Solutions - PHY3054 - Final - Fall 1995

1. The forces due to the $-1\mu C$ and $2\mu C$ charges need to be added together as vectors.

$$
\begin{aligned}
F_x \text{ on } 1\mu C \text{ due to } 2\mu C &= -k\frac{|1\mu C||2\mu C|}{(0.1m)^2} \\
F_x \text{ on } 1\mu C \text{ due to -}1\mu C &= k\frac{|1\mu C||\text{-}1\mu C|}{(0.1m)^2}\cos(60) \\
F_x \text{ net} &= 0.45N\text{-}1.80N = \text{-}1.35N
\end{aligned}
$$

$$
\begin{aligned}
F_y \text{ on } 1\mu C \text{ due to } 2\mu C &= 0 \\
F_y \text{ on } 1\mu C \text{ due to -}1\mu C &= k\frac{|1\mu C||\text{-}1\mu C|}{(0.1m)^2}\sin(60) \\
F_y \text{ net} &= 0.78N
\end{aligned}
$$

To determine the any use

$$\theta = \arctan(F_y/F_x) = -30^o \text{ or } +150^o.$$

By drawing a vector diagram, you can see that $\mathbf{+150^o}$ is the correct answer.

2. The magnitude of the net force is

$$|F| = \sqrt{(F_x)^2 + (F_y)^2} = 1.56N \approx \mathbf{1.6N}.$$

3. According to Gauss's Law, the flux $(= E_\perp A)$ is equal to the charge enclosed divided by ϵ_o.

$$
\begin{aligned}
\text{Flux} &= (|E_1| - |E_2|)A = \frac{\sigma A}{\epsilon_o} \\
\rightarrow \sigma &= (|E_1| - |E_2|)\epsilon_o = \mathbf{-8.85 \times 10^{-6} C/m^2}
\end{aligned}
$$

4. There are three forces acting on the ball: gravity (downwards), the electrostatic force qE (to the right), and the tension. Since the net force on the ball is zero,

$$
\begin{aligned}
T\cos(X) &= mg \\
T\sin(X) &= qE \\
\rightarrow \tan(X) &= \frac{qE}{mg} \\
\rightarrow X &= 13.76^o = \mathbf{14^o}.
\end{aligned}
$$

5. The electric potential due to a point charge is $V = kQ/R$. It is inversely proportional to R. Thus, if R is doubled, V goes down by $2 \rightarrow$ **0.5V**.

6. Since the potential due to one point charge is $V = kQ/R$, we can work out the following limits:

$$\begin{aligned} \text{very near the}\, 3\mu C\, \text{charge} &: V > 0 \\ \text{very near the} - 2\mu C\, \text{charge} &: V < 0. \end{aligned}$$

   When one is very far away from both charges, it looks as if one has one $3\mu C - 2\mu C = 1\mu C$ charge so $V > 0$.

   In between the $3\mu C$ charge ($V > 0$) and the $-2\mu C$ charge ($V < 0$), the potential must change sign.

   Also, in between the $-2\mu C$ charge ($V < 0$) and + infinity ($V > 0$), the potential must change sign.

   In Region I, the electrical potential is always positive.

   $\Rightarrow$ **Regions II and III**

7. The effective capacitance of the $2\mu F$ and $6\mu F$ capacitors in parallel is $8\mu F$. The charge on the effective $8\mu F$ capacitor is $2\mu C$ because it is in series with the $1\mu F$ capacitor. Consequently, the voltage across the $8\mu F$ capacitor is $V = Q/C = 0.25V$. For parallel capacitors the voltage is the same so the charge on the $2\mu F$ capacitor is $Q = CV =$ **0.5μC**.

8. Capacitors in parallel have the same voltage drop, while the voltage drops add for capacitors in series. Consequently, A, B, and C, which are in parallel, have the same voltage drop. D and E, which are in series with each other, have half the voltage drop. (**A, B, and C**)

9. In computing the effective resistance start with the inner most circuit elements and work outwards. Adding the two 1Ω resistors in parallel yields 0.5Ω. Next adding the resistor in series gives 1.5Ω. Finally, add the remaining resistor, which is in parallel with the other resistors $\rightarrow$ **0.6Ω**.

10. Let R be the resistance of one resistor and V be the voltage of the battery. With the switch open, the current in the circuit is $V/(2R)$. With the switch close, the current, I, is $V/(2R/3) = 3 \cdot V/(2R)$. **I goes up by a factor of 3.**

11. This is a Kirchoff's Law problem. Let I be the current flowing counterclockwise around the square. We are given that there is no current along the diagonal. Use the two triangles as the two loops:

$$\begin{aligned} 0 &= 3V - 2\Omega I - 0 \\ 0 &= 2V - 0 - IR \end{aligned}$$

    Solving the first equation gives $I = 1.5A$. Substituting into the second equation yields **$\mathbf{R} = 1.33\Omega$**.

12. With B going into the plane of the page, a positively charge particle will travel counterclockwise ($\mathbf{Q_1 > 0}$), an uncharged particle will travel in a straight line ($\mathbf{Q_2 = 0}$), and a negatively charged particle will travel clockwise ($\mathbf{Q_3 < 0}$).

13. The wire produces a magnetic field going into the plane of the page at the position of the electron. This field has magnitude

$$B = \frac{\mu_o(5A)}{2\pi(2\times 10^{-6}m)} = 5T.$$

The electron experience a force **downwards** with magnitude

$$|F| = |q||v||B| = \mathbf{2.4\times 10^{-14}N}.$$

14. Use the superposition principle to add the magnetic fields due to the two wires.

$$\begin{aligned} B &= \frac{\mu_o(3A)}{2\pi(3m)}\,\text{in} + \frac{\mu_o(4A)}{2\pi(1m)}\,\text{out} \\ &= \mathbf{6\times 10^{-7}T\,out\ of\ the\ page} \end{aligned}$$

15. Use the superposition principle to add the magnetic fields due to the six wires. The magnetic field will go clockwise around the wires with current flowing into the plane of the page and counterclockwise around the wires with current flowing out of the plane of the page. Wires on opposite corners with currents flowing in the same direction produce canceling magnetic fields. $\rightarrow$ **210 degrees**

16. As the loop goes from the $3T$ region to the $1T$ region the flux into the page decreases. Consequently, the induced current produces a flux going into the page (**clockwise**). The magnitude of the induced current is

$$\begin{aligned} I &= \frac{1}{R}\left(\frac{v\Delta t(0.01m)(3T)}{\Delta t} - \frac{v\Delta t(0.01m)(1T)}{\Delta t}\right) \\ &= \mathbf{25mA}. \end{aligned}$$

17. When the switch is closed the induced current flows clockwise, producing a magnetic field going into the plane of the page. To oppose this change in flux, the induced current produces a flux going out of the plane of the page (**counterclockwise**).

18. The speed of light in vacuum is $c = 3\times 10^8 m/s$. The time it takes for light to reach the earth from the sun is

$$t = \frac{d}{v} = \frac{1.5\times 10^11m}{3\times 10^8 m/s} = \mathbf{500\,s}.$$

19. The angle of incidence at the first interface is 80^o. The angle of the transmitted light with the normal to the first interface is thus

$$1\sin(80) = 1.5\sin(\theta') \rightarrow \theta' = 41^o.$$

The angle of incidence at the second interface is $90 - 41 = 49^o$. To determine the angle of the transmitted light with the normal to the second interface we must solve

$$1.5\sin(49) = 1\sin(\theta'').$$

There is no solution to this equation because $1.5\sin(49) > 1$. **No light is transmitted.**

20. Light emerging from the second polarizer is polarized at a 45^o angle. In order for no light to emerge from the second polarizer, the right most polarizer must be polarized perpendicular to 45^o, i.e., at **135 degrees**.

21. There are two surfaces in this problem. First compute the image due to the planar surface:

$$\frac{1}{2m} + \frac{1.5}{s'} = 0 \rightarrow s' = -3m.$$

This means that the image is $3m$ to the left of the planar surface or $4m$ to the left of the spherical surface. Next compute image due to the spherical surface:

$$\frac{1.5}{4m} + \frac{1}{s'} = \frac{1 - 1.5}{-1m} \rightarrow s' = 8m.$$

This corresponds to $\mathbf{x = 8m}$.

22. The reflection of the fish is the mirror is a virtual image 1m behind the mirror or 4m below the surface of the water.

$$\frac{1.33}{4m} + \frac{1}{s'} = 0 \rightarrow s' = -3m.$$

The fish appears to be **3m** below the surface of the water.

23. **Point A** The principle rays are

    - A ray coming in horizontally diverges as if it had come from the focal point on the left.
    - A ray through the center of the lens will come out straight.
    - A ray going towards the focal point on the right hand side will come out horizontally.

24. **Point C** The principle rays are:

    - A ray coming in horizontally will go through the focal point.
    - A ray going through the focal point will come out horizontally.
    - A ray hitting the vertex will be reflected back at the opposite angle.
    - A ray going through the center of curvature will be reflected back upon itself.