PHY3054 - Exam 1 - Solutions - Fall 1995 - Selman Hershfield

1. The direction of the force is determined using opposite charges attract and like charges repel. The magnitude of the force is determined using Coulomb's law.

$$\begin{aligned}
\text{F on } Q_3 \text{ by } Q_1 &= \frac{k|-3\mu C||2\mu C|}{(2m)^2} \text{ in } -\text{x direction} = -0.0135N \\
\text{F on } Q_3 \text{ by } Q_2 &= \frac{k|-3\mu C||-1\mu C|}{(1m)^2} \text{ in } +\text{x direction} = +0.0270N \\
\Rightarrow \text{F net on } Q_3 &= +0.0135N \approx \mathbf{0.01N}
\end{aligned}$$

2. The positive rod attracts some of the negative charges from the electroscope to the ball at the top of the electroscope. This leaves the foil positively charged. Grounding the electroscope causes the foil to be neutralized as indicated by the leaves coming together in Fig. 1.2. The sphere at the top of the electroscope is still negatively charged because the negative charges are attracted to the positive rod. When the ground is removed, the electroscope remains negatively charged. Once the rod is removed, this negative charge is distributed over the entire electroscope because there is no reason for it to stay just at the top of the electroscope. **The overall charge on the electroscope is negative, and the foil is negatively charged.**

3. The center $-1\mu C$ charge will be repelled by the $-1\mu C$ charge on the bottom and attracted to the $+1\mu C$ charge on the top, so the vertical component of the total force will be upward. The charges to the left and right of the center charge are both positive, and will both attract the $-1\mu C$ charge; however, the $3\mu C$ charge is larger, and hence the center charge will be pulled to the right. The net force is up and to the right. The only choice in this quadrant is **45 degrees**.

4. To do this problem, we add the forces like vectors using Coulomb's law to determine the magnitude. The direction is determined by opposite charges attract and like charges repel.

Force	x component	y component
F on center by left charge	–0.018 N	0
F on center by right charge	+0.054 N	0
F on center by top charge	0	+0.018 N
F on center by bottom charge	0	+0.018 N
F net on center	+0.036 N	+0.036 N

The magnitude of the net force is

$$F = \sqrt{F_x^2 + F_y^2} = \mathbf{0.05N}.$$

5. **Done correctly.**

6. The electric field in Fig. 3 is the sum of the electric fields due to the two plates. If the plates have area, A, then the magnitude of the electric field due to plate 1 is $|E_1| = |Q_1|/(2\epsilon_o A)$ and the magnitude due to plate 2 is $|E_2| = |Q_2|/(2\epsilon_o A)$. To get the direction of the electric field, use the fact that the electric field goes away from positive charges and towards negative charges. With the convention that E is positive to the right and negative to the left, the net electric field in the three regions is

$$\begin{aligned} \text{E on left side} &= -|E_1| + |E_2| \\ \text{E in middle} &= +|E_1| + |E_2| \\ \text{E on right side} &= +|E_1| - |E_2|. \end{aligned}$$

If $|Q_1| = |Q_2|$, then $|E_1| = |E_2|$, and the electric field is zero on the left side and right sides (Fig. 3.1). On the other hand, if $|Q_1| > |Q_2|$, then $|E_1| > |E_2|$, and the electric field is to the right on the right side and to the left on the left side (Fig. 3.2). Finally, if $|Q_1| < |Q_2|$, then $|E_1| < |E_2|$, and the electric field is to the right on the left side and to the left on the right side (Fig. 3.3). $\Rightarrow$ **All can be correct.**

7. Using the convention that a positive electric field is up and a negative electric field is down, the net electric field at P is

$$E_P = -\frac{|3 \times 10^{-9} C/m^2|}{2\epsilon_o} + \frac{|2 \times 10^{-9} C/m^2|}{2\epsilon_o} + \frac{|1 \times 10^{-9} C/m^2|}{2\epsilon_o} = \mathbf{0V/m}.$$

8. Applying Gauss's law to a sphere going through point Q yields

$$\text{Q(sphere) + Q(shell)} = \epsilon_o E_\perp A = \epsilon_o (2000V/m)\, 4\pi (2m)^2 = 0.89\mu C.$$

Applying Gauss's law to a sphere going through point P yields

$$\text{Q(sphere)} = \epsilon_o E_\perp A = \epsilon_o (1000V/m)\, 4\pi (1m)^2 = 0.11\mu C.$$

Thus, the charge on the shell is $0.89 - 0.11\mu C \approx \mathbf{1\mu C}$.

9. The electric field due to the $1C$ charge is in the positive x-direction at $x = 1m$. In order to cancel this field, the charge at $x = 3m$ must be positive so that its electric field is in the negative x-direction at $x = 1m$. The two fields will cancel if they have the same magnitude.

$$\frac{k(1C)}{(1m)^2} = \frac{kQ}{(2m)^2} \Rightarrow \mathbf{Q = 4C}$$

10. The electric field goes away from the positive charges and towards the negative charges. If you draw a vector diagram for the electric field at the center, you will see that the net electric field is at **126 degrees.**

11. The net electrical potential (voltage) is the sum of the potentials, $V = kQ/r$, due to each of the point charges. Since $r = 1m$ is the same for each point charge, the sum of the potentials is proportional to the sum of the charges. $\Rightarrow V = k(2\mu C)/(1m) = \mathbf{18kV}$.

12. This is a conservation of energy problem. Since the initial velocity is $5 \times 10^5 m/s$, the electron's initial kinetic energy is $K_i = 0.5mv^2 = 1.14 \times 10^{-19} J$. Since the electron is accelerated, its kinetic energy increases and its potential energy decreases: $U_i - U_f = |e|(2V) = 3.2 \times 10^{-19} J$. The final kinetic energy is

$$K_f = K_i + U_i - U_f = 4.34 \times 10^{-19} J \Rightarrow v_f = 9.76 \times 10^5 m/s \approx \mathbf{10 \times 10^5 m/s}.$$

13. The electron is attracted to the positive plate and repelled from the negative plate. Thus, when it is moved from the positive plate to the negative plate, work must be done and its potential energy is increased. The electrical potential or voltage, $V = U/q$, decreases because the electron has a negative charge and U increases. **The potential energy increases, and the electrical potential decreases.**

14. The proton is moving in parabolic motion. Let the initial direction of the proton's motion be in the x direction and the electric field be in the y direction. Since the force is in the y direction, the velocity in the x direction remains constant: $v_x = 5 \times 10^5 m/s$. The initial velocity in the y direction is zero, and the final velocity is:

$$(7.6 \times 10^5 m/s)^2 = (5 \times 10^5 m/s)^2 + (v_{f,y})^2 \Rightarrow v_{f,y} = 5.7 \times 10^5 m/s.$$

From this we can determine the acceleration, force, and electric field in the y direction:

$$a_y = \frac{v_{f,y} - v_{i,y}}{t} = \frac{5.7 \times 10^5 m/s}{0.002 s} = 2.86 \times 10^8 m/s^2 = \frac{F_y}{m_p} = \frac{eE_y}{m_p}.$$

The magnitude of the electric field is **3 V/m**.

15. The potential energy of a set of point charges is the sum of the potential energies of each pair: $U_{ij} = kQ_iQ_j/r^2$. In this problem, the only potential energy contributions which change are those involving the third charge, which is initially at $x = 2m$ and finally at $x = 3m$.

$$U_f - U_i = k(1\mu C)^2 \left(\frac{1}{3m} + \frac{1}{2m} + \frac{1}{1m}\right) - k(1\mu C)^2 \left(\frac{1}{2m} + \frac{1}{1m} + \frac{1}{2m}\right) = \mathbf{-1.5\ mJ}$$

16. The effective capacitance of the part of the circuit containing the $1\mu F$ capacitors is $(4/3)\mu F$. Combining this in series with the $4\mu F$ capacitor gives an effective capacitance of $1\mu F$. When combined in parallel with the $2\mu F$ capacitor, the effective capacitance of the circuit is **3μF**.

17. The connecting lines between capacitors in circuits are wires in real life. These wires can be bent in any manner so long as the connections are not broken. Fig. 9.1 is different from Figs. 9.2 and 9.3 because it has five capacitors while the other two have four capacitors. Fig. 9.2 can be deformed into Fig. 9.3 as can be seen by labelling points with the same potential in the two circuits. This is done in the figure below. Points with the same potential are labelled by numbers, and the letter C indicates a capacitor. **Fig. 9.1 is different. Figs. 9.2 and 9.3 are the same.**

```
        3                  2--C--3
       /|\            1--C--|      |--3
      C | \           |     2--C--3  |
     /  |  \          |              |
--1     C    3--   --1------C-------3--
     \  |  /
      C | C
       \|/
        2
   Fig. 9.2           Fig. 9.3
```

18. Let C be the capacitance of each of the capacitors. The effective capacitance of Fig. 10.1 is $C/3$, and the effective capacitance of Fig. 10.2 is $3C$. Since the potential energy stored in the capacitor network is just $U = 0.5C_{eff}V^2$, the ratio of the energy stored in the two circuits is

$$\frac{U \text{ in Fig. 10.1}}{U \text{ in Fig. 10.2}} = \frac{0.5(C/3)(9V)^2}{0.5(3C)(9V)^2} = \mathbf{1/9}.$$

19. Since capacitors in series have the same charge, both the $1\mu F$ and $3\mu F$ capacitor have a charge of $1\mu C$. The effective capacitance of the $1\mu F$ and $3\mu F$ capacitors in series is $0.75\mu F$ so the voltage across them is $(1\mu C)/(0.75\mu F) = (4/3)V$. This is also the voltage across the right half of the circuit because capacitors in parallel have the same voltage drop. The effective capacitance of the $2\mu F$, $4\mu F$, and $4\mu F$ capacitors in series is $1\mu F$ so the charge on this capacitor is $(1\mu F)(4/3\ V) = (4/3)\mu C$. This is the charge across each of the capacitors in series. The voltage drop across the $2\mu F$ capacitor is

$$\frac{(4/3)\,\mu C}{2\mu F} = \mathbf{(2/3)V}.$$

20. The energy stored in a parallel plate capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{K\epsilon_0 A}{d}V^2,$$

where the dielectric constant, K, is equal to 1 when there is only air between the plates. If K is changed to 2, then **the energy will go up by a factor of 2.**