert Einstein was only able to predict nearly a century ago in his General Theory of Relativity. That is, he predicted that gravitational waves are a co-product of colliding accelerated super masses.

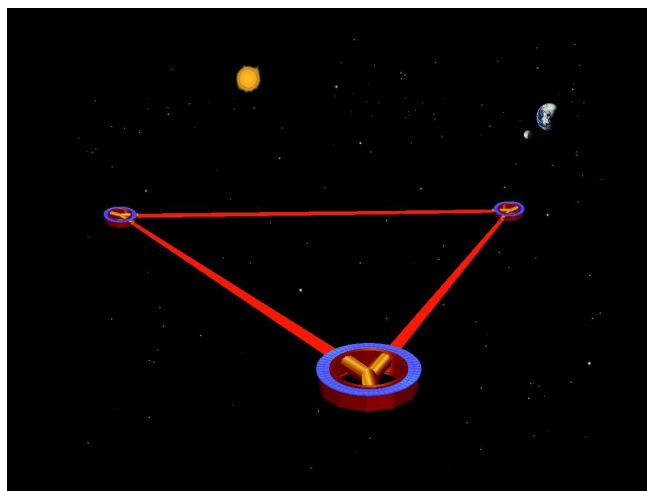
In the 1970s, Joseph Weber introduced the scientific community to a resonating bar which gave the first glimpse of hope to the possibility of detecting a gravitational wave. On this bar, he attached several piezoelectric crystals and was able to detect vibrations of a possible gravitational wave from a very powerful source.

Measuring gravitational waves has proven to be a very difficult endeavor. Only within the past decade has there been sufficient technology capable of detecting these high-energy, low-frequency, microscopic waves. Einstein was only able to theorize these waves, calculating that ambition was no match for the technology of his day.

However, with the advent of more sophisticated techniques of laser interferometry, scientists and labs around the world are now able to measure the changing of distances down to 10 pm, the theorized sensitivity needed to observe gravitational waves near earth. This picometer variable change in distance is the change in the relative ripple of space-time itself and is considered to be the greatest strength able to be observed. (Thorne 1994)

There are several sources of gravitational waves, including the collisions of neutron stars, black holes, or any other rapidly accelerating binary body. It is within this collision that gravitational waves propagate and, similar to a shock wave, ripple and vibrate through space-time. Unfortunately, the further they travel, the weaker they become. This is where LISA can prove to be most effective. While there are highly-advanced and sophisticated ground-based gravitational wave detectors around the world, they are limited to a select band of frequencies. LISA will be able to escape the terrestrial noise that the gravitational waves may be laying in and in doing so, “unearth” the hidden signals. LISA will be able to detect gravitational waves at a frequency in the mHz-Hz range with an accuracy of:

$$10 \text{ pm} / \sqrt{\text{Hz}}$$



**Figure 1** LISA triangular interferometer setup.

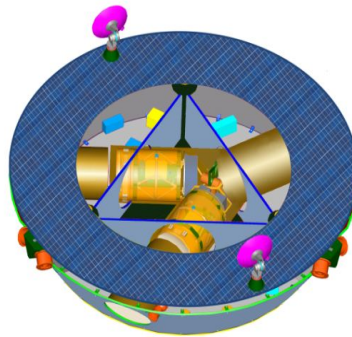
## LISA Mission Concept

The LISA satellites are set up as a triangular Michelson Interferometer . The three spacecraft orbit behind the earth by 20 degrees and are separated in space from each other by 5 million km.

The laser light that is received after traveling 5 million km is very low in intensity. This low intensity requires back-reflection. Back-reflection occurs onboard a secondary spacecraft's local laser. That is, the far laser transfers its phase information onto this secondary local laser.

The two arms lengths change over time creating a source of laser phase noise. Time Delay Interferometry is a simulated equal-arm interferometer and effectively cancels out the phase noise that occurs. How this works is through an estimation of the arm lengths via giving each beam a PRN code. This PRN code can be detected and tracked on the other satellite.

The PRN codes that are used by the LISA mission are generated by six lasers, each one unique from the other. The codes are generated prior to the launch and consist of alternating 1's and -1's. They are added onto the laser beam by a look-up table (LUT). The LUT is essentially a block of memory that contains the information of the code. The laser is ran at the sampling heterodyne frequency and then passed through a running counter that counts up to the full length of a PRN code, 32768 bytes. This is then added to a sample-based sinusoid. This signal is passed through a LUT where the information of the PRN is added on. At any time, before or after the two lasers are locked together, a PRN is added onto each beam.



**Figure 2** Illustration of one spacecraft showing optical bench, test mass and two lasers.

For simplification, only the relationship between two satellites will be discussed. It is the same for each arm.

After the correct orbital alignment is achieved, the first spacecraft transmits the first laser beam to a far satellite. The far satellite sees this incoming beam and mixes it with its own local signal. A beat signal results from this mixing and a phase relationship between the lasers emerges. This is phase-locking. However, the incoming beam can only interfere with a small portion of the light at the local laser because of the distances between the satellites. This small interference is where the beat note is generated.

The photo detector (PD) records this beat note. More importantly, however, is that the information of the PD comes from TDI. The phase of this incoming signal and the varying length of the interferometer arm are recorded with the PLL and the DLL, respectively. The PMS must measure the variable length changes down to an accuracy of  $10\text{ pm}/\sqrt{\text{Hz}}$ . However, the detection of a gravitational wave will occur when the phases of the traveling beams are slightly disturbed and the two beams will

interfere constructively. The photodetector is able to see this signal and, in the PMS, subtract the two phases leaving a beat signal. The resulting beat signal will contain information of the relative phases of the two beams.

This beat signal must be mixed again with a sinusoid of the same frequency. It is only after this mixing that the wanted signal is isolated, that is, the signal with phase information of the PRN and the Interferometer. This is essentially the PLL.

The DLL processes this code unfiltered. It looks for the incoming code, correlates it with its own local code and is able to gather information about the delay. This delay is used to offset the code by an amount that will later be used to know how far the beam has travelled.

The DLL takes this phase information as an input and isolates it by low-pass filtering and down-sampling. This is accomplished through the DLL integrators and the information about the delay of the beams is able to be retrieved.

There is also some phase information that needs to be recorded from the PLL in order to compare the the phases of the two arms. The phases of both of the arms of LISA are needed in order to find the gravitational wave and relating the difference of the two phases shows the magnitude of the relative length changes. This signal is not ideal, however, as the noise levels are not sensitive enough. Time Delay Interferometry (TDI) is a secondary type of interferometry that will reduce the noise level to its required sensitivity through data post-processing.

TDI utilizes heterodyne interferometry with unequal arms. With two arms and two delays, this special type of interferometry needs an equation accounting for the phase readouts of the two arms as a result of the heterodyne interferometry and the effects of Doppler-shifting. LISA implements TDI simply as a method of processing the recorded data.

As the spacecraft orbit behind Earth, Doppler-shift affects the laser beams on the satellites. This is because the space-time they are orbiting consists of tiny ripples. Doppler-shift is mainly a phenomenon that affects waves because of these tiny ripples.

It is important for the PLL to track this shift in frequency and the DLL the corresponding change in delay.

## Ranging

The concept of ranging is very important as it allows for the modulated PRN to be used for modulating the phase of the beat signal to find the changing distances between the satellites. Ranging employs a phase measurement system (PMS), in which a phase-locked loop (PLL) and a delay-locked loop (DLL) account for the change in phase and delay of an incoming signal and compares it to an on-board signal.

Each laser transmits a PRN code at the running heterodyne frequency to the far satellite where it is received, back-reflected, and transmitted back to its source. Each satellite records an incoming signal. It compares a far laser beam to its own local signal onboard. The phase information of the incoming laser is encoded onto the local laser and returned to the original satellite. The signal that is returned, however, must have a slightly different frequency in order for this to work. This frequency/phase shift is recorded by the PMS. The PLL takes the incoming signal and with a Local Oscillator (LO) sinusoid running at the same frequency it mixes the two sinusoids, effectively canceling out the differing phases. What is left, most importantly, is purely the phase information of the two signals, plus the PRN code. The beat note must be filtered away for it to transfer to the DLL.

This differential phase error is inputted to the DLL where the delay of the signals is tracked against a local copy of the same PRN and their correlation is extracted as an output. However, this is not nearly sensitive enough to estimate the parameters for TDI, as the output of the DLL is in the meter range. The noise levels of this output will show no data at the required bandwidth. By filtering through post-processing, the noise can be reduced to the required levels for TDI.

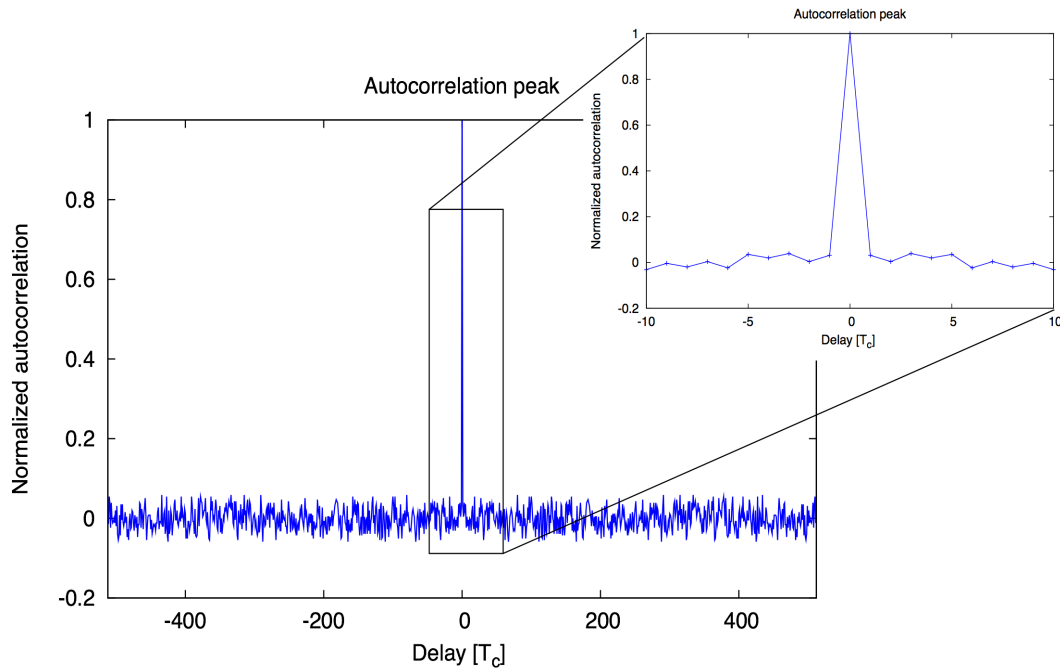
Ranging depends on the frequency of the noise, the delay of the signals, TAU, and a precision of the relative distances of the spacecraft.

## Tracking the Signal

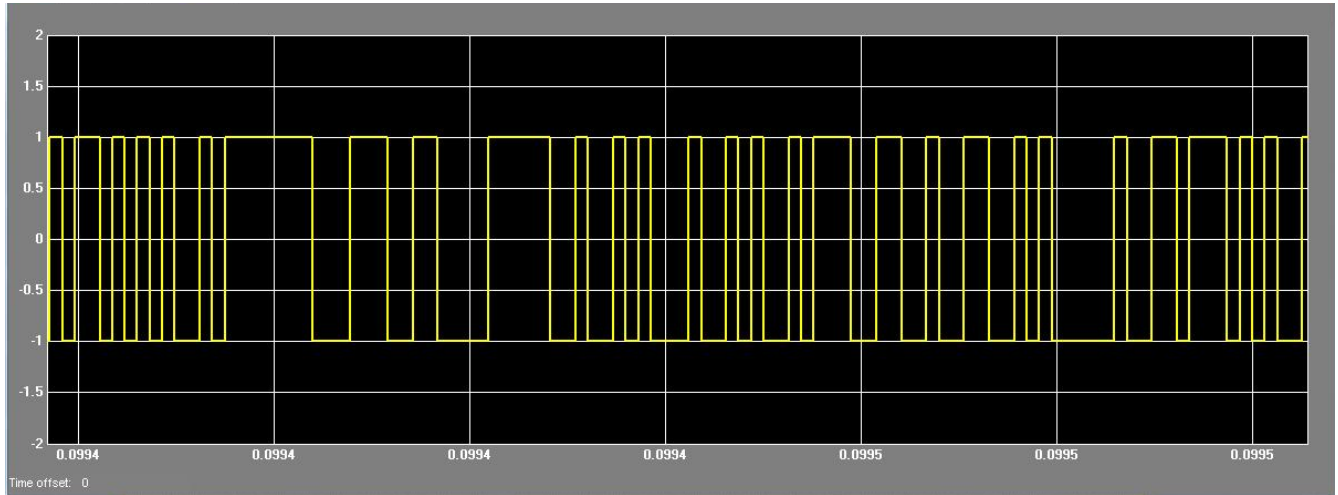
The two PRN codes are mixed together. As they travel through the PMS onboard each satellite, the system is considered to be in acquisition mode. Once these two signals lock to each other, it is considered to be in tracking mode and a correlation peak appears. The total time that it takes for the system to track is estimated by the function:

$$Time = \left( \frac{Length\ of\ PRN * Delay}{F_s} \right)$$

The frequency is a dynamic heterodyne frequency. The correlation data appears as a delta function. A correlation peak of 1.0000 is ideal, but require some conditions that do not take real LISA effects into consideration. (Esteban plotsacq.doc)



**Figure 3** Ideal Auto-Correlation shows delta function with zero delay.



**Figure 4** Time series illustration of one proposed PRN sequence

## Project: Ranging Accuracy

The PRN is generated onto the phase of the counter-sinusoid. The PRN code created on a local satellite is affected by Doppler-shifting as it is transmitted to a far satellite. The PLL onboard corrects for the beat signal and doppler phase shifting, the DLL corrects for this change in delay and by continuously adjusting the outputs by adding delays, the system is able to track.

The Doppler shift is simulated by adding in a slowly changing time-based sinusoid running at the relative velocity of the sampling frequency. This is added to the beat note frequency (NCO) of the local satellite. The sinusoid is given an amplitude that allows for one code length of PRN. The frequency of the Doppler-shift is the relative velocity of the initial heterodyne frequency:

$$f_{rec} = f_{init} \left( 1 + \frac{v_{het}}{c} \right)$$

The local PRN is simulated with a sample-based sinusoidal added onto a counter modulated with an NCO frequency of 32768 and then passed through a Lookup Table (LUT) containing the PRN code. The length of one PRN code is 1024 chips, with one chip equivalent to 32 bits (**Figure 4**). The Doppler-shifted signal and the PRN are combined before passing through the PLL, which will lock to the beat signal and the extraneous high-frequency signals will be filtered out through post-processing. The DC component of the error signal, formed after mixing, passes into the DLL. The DLL compares the incoming signal to its own local signal and adds in a delay to adjust as necessary. It measures the correlation between the incoming PRN sequence and the onboard sequence which is tracked and outputs the delay (TAU). This adjustment is the basis of ranging.

The transfer function of the Phase meter is estimated by the following formula:

$$G(z) = \left( \frac{1 + \alpha_1 z^{-1} + \alpha_2 z^{-2}}{1 + \beta_1 z^{-1} + \beta_2 z^{-2}} \right) C_1 * \left( \frac{1 + \alpha_3 z^{-1} + \alpha_4 z^{-2}}{1 + \beta_3 z^{-1} + \beta_4 z^{-2}} \right) C_2 * \left( K_P + K_I \frac{T_s}{z-1} \right) * \left( \frac{T_s}{z-1} \right) * \left( \frac{1}{2} \right)$$

And carried out in MATLAB with the following script:

```
%%
x=1/32768:1/32768:1;
f1=zeros(1,length(x));
f2=zeros(1,length(x));
g=zeros(1,length(x));
h=zeros(1,length(x));
H=zeros(1,length(x));
E=zeros(1,length(x));

a1=-1.999169792572207;
a2=1;
b1=-1.998255701746941;
b2=0.998294671141222;
c1=-1.995262140228550;
c2=1;
d1=-1.995727607810404;
d2=0.995738914821589;
g_p=2e7;
g_i=5e11;
%%
for i=1:length(x)
    f1(i)=(1+a1*exp(-2i*pi*x(i))+a2*exp(-4i*pi*x(i)))*1.055069254858308e-005/
    (1+b1*exp(-2i*pi*x(i))+b2*exp(-4i*pi*x(i)));
    f2(i)=(1+c1*exp(-2i*pi*x(i))+c2*exp(-4i*pi*x(i)))*0.044585330459528/(1+d1*exp(-
    2i*pi*x(i))+d2*exp(-4i*pi*x(i)));
    g(i)=(g_p+g_i/50e6/(exp(2i*pi*x(i))-1))/(exp(2i*pi*x(i))-1)/50e6/2;
    h(i)=f1(i)*f2(i)*g(i);
    H(i)=h(i)/(1+h(i));
    E(i)=1/(1+h(i));
end
```

The output of this estimation is shown in Figure 8(c).

The closed loop transfer function is estimated by:

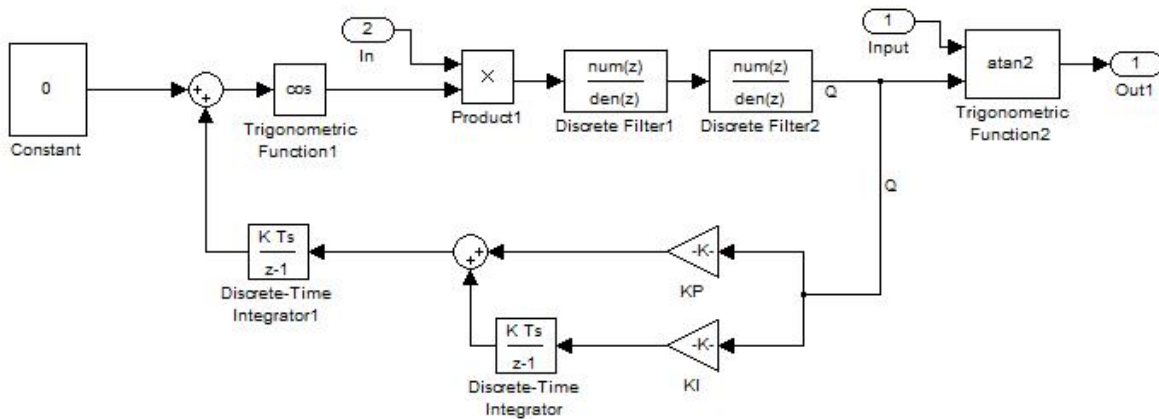
$$H(z) = \left( \frac{G(z)}{1 + G(z)} \right)$$

The error transfer function is estimated by:

$$E(z) = \left( \frac{1}{1 + G(z)} \right)$$



The PRN is modulated by a slowly-changing sinusoid with an input heterodyne frequency of 12.5 MHz. This is equivalent to the causes of Doppler-shifts upon the PRN data stream as it travels to its intermediate satellite. The sinusoid changes the amplitude and the phase of the PRN code over, but not its frequency. It is a data sequence of pre-assigned and alternating 1's and -1's. There are six codes, one for each direction of each arm, and every code is unique. The heterodyne frequency will run from 2-20 MHz, but the code is generated at a frequency of 12.5 MHz. The signal is downsampled through post-processing to a sampling frequency of a few mHz to Hz.

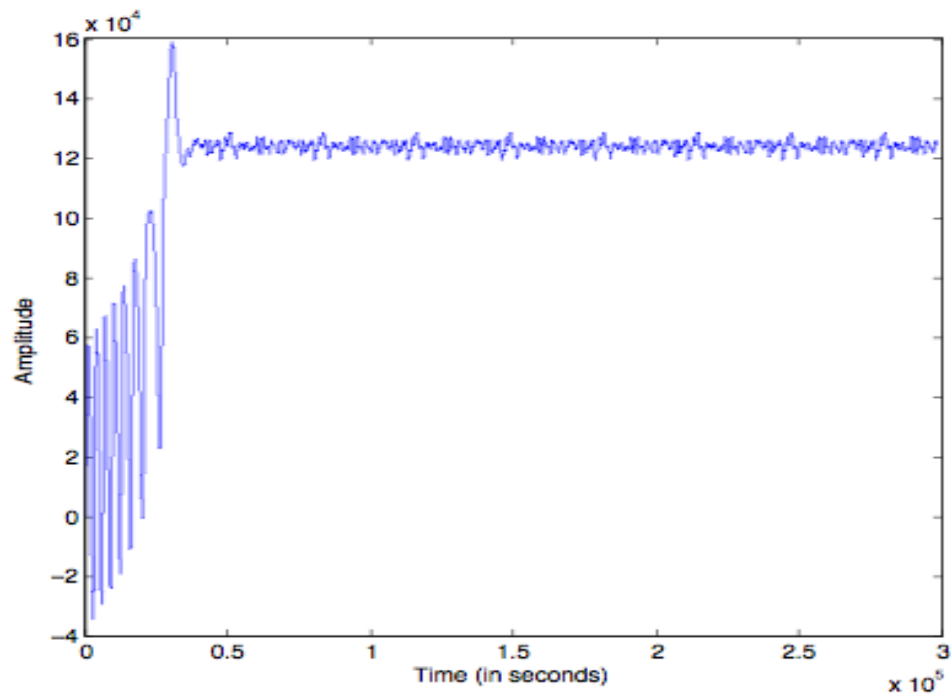


**Figure 5** Quadrature path of phase meter.

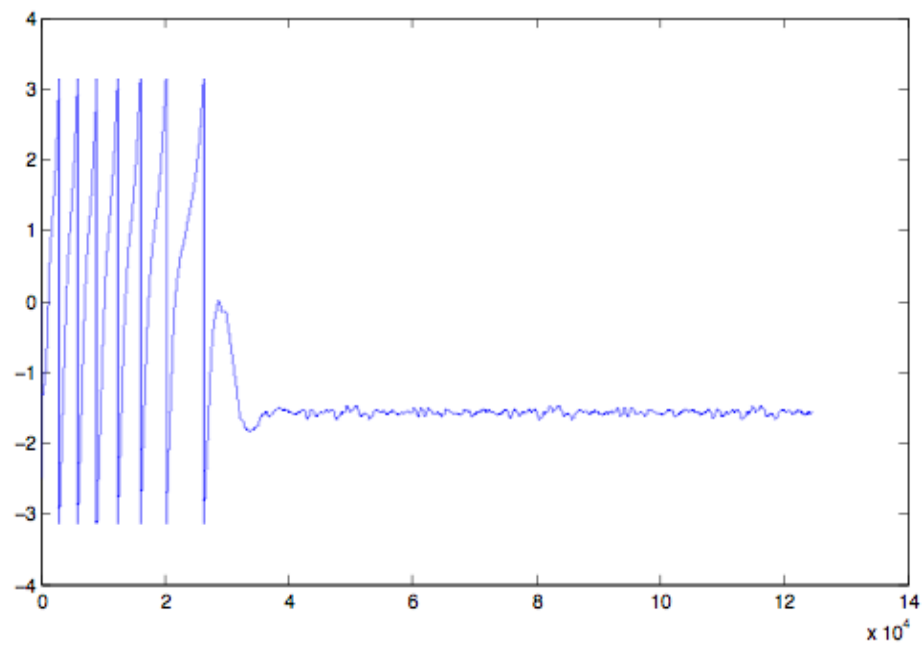
Figure 5 shows the Quadrature path of the phase meter. The input (2) is mixed with the cosine wave of an numerically controlled oscillator (NCO) and is filtered by down-sampling discrete filters. This residual phase is fed into a proportional-integral (PI) controller (**Figure 6**), resulting in an error signal. This error signal is returned and fed back into the PLL, adjusting the phase. This error can then phase lock the NCO frequency to the incoming signal. This locked signal is transferred to the DLL after the mixer.

What is not shown is the In-phase path. This contributes to the input (1) of the arctangent function. This path gives the amplitude of the signal. It is fed into the arctangent function to resolve for the sin error that is introduced in the initial NCO (the IQ block, or the sine and cosine waves). The two signals in the NCO, multiplied by a cos and sin, are filtered through discrete filters and then processed by an arctangent. The arctangent [ $\arctan = (I/Q)$ ] is used to recreate the phase and allows for the sin error to be removed (**Figure 7**).

## Results



**Figure 6** PI controller output.



**Figure 7** Arctangent output: tracking, variable phase shifting due to Doppler(Phase vs. Time)

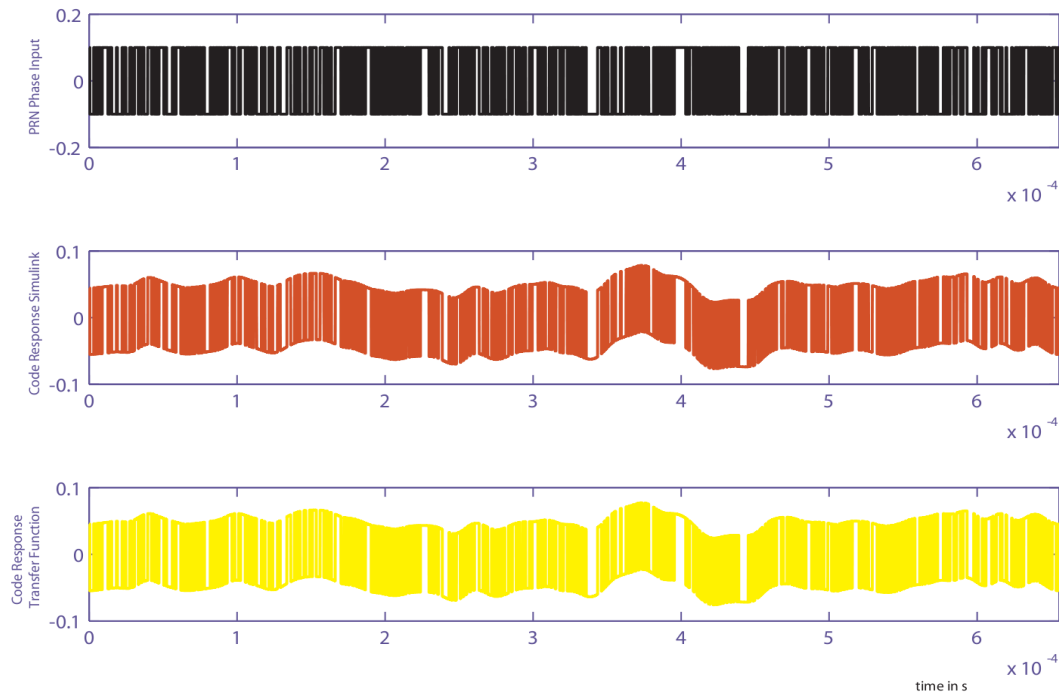


Figure 8 (a) PRN code (b) PRN code response (c) estimated code response

Figure 6 shows the output of the PI controller, amplitude over time. The signal successfully tracks in the PLL.

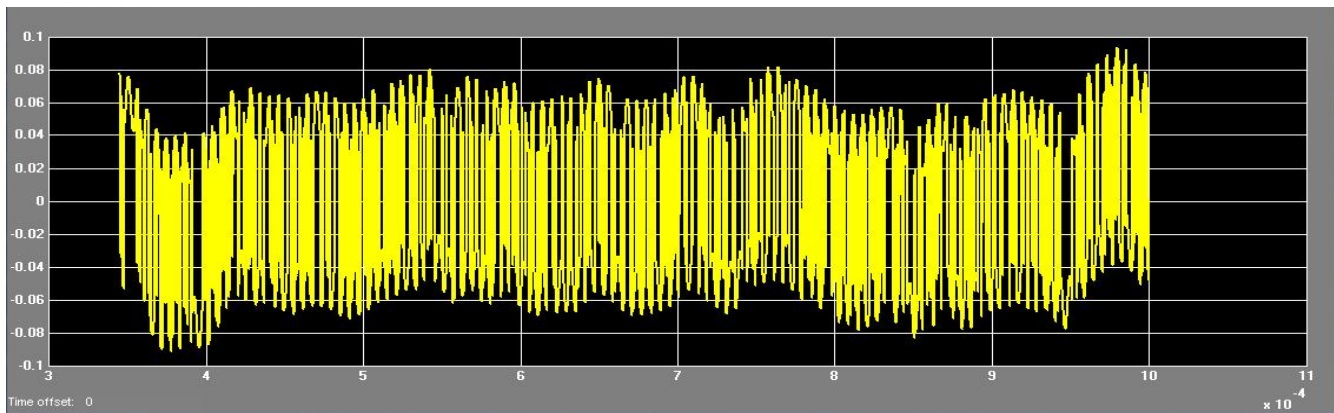
Figure 7 shows the arctangent function ( $\arctan = (I/Q)$ ) as phase over time. This signal was also successfully tracked. The phase shifting is also shown in the readout.

Figure 8 shows the relationship between a PRN code, how this code responds in the PLL of the PMS and how the transfer function estimated this relationship. The transfer function estimation is equivalent to the response in the PLL.

Figure 9 shows the effects of the mixing doppler shift and the PRN code. The original PRN is still present in the output, which means that the PMS can effectively add in Doppler shifts and still be able to track the relative changes in distances between the spacecraft.

## Conclusion

Although there were a few initial start-up difficulties relating to technological errors, I was still able to run the phase meter and create outputs that showed the effects of Doppler into the PMS. There is still much work to be completed in this area



**Figure 9** Effects of sinusoidal Doppler shift and sinusoidal PRN code on phase readout.

## Hardware and Software Implementation

The Hardware Implementation used for the optical experiment consists of FPGA (Field Programmable Gate Array) processors that are programmed via VHDL code.

The types of software used for the phase measurement system and adding in Doppler were Simulink models, Matlab, and C programming. The FPGA boards use VHDL coding, which is similar to C programming and is the standardized code for the processors.

## Future Experiments/ Further Research

Further Experiments include tracking the phase difference and delay with a second PRN, which is also affected by Doppler-shifts. My project accounted for only one PRN code and how Doppler affects this signal as well as improving the input signal and maximizing the correlation. Once a second PRN sequence has a Doppler-shift added on and it is tracking with an original PRN code locally, then data could be transferred over the beams.

## Acknowledgements

I would like to thank Prof. Guido Mueller and Prof. Bernard Whiting, for allowing me to take part in such a wonderful learning and growing experience and to Esteban, for guiding and mentoring me throughout this whole process.

The greatest thanks goes out to Johannes Eichholz who spent most of his time answering every single one of my questions, for letting me join his lunch crew when everyone left for the Amaldi Conference, and for being a good friend in general. Also, to Anneke Monsky and Frank Steier, thank you for making me feel at home immediately, for all of our endless, meaningful talks, and, of course, all of the many excursions around Hannover. It would not have been nearly the same without you. To Jan Pold, and to Simon Barke, for being the greatest friend, even across an ocean. And last, but not least, to everyone at the AEI, for allowing my experience to be so rewarding both academically and personally, the National Science Foundation and the University of Florida.

## Sources

Smith S., "The Scientist and Engineer's Guide to Digital Signal Processing", California Publishing, pp. 1-350 (1997)

Delgado J., Marin A., Bycov I., Heinzl G., Danzmann K., "Free-space laser ranging and Data Communications", Institute for Gravitational Physics of the Leibniz University Hannover and Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Germany

Brown S., Vranesic Z, (2nd Ed) "Fundamentals of Digital Logic With VHDL Design", McGraw-Hill, pp. 58 (2005)

Delgadillo R., Wu W., Thorpe I., Mitryk S., Quetschke V., Mueller G., "Laser communication, clock synchronization, and ranging in interferometric space missions.", University of Florida

"Global Positioning System: Theory and Applications, (Volume I), American Institute of Aeronautics and Astronautics, Inc (1996)

Gardner, Floyd M., "Phaselock Techniques, Third Edition," John Wiley & Sons, Inc. (2005)

Wand, V. "Interferometry at Low Frequencies: Optical Phase Measurement for LISA and LISA Pathfinder," Ph.D. Thesis, Universität Hannover (2007)

Heinzel, G., Ruediger, A., Shilling, R., "Spectrum and spectral density estimation by the Discrete Fourier transform (DFT), including a comprehensive list of window functions and some new flat-top windows," Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) Teilinstitut Hannover (2002)

Steier, F., "Interferometry techniques for spaceborne gravitational wave detectors," Ph.D. Thesis, Von der Fakultät für Mathematik und Physik der Gottfried Wilhelm Leibniz Universität Hannover (2008)

Thorne, K., "Black Holes & Time Warps: Einstein's Outrageous Legacy," W.W. Norton & Company pp. 357-396 (1994)