Properties of valence-bond stripes in cuprates
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Stripes, dominated by bond order and competing with superconductivity.
1. **Valence-bond stripes, neutrons & STM**
   Stripes co-exist with nodal quasiparticles below $T_c$

2. **Fermi surface reconstruction and Nernst effect**
   Low-temperature Nernst effect from stripes

3. **Interlayer Josephson tunneling**
   Could a uniform condensate be compatible with quasi-2d pairing?

Stripy review: arXiv:0901.3145
Valence-bond stripes
Quantum fluctuations

Bond order

Neel order

Hole density
Quantum fluctuations

Sp($N$) theory of $t$-$J$ model: bond order coexists with $d$-wave superconductivity

Quantum fluctuations

Neel order

Bond order

Stripe order originates from formation of valence bond solid (VBS)

Hole density $\delta$

Sp($N$) theory of $t$-$J$ model: bond order coexists with $d$-wave superconductivity


Red lines: triplon excitation of a 2 leg ladder with exchange $J=100$ meV

La$_{15/8}$Ba$_{1/8}$CuO$_4$ : Neutron scattering

Minimal model: Coupled spin ladders

Uhrig et al., PRL 93, 183004 (2004)
Results strongly suggest that stripes are bond-centered!


What about fluctuating stripes?
Spin excitations in a dynamically fluctuating stripe phase

Spin fluctuations near \((\pi,\pi)\) described by \(\varphi^4\) theory on a lattice:

\[
S_0 = \int d\tau \sum_j \left[ \frac{1}{2} \left( \frac{\partial \varphi_j^\alpha}{\partial \tau} \right)^2 + \frac{s}{2} \varphi_j^\alpha + \frac{u}{4} (\varphi_j^\alpha)^2 \right] + \int d\tau \sum_{\langle jj'\rangle} \frac{c^2}{2} (\varphi_j^\alpha - \varphi_{j'}^\alpha)^2
\]

coupled to local charge density \(Q\):

\[
S_x = \int d\tau \sum_j \left[ \lambda_1 Q_x(r_j)\varphi_j^\alpha + \lambda_2 Q_x(r_j+\pi/2)\varphi_j^\alpha\varphi_{j+x,\alpha} + \lambda_3 Q_x(r_j)\varphi_{j-x,\alpha}\varphi_{j+x,\alpha} + \lambda_4 Q_x(r_{j+y/2})\varphi_j^\alpha\varphi_{j+y,\alpha} \right]
\]

\(Q\) is parametrized as

\[
Q_x(r) = \phi_x(r)e^{iK_x \cdot r} + \phi_x^*(r)e^{-iK_x \cdot r}
\]

Static charge order: \(\phi = \text{const}\)

Vojta / Vojta / Kaul, PRL 97, 097001 (2006)
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\]

Static charge order: \(\phi = \text{const}\)

Fluctuating charge order:

\[
S_\phi = \int d\tau d^2r \left[ |\partial_\tau \phi_x|^2 + |\partial_\tau \phi_y|^2 + c_1^2 |\partial_x \phi_x|^2 + c_2^2 |\partial_y \phi_x|^2 + c_3^2 |\partial_y \phi_y|^2 + c_4^2 |\partial_x \phi_y|^2 + i\delta \phi_x^* \partial_x \phi_x \\
+ i\delta \phi_y^* \partial_y \phi_y + s_1 (|\phi_x|^2 + |\phi_y|^2) + u_1 (|\phi_x|^4 + |\phi_y|^4) + v_1 (|\phi_x|^2 |\phi_y|^2) + w (\phi_x^4 + \phi_x^4 + \phi_y^4 + \phi_y^4) \right]
\]

Decides between stripes and checkerboard!

Vojta / Vojta / Kaul, PRL 97, 097001 (2006)
Spin excitations in a dynamically fluctuating stripe phase

Vojta / Vojta / Kaul, PRL 97, 097001 (2006)
Evolution with stripe correlation length

Strongly „repulsive“ x/y stripes

55% $E_{\text{res}}$

$\xi \sim 50$

$\xi \sim 30$

$\xi \sim 15$

$\xi \sim 10$

Correlation length $\xi$ will be temperature-dependent
De-twinned YBCO: Adding in-plane anisotropy

Ratio of charge gradient terms:

1.005 1.01 1.02

55% $E_{\text{res}}$

200% $E_{\text{res}}$

Low-energy part of spin excitations is very sensitive to anisotropy!
(Charge sector close to spontaneous breaking of rotation symmetry)
STM: Local static order in superconducting state

R map (asymmetry) at 150 meV
STM: Local static order in superconducting state

R map (asymmetry) at 150 meV

Period-4 nanodomains with contrast on Cu-Cu bonds: Valence bond solid (glassy)

Kohsaka et al., Science 315, 1380 (2007)
Order has stripe character!

Unconventional order: „$d$-wave“ stripes

Vojta / Rösch, PRB 77, 094504 (2008)
„$d$-wave“ stripes

\[
\phi_1(k) = \langle c_{k\uparrow} c_{-k\downarrow} \rangle \\
\phi_2(k) = \langle c_{k+Q,\sigma}^{\dagger} c_{k\sigma} \rangle \\
\phi_3(k) = \langle c_{k+Q,\uparrow} c_{-k\downarrow} \rangle
\]

Homogeneous pairing  
Charge/bond modulation  
Modulated pairing (FFLO)  

$\sim \cos k_x - \cos k_y$

Vojta / Rösch, PRB 77, 094504 (2008)
Order is static, but short-ranged (random field pinning). Order coexists with well-defined low-energy quasiparticles.
Monte-Carlo simulation of short-range ordered bond-centered $d$-wave stripes

Homogeneous $d$-wave superconductor:

\[ \mathcal{H}_0 = \sum_k (\epsilon_k - \mu) c_k^\dagger c_k + \Delta_k (c_{k\uparrow} c_{-k\downarrow} + h.c.) \]

coupled to local „charge density“ order parameter $Q$:

\[ \mathcal{H}_x = \sum_i \kappa_1 Q_x(r_i) c_i^\dagger c_i + \kappa_2 Q_x(r_i + x/2) c_i^\dagger c_{i+x} + \kappa_3 Q_x(r_i + y/2) c_i^\dagger c_{i+y} + \kappa_4 Q_x(r_i + x/2) c_i^\dagger c_{i+x} + h.c. + \kappa_5 Q_x(r_i + y/2) c_i^\dagger c_{i+y} + h.c. \]
Monte-Carlo simulation of short-range ordered bond-centered $d$-wave stripes

Real-space density
(one run)

Fourier-transformed density
(config average)

Vojta, PRB 78, 144508 (2008)
Add impurities

LDOS $\rho(-)$  |  LDOS $\rho(+)$  |  $Z$
--- | --- | ---
50 meV  |  | |
24 meV  |  | |
8 meV  |  | |

Short-range valence-bond ($d$-wave) stripes reproduce salient features of experiment. Nodal QP are "protected".

LDOS shows **quasiparticle interference** features at low energies ($<<$ gap), but stripe signatures at high energies.

Vojta, PRB 78, 144508 (2008)
Fermi-surface reconstruction
and Nernst effect
Enhancement of the Nernst effect by stripe order in a high-$T_c$ superconductor

Olivier Cyr-Choinière\textsuperscript{1,*}, R. Daou\textsuperscript{1}, Francis Laliberté\textsuperscript{1}, David LeBoeuf\textsuperscript{1}, Nicolas Doiron-Leyraud\textsuperscript{1}, J. Chang\textsuperscript{1}, J.-Q. Yan\textsuperscript{1}, J.-G. Cheng\textsuperscript{1}, J.-S. Zhou\textsuperscript{1}, J. B. Goodenough\textsuperscript{2}, S. Pyon\textsuperscript{1}, T. Takayama\textsuperscript{3}, H. Takagi\textsuperscript{1,4}, Y. Tanaka\textsuperscript{1,3} & Louis Taillefer\textsuperscript{1,6}

Nernst signal shows two „peaks“:

1) Superconducting fluct at low $T$

2) Fermi surface reconstruction at higher $T$
Mean-field/Boltzmann calculation

Mean-field stripe Hamiltonian (CDW+SDW)

Boltzmann equation for transport coefficients, relaxation time approx. with $k$-independent $\tau$ (for impurity-dominated scattering)

Linear response

\[
\begin{pmatrix}
\vec{J} \\
\vec{Q}
\end{pmatrix}
\begin{pmatrix}
\hat{\sigma} \\
T \hat{\alpha}
\end{pmatrix}
= 
\begin{pmatrix}
\vec{E} \\
-\vec{\nabla} T
\end{pmatrix}
\]

Nernst signal:

\[
\vec{E} = -\hat{\partial} \nabla T \quad \text{(no charge current, } B \text{ field } || z)\]

\[
\hat{\partial}_{yx} = -\frac{\sigma_{xx} \alpha_{yx} - \sigma_{yx} \alpha_{xx}}{\sigma_{xx} \sigma_{yy} - \sigma_{xy} \sigma_{yx}}
\]

Nernst coefficient:

\[
\nu = \hat{\partial}_{yx} / B \quad \text{(~} T \text{ at low } T)
\]

Hackl / Vojta / Sachdev, unpublished
Fermi surface reconstruction: CDW only (period 4)
Fermi surface reconstruction: CDW + SDW (period 8)
Nernst signal for CDW+SDW (period 8)

\[ <S_{\text{max}} > \sim 0.15 \]
Nernst signal for CDW+SDW (period 8)

$\langle S_{\text{max}} \rangle \sim 0.15$

Only electron-like pockets cause large positive Nernst signal!
Nernst signal for CDW+SDW (period 8)

Assuming a mean-field dependence of the stripe order parameter on doping

\[ V_s(x) = V_0 \sqrt{1 - \frac{x}{x_c}} \]

Hackl / Vojta / Sachdev, unpublished
Nernst signal for CDW+SDW (period 8)

Assuming a mean-field dependence of the stripe order parameter on temperature:

\[ V_s(x) = V_0 \sqrt{1 - \frac{T}{T_{sp}}} \]

Hackl / Vojta / Sachdev, unpublished
Eletron pockets are needed for positive Nernst signal!

Period-10 stripe order with 

\[ V_s(x) = V_0 \sqrt{1 - T/T_{sp}} \]

Hackl / Vojta / Sachdev, unpublished
Inter-layer Josephson coupling
Fluctuating 2d pairing in the presence of stripes?
Inter-layer Josephson coupling

Inter-layer tunneling: \[ t_{\perp}(\mathbf{k}) = \frac{t_{\perp}}{4}(\cos(k_x) - \cos(k_y))^2 \]

Calculate free-energy correction from \( t_{\perp} \), with phase difference \( \Delta \theta \)
\[ \Delta F^{(2)}(\Delta \theta) = -J_J(1 + \cos(\Delta \theta)) \]

Quasiparticle calculation:
\[
\Delta F^{(2)} = \frac{1}{\beta N} \sum_{\mathbf{k}n} t_{\perp}(\mathbf{k})^2 \sum_{\alpha, \beta = 0} (-)^{\alpha+\beta} G_{\Psi \mathbf{k}n}^{1,\alpha\beta} G_{\psi \mathbf{k}n}^{2,\beta\alpha}
\]
Inter-layer Josephson coupling: Results

Charge modulation of about 25%

Momentum-resolved contributions to $J_J$

pos

neg

Wollny / Vojta, unpublished
Inter-layer Josephson coupling: Results

Orthogonal stripes with primarily uniform pairing show strongly reduced inter-layer Josephson coupling.

But: Effect may be too small ...

Caveat of mean-field calculations: Assume coherent antinodal QP.
Conclusions

1. Tendencies toward **stripe/bond order** common to underdoped cuprates; often static component is impurity-pinned and weak

2. Proposal:
Nodal quasiparticles are protected by **$d$-wave form factor** of order.
Microscopically: The action is on the oxygen!!!

3. Pocket-induced Nernst signal and reduced inter-layer Josephson coupling may be relevant for explaining various experiments.

Stripy review: arXiv:0901.3145