November 6, 2003

Calculations in Perturbative String Field Theory*

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ABSTRACT

We discuss methods for evaluating the Feynman diagrams of string field theory, with particular emphasis on Witten's version of open string field theory. We explain in some detail how the rules stated by Giddings and Martinec for relating a given diagram to a Polyakov path integral emerge from the Feynman rules.

* To appear in the proceedings of the conference "Perspectives in String Theory" held at the Niels Bohr Institute in Copenhagen, Denmark, 12 - 16 October 1987.

Today I would like to discuss work done in collaboration with Freedman, Giddings and Shapiro[1]. We have developed general techniques for the study of the off-shell dynamics of a string field theory. Although our primary motivation was to understand how Witten's open string field theory describes closed strings, the more general idea was to set up a formalism for studying string dynamics which was not tied to a study of scattering amplitudes.

In ordinary point field theory the effective action formalism provides a valuable tool for such studies. One can calculate the effective action for point field theory as a series in powers of the effective fields wherein the coefficient of the n^{th} power of fields is just the sum of one-particle-irreducible n point functions. The analogous quantity in an open string field theory is a series in powers of string fields, where the coefficients are just one-open-string-irreducible (1OSIR) n string functions. Note that such quantities possess single closed string poles but not single open string poles.

Since Shapiro has presented and discussed our major results on closed strings, including the story about our discovery of closed string poles in unphysical amplitudes at unexpected locations, I shall concentrate in this lecture on explaining the general methods we brought to bear on our problem. Of particular interest is the detailed derivation of the rules stated by Giddings and Martinec[2] for evaluating diagrams in Witten's string field theory. We shall see, for example, that the rule for inserting a factor of the ghost field c along with each physical vertex insertion fixes the overall phase of each diagram in precisely the manner prescribed by the statistics assigned to second quantized ghost fields in the process of gauge fixing[3]. In addition we stress that the rules for calculating the nonzero winding sectors of bosonized ghost fields follow unambiguously from perturbative string field theory. Thus Witten's string field theory provides a direct (brute force) solution to the problem of bosonizing a chiral fermion on any Riemann surface with at least one boundary.

As a simple example of a 1OSIR diagram, consider the one loop tadpole diagram. By definition of irreducible diagrams the external open string propagator is amputated. The diagram is therefore a right circular cylinder of length π, circumference T, and with a "notch" formed by cutting a slit half the length of the cylinder perpendicular to the circular boundaries. The edge of the notch forms a parameter interval $0 \leq \sigma \leq \pi$, the vertex of the notch being at $\sigma = \frac{\pi}{2}$. The diagram depends on the values of coordinate and bosonized ghost fields $x^\mu(\sigma)$ and $\phi(\sigma)$ defined on this interval. A general N string diagram would possess N such notches. The building blocks of each such diagram are open string propagators joined by the familiar Witten vertices. For example, for the tadpole there is a single open string propagator and the vertex identifies the lower half of each propagator end, leaving the notch to carry the labels of the open string entering the diagram. Each open string propagator is given the Schwinger representation

$$\langle 1|\langle 2|\frac{b_0}{\mathcal{L}_0^{(1)}}|\mathcal{V}_2\rangle = \int_0^\infty dT \langle 1|\langle 2|b_0 \exp\{-\mathcal{L}_0^{(1)}T\}|\mathcal{V}_2\rangle$$

where $\mathcal{L}_0^{(1)}$ is the open string zero-moded Virasoro generator for string 1, with ghosts and zero-point energy included. The Schwinger parameters T_i are each integrated from 0 to ∞. The tadpole has only one Schwinger parameter T.

To interpret each diagram as a functional integral defined on a Riemann surface, one simply writes the integrand of each propagator $\langle f|b_0 \exp\{-\mathcal{L}_0 T\}|i\rangle$ as a functional integral defined on a strip of length T and width π, exploiting the fact that $\mathcal{L}_0$ is the Hamiltonian for free quantum fields $x^\mu(\sigma,\tau)$ and $\phi(\sigma,\tau)$. Thus one obtains the diagram as a functional integral over $x^\mu(\sigma,\tau)$ and $\phi(\sigma,\tau)$ defined on a Riemann surface equipped with the trivial metric $g_{ij} = \delta_{ij}$ everywhere away from the vertices. The Witten vertex introduces a conical curvature singularity with angular excess of 180 degrees, corresponding to ghost number $\frac{3}{2}$, at its midpoint. Furthermore the antighost insertion b_0 for each propagator provides precisely the slice independent measure associated with the choice of the T_i as moduli for the Polyakov path integral with this metric[4,2].

We set ourselves the problem of evaluating these diagrams for arbitrary string states entering each notch. To exploit powerful conformal mapping techniques developed by Mandelstam[5], we first attach a strip of width π and length T_e to each notch, and specify the external string variables at the strip end rather than at the notch. We must, of course, eventually set each T_e to 0. Next we label each external string state using an eigenbasis of $\mathcal{L}_0^{(e)}$, so that for each basis element with eigenvalue λ_e the T_e dependence is just the numerical factor $\exp\{-\lambda^e T_e\}$. It is only for $T_e \to \infty$ that one can conformally map the Witten diagram to a figure for which the Neumann function can be simply written down. It is because the T_e dependence of each diagram is exactly known for $\mathcal{L}_0^{(e)}$ eigenstates that we can infer the $T_e \to 0$ limit from the $T_e \to \infty$ limit.

Let me summarize our method. We take each T_e large but finite and map the resulting figure to one for which the Neumann function is known in the limit $T_e \to \infty$. The Neumann function for large but finite T_e can be expanded in the normal modes of each external strip. The coefficients of this latter expansion are then simply related at large T_e to the corresponding normal mode coefficients for the known Neumann function for the limiting figure as $T_e \to \infty$. The exact state dependence of the integrand of each diagram can be evaluated in terms of these expansion coefficients as described in Ref[5]. This procedure determines the integrand up to a state independent function of the T_i, which is proportional to the determinant of the Laplacian on the initial figure. We can compute this factor by using the known transformation properties of such determinants under Weyl transformations to reduce it to the determinant of the Laplacian on the simpler figure. Because of the intricate nature of the conformal mappings, this relationship is quite complicated. However, the slice independence of the Polyakov path integral implies (with a mild caveat) that, at least for external string states with no ghost excitations, these complicated factors are cancelled by the Jacobian arising from the change of variables from T_i, the moduli for the initial conformal frame, to the natural moduli for the final frame. Even so, our final expressions are highly implicit. For example, when we study the closed string poles in the nonplanar one-loop diagram, we obtain explicit results only for the residues of the first few poles.

I would like to devote the remainder of this lecture to a discussion of a few of the trickier points in the above analysis. The first point concerns the pair of 90 degree corners that terminate each of the external string strips. It is important not to round these since we want to maintain the closure property of path integrals. Otherwise we would not have a simple rule for constucting larger diagrams from their irreducible parts. The Weyl transformation for determinants of Laplacians in the presence of sharp corners involves extra terms[6] not included, for example, in the formula given by Alvarez[7]. The correct transformation law is

$$\left(\det \Delta_{\bar{g}}\right)^{-d/2} = e^{-S_{conf}} \left(\det \Delta_{g}\right)^{-d/2} e^{\frac{d}{48}\sum_{corners}\sigma(z_i)} \tag{1}$$

where

$$S_{conf} = -\frac{d}{12\pi}\left[\frac{1}{2}\int_{\mathcal{M}} dA\,\nabla_a\sigma\nabla^a\sigma + \int_{\partial\mathcal{M}} dl\,k\sigma + \frac{1}{2}\int_{\mathcal{M}} dA\,R\sigma\right] \tag{2}$$

and where $\Delta_{\bar{g}}$ is the Laplacian defined with metric $\bar{g} = e^{2\sigma}g$ and Δ_g is that defined with metric g. The last factor in Eq.(1) has been specialized to the case of 90 degree corners between two boundaries with like boundary conditions (e.g. Neumann conditions on both). Also k, the extrinsic curvature of the boundary, contains δ function contributions due to the corners, so that, for example, the Euler character

$$\chi = \frac{1}{2\pi}\int d\ell k\sigma + \frac{1}{4\pi}\int dAR\sigma \tag{3}$$

for a rectangle is 1. The total corner contribution to the logarithm of Eq.(1) is thus

$$\frac{d}{16}\sum_{corners}\sigma(z_i)$$

for Neumann-Neumann or Dirichlet-Dirichlet corners. This contribution reverses sign for Dirichlet-Neumann corners.

To illustrate the importance of the corner contribution, we consider the values of $\ln \det \Delta$ for a rectangle of width L, length T, and various boundary conditions[8]:

$$\ln \det \Delta_{NDND} = -\frac{\pi}{12}\frac{T}{L} + \sum_{k=1}^{\infty} \ln(1 - e^{-\frac{2\pi k T}{L}}) + \frac{1}{2}\ln\frac{T}{L} + \frac{1}{2}\ln 2T \tag{4}$$

$$\ln \det \Delta_{DDDD} = -\frac{\pi}{12}\frac{T}{L} + \sum_{k=1}^{\infty} \ln(1 - e^{-\frac{2\pi k T}{L}}) + \frac{1}{2}\ln\frac{T}{L} - \frac{1}{2}\ln 2T \tag{5}$$

$$\ln \det \Delta'_{NNNN} = -\frac{\pi}{12}\frac{T}{L} + \sum_{k=1}^{\infty} \ln(1 - e^{-\frac{2\pi k T}{L}}) + \frac{1}{2}\ln\frac{T}{L} - \frac{1}{2}\ln 2T \tag{6}$$

where Δ_{NDND} is the Laplacian for Neumann conditions on the sides of length T and Dirichlet conditions on those of length L. In $\det \Delta'_{NNNN}$, the zero eigenvalue of the Laplacian has been replaced by $\frac{1}{4LT}$, in accord with the definition of the string propagator between momentum eigenstates as the Fourier transform of that between position eigenstates. The $\ln 2T$ terms are consistent with Eqs.(1) and (2) for constant scale transformations.*

We shall typically map the ends of the external string strips to arcs which become semi-circular and infinitesimally small in the limit $T_e \to \infty$. These arcs meet the image of the open string boundaries at 90 degrees and these sharp corners are responsible for a singular behavior in $\det \Delta$ for the new region. To see this, we can apply Eqs (1) and (2) to the determinant for a rectangle to get

$\star$ In Ref[8] the functional integrals and determinants are defined on a lattice so that closure is manifest. Then T and L enter in the combinations $\frac{T}{a}$ and $\frac{L}{a}$ where a is the lattice spacing. The spacing a cancels in the ratio of T to L, but remains in the last terms of Eqs.(4, 5, 6) which should then involve $\ln \frac{2T}{a}$. This uncancelled a dependence is necessary for the proper normalization of position eigenstates. For properly normalized eigenstates of $\mathcal{L}_0$, the a dependence of each diagram cancels in the critical dimension.

$\det\Delta$ for a semi-annulus of radii ϵ and R:

$$\det\Delta^{ann}_{DDDD} = \det\Delta^{ann}_{NNNN} = (const)\epsilon^{-\frac{1}{12}}R^{-\frac{5}{12}}\prod_{k=1}^{\infty}\Big(1-(\frac{\epsilon}{R})^{2k}\Big). \tag{7}$$

From this equation we see that the limit $\epsilon \to 0$ or $R \to \infty$ induces singular behavior in the determinant. In the general situation where there are several small and perhaps one large semi-circle, it is clear from the local nature of path integrals that the leading behavor as the radii go to 0 or ∞ will factorize:

$$\det\Delta(\{\epsilon\}, R) \underset{\epsilon\to 0}{\widetilde{\quad}} (const)\det\Delta_0\prod_a \epsilon_a^{-\frac{1}{12}}R^{-\frac{5}{12}} \tag{8}$$

where we have taken Neumann boundary conditions everywhere, and where $\det\Delta_0$ is the determinant of the Laplacian for the limiting region with the semi-circles ironed out.

Now let's apply these ideas to our problem. We can choose a standard coordinate chart for each external strip: $\rho_e = \tau_e + i\sigma_e$, where $0 \le \tau_e \le T_e$ and $0 \le \sigma_e \le \pi$. Then the required map will have the behavior

$$z(\rho) \underset{\tau_e\to\infty}{\widetilde{\quad}} z_e + \xi_e e^{-\rho_e} \tag{9}$$

so the radius of the small semi-circle will be

$$\epsilon_e = |\xi_e|e^{-T_e}. \tag{10}$$

Thus the determinant of the Laplacian for the target region will have the behavior

$$\det\Delta_g \underset{T_e\to\infty}{\widetilde{\quad}} \prod_e\Big(|\xi_e|e^{-T_e}\Big)^{-1/12} \det\Delta_0. \tag{11}$$

On the other hand $\det\Delta_{\bar g}$, the determinant in the initial conformal frame is related to this one by Eqs(1) and (2) where the scale factor is given by

$$e^{2\sigma} = \left|\frac{d\rho}{dz}\right|^2 \quad \widetilde{\rho = T_e + i\sigma \to \infty} \, |\xi_e|^{-2} e^{2T_e}$$

and so in the limit the last factor in Eq(1) cancels the prefactors in Eq (11). Thus we may summarize the discussion of corner effects by rewriting Eq(1) as

$$(\det \Delta_{\bar{g}})^{-d/2} \quad \widetilde{T_e \to \infty} \quad e^{-S_{conf}} (\det \Delta_0)^{-d/2} \tag{12}$$

with S_{conf} still given by Eq(2), and where Δ_0 is the Laplacian on the new region with the infinitesimal semi-circles ironed out.

As pointed out in Ref[2], it is convenient to solve for σ in terms of the curvatures $\bar{R}$, R, $\bar{k}$, and k of the initial and final conformal frames. Inserting this solution into S_{conf} leads to

$$\begin{aligned} S_{conf} = \frac{d}{24\pi^2}\Big[\frac{1}{8}\int d\bar{A}\, d\bar{A}'\, \bar{R}(z)\bar{R}(z')N(z,z') \\ + \frac{1}{2}\int d\bar{A}\, d\bar{l}'\, \bar{R}(z)\bar{k}(z')N(z,z') + \frac{1}{2}\int d\bar{l}\, d\bar{l}'\, \bar{k}(z)\bar{k}(z')N(z,z') \\ - \frac{1}{8}\int dA\, dA'\, R(z)R(z')N(z,z') - \frac{1}{2}\int dA\, dl'\, R(z)k(z')N(z,z') \\ - \frac{1}{2}\int dl\, dl'\, k(z)k(z')N(z,z') - 4\pi^2\chi \int dA\, \mu(z)\sigma(z)\Big]. \end{aligned} \tag{13}$$

where N is the Neumann function:

$$\Delta_z N(z,z') = -2\pi\delta^2(z,z') + 2\pi\mu(z). \tag{14}$$

with μ a background charge density to handle the outflow of flux. One can then consider the Weyl transformation of $(\det \Delta_{\bar{g}})^{-d/2} Z_{ghost}(\bar{J}_e)$ where the generating functional for bosonized ghosts is given by

$$Z_{gh}[J_e] = \int \mathcal{D}\phi \exp\Big\{-\frac{1}{4\pi}\int dA\,\phi\Delta\phi \\ - \frac{3i}{2\sqrt{2}\pi}\left[\frac{1}{2}\int dA\,R\phi + \int dl\,k\phi\right] + i\int dA\,J_e\phi\Big\}. \tag{15}$$

and J_e describes the "non-geometrical" part of the bosonized ghost dependence of the string field theory diagram. It includes chiral point insertions of ghost number -1 arising from the b_0 insertions on each propagator. It also includes a distribution of ghost number density $g(\sigma)$ at the end of each external strip. However, the corner δ function contributions to k have included part of $g(\sigma)$ in S_{ghost}, so J_e has this part subtracted out:

$$J_e(\rho) \underset{\rho \to T_e + i\sigma}{\widetilde{\quad}} \{g(\sigma) + \frac{3}{4}[\delta(\sigma + \delta(\pi - \sigma)]\}\delta(\tau - T_e). \tag{16}$$

One then finds "Weyl invariance" of the combination $(\det\Delta_{\bar{g}})^{-d/2} Z_{ghost}$ provided $d = 26$:

$$\big(\det\Delta_{\bar{g}}\big)^{-13} Z^{\bar{g}}_{gh} \underset{T_e \to \infty}{\widetilde{\quad}} \big(\det\Delta_0\big)^{-27/2} \exp\Bigg\{+\frac{9}{16\pi^2}\Big[\frac{1}{4}\int dA\,dA'\,RR'N(z,z') \\ + \int dA\,dl'\,Rk'N(z,z') + \int dl\,dl'\,kk'N(z,z')\Big] \\ - \frac{3}{2\sqrt{2}\pi}\int d\bar{A}\,\bar{J}_e\Big[\frac{1}{2}\int dA'\,R'N(z,z') + \int dl'\,k'N(z,z')\Big] \\ + \frac{3}{\sqrt{2}}\int d\bar{A}\,\sigma(z)\bar{J}_e + \frac{1}{2}\int d\bar{A}\,d\bar{A}'\,\bar{J}_e\bar{J}'_e N(z,z')\Bigg\}. \tag{17}$$

This formula is quite general but, in the following, I shall discuss it in the context of one loop calculations. In this case one maps the Witten diagrams to a cylinder: i.e. a rectangle in the complex z plane with a pair of opposite sides identified. Then the ends of the external strips map in the limit $T_e \to \infty$ to semi-circular arcs centered on points on the top and bottom boundaries. Here we take

$R = 0$, and $k \neq 0$ only on the semi-circles and at the corners where the semi-circles join the rectangular boundaries. Eq(17) shows that the effect of k is to shift $\bar{J}_e$ in the second and last terms of the exponent by an amount which cancels the corner δ functions in Eq(16) and adds to $g(\sigma)$ a uniform ghost number density with ghost number $+\frac{3}{2}$. Since physical states in the cohomology of the world-sheet BRST charge Q have ghost number $-\frac{1}{2}$, these will be described in the new frame by ghost number $+1$ insertions. For unphysical states, we get dramatic simplifications for those with ghost number $-\frac{1}{2}$ and no ghost excitations, and we restrict consideration to this sector. In this case the exponential in Eq(17) reduces to a product of factors

$$\left(\frac{dz}{d\rho_i}\right)^2 \qquad or \qquad \left(\frac{d\bar{z}}{d\bar{\rho}_i}\right)^2 \tag{18}$$

for each b or $\bar{b}$ insertion on the propagators and a product of factors

$$\frac{1}{|\xi_e|}e^{T_e} \tag{19}$$

for each external line. The ξ in Eq(19) is defined in Eq(9). The factor e^{T_e} for each external line simply provides the familiar zero-point energy of the open string tachyons, i.e. the amplitude has the behavior $e^{-(L_0-1)T_e}$ as each $T_e \to \infty$ where L_0 is the coordinate part of $\mathcal{L}_0$. (Recall that we have restricted to states with no ghost excitations.) The factors in Eq(18) are precisely the factors expected from the tensor transformation properties of the fermionic anti-ghost fields. If the external strings have noghost excitation, we can therefore interpret the RHS of Eq(17) as $(\det \Delta_0)^{-13}$ times the ghost field correlator

$$\left\langle \cdots \left(\frac{dz}{d\rho_i}\right)^2 b(z_i) \cdots \frac{1}{|\xi_e|} e^{T_e} e^{+\frac{i\phi(z_e)}{\sqrt{2}}} \cdots \right\rangle. \tag{20}$$

This is not quite the rule stated by Giddings and Martinec: they prescribe in place of the $e^{i\phi/\sqrt{2}}$ a chiral $c(z_e)$ insertion for each external line. The relation between these insertions is given by the standard bosonization rules for free fermions on a strip:

$$c_\pm(z) =: \exp \frac{i}{\sqrt{2}} \left[\phi(z) \pm i \int\limits_0^y dy' \, \partial_x \phi(x, y') \right] : \tag{21}$$

Clearly for the boundary at $y = 0$, they are the same. But on the other boundary they differ

$$c_\pm(x, \pi) =: e^{i\phi(x,\pi)/\sqrt{2}} : (-)^{g_0 + 1/2} \tag{22}$$

where we take an open string defined on a strip of width π. The difference is significant since the two operators weight circulating string states of different ghost number differently. In fact we have not yet described the detailed sign assignments that follow from perturbative string field theory as discussed in Ref[3]. As we shall see they have a simple interpretation. They prescribe the c insertion as well as an unambiguous treatment of ghost winding number.

For simplicity we consider these issues only at one loop. Then we can represent a one loop string field theory diagram as a supertrace

$$\mathcal{M}_{1 \ loop} = \mathrm{Tr} \Big\{ (-)^{g_0 + 1/2} V_{e_1} D V_{e_2} \cdots V_{e_M} D \Big\} \tag{23}$$

where we have simplified life by taking only ghost number $2n - 1/2$ external strings. The factor $(-)^{g_0+1/2}$ is the standard minus sign required when ghost number $2n + 1/2$ strings circulate the loop: these are space-time fermions since they are Feynman-Fadeev-Popov ghosts[3]. To convert (23)to a path integral, we first represent

$$\mathcal{M}(\phi_f, \phi_i) = \langle \phi_f | \, V_{e_1} D V_{e_2} \cdots V_{e_M} D \, | \phi_i \rangle \tag{24}$$

as a path integral and pass to ghost number density (momentum) representation by Fourier transform

$$\tilde{\mathcal{M}}(g_f, g_i) = \int d\phi_f\, d\phi_i\; \exp\left\{-i\int d\sigma\, \frac{g_f(\sigma)\phi_f(\sigma) - g_i(\sigma)\phi_i(\sigma)}{\sqrt{2}}\right\}\mathcal{M}(\phi_f, \phi_i). \tag{25}$$

Then a straightforward calculation yields

$$\begin{aligned}\mathcal{M}_{1\ loop} =& \sum_{g_0\in Z+1/2}(-)^{g_0+1/2}\int d\hat{g}\tilde{\mathcal{M}}(g,g)\\ =&2\pi\sum_{m\in Z}\int d\phi_f\, d\phi_i\delta\Big(\frac{\phi_f(\sigma)-\phi_i(\sigma)}{\sqrt{2}} - (2m+1)\pi\Big)e^{i\pi(m+1/2)}\mathcal{M}(\phi_f, \phi_i).\end{aligned} \tag{26}$$

Thus we have derived a winding number interpretation: one integrates over fields satisfying the winding condition

$$\phi_f(\sigma) - \phi_i(\sigma) = (2m+1)\pi\sqrt{2} \tag{27}$$

and sums over winding sectors m with a prescribed phase.

In addition to the phases in Eq(26), perturbative string field theory also prescribes additional phases such that the effective twist operator is $(-)^{R_f}$ where

$$R_f = \sum_{n=1}^{\infty} n(c_{-n}b_n + b_{-n}c_n) = \sum_{n=1}^{\infty} g_{-n}g_n + \frac{g_0^2 - 1/4}{2}. \tag{28}$$

Thus if we identify the vertex insertions $e^{+i\phi/\sqrt{2}}$ at the lower boundary with c, the string field theory prescribes that those at the upper boundary are

$$(-)^{R_f}\, e^{i\phi(\sigma)/\sqrt{2}}\, (-)^{R_f} = e^{i\phi(\pi-\sigma)/\sqrt{2}}\, (-)^{g_0+1/2}. \tag{29}$$

since $e^{i\phi(\sigma)/\sqrt{2}}$ carries ghost number $+1$. The net effect of the signs in (26) and (29) is therefore to replace each external line insertion $e^{i\phi(z_e)/\sqrt{2}}$ by $c(z_e)$ in accord with the rules stated by Giddings and Martinec. It is amusing how the field theory supplies the signs necessary for this interpretation: As stressed in Ref[3], they are supplied by the usual Fermi statistics assigned to Feynman-Fadeev-Popov ghosts.

Finally, let me summarize the results of our analysis. A given diagram can be expressed as an integral over the propagator lengths T_i which serve as moduli for the associated Riemann surface. For external string states with no ghost excitations and ghost number $-\frac{1}{2}$, the integrand can be put in the form:

$$\prod_e \Big[\frac{1}{|\xi_e|} e^{T_e}\Big] \Big\langle \prod_e c^z(x_e) \prod_i \int d\sigma_i \Big[\big(\frac{dz}{d\rho_i}\big)^2 b_{zz} + \big(\frac{d\bar{z}}{d\bar{\rho}_i}\big)^2 b_{\bar{z}\bar{z}}\Big] \Big\rangle \big(\det \Delta_0\big)^{-13} \langle F| \,. \quad (30)$$

Here $\langle F|$ is a bra in the multi-string state space, and has the general form

$$\langle F| = \langle 0| \exp\Big\{ \frac{1}{2} \sum a_m^r \cdot a_n^s N_{mn}^{rs}(T_i) \Big\} e^{-\sum_e T_e L_0^e}, \quad (31)$$

where the N_{mn}^{rs} are suitable coefficients in the normal mode expansion of the Neumann function

$$N(\rho_r, \rho_s) = N_{Cylinder}\big(z(\rho_r), z(\rho_s')\big)$$

on the r^{th} and s^{th} external strings. In (31) L_0^e is just the coordinate part of $\mathcal{L}_0^e$:

$$L_0^e = \sum_{n=1}^{\infty} a_{-n} \cdot a_n + p^2.$$

The map from the Witten diagram to the cylinder is given by the function $z(\rho)$. The derivative $d\rho/dz$ is an explicit function of z which must be integrated and inverted to obtain $z(\rho)$. Although this procedure cannot be exactly carried out for general values of the moduli, we can carry it out in certain limits. In particular closed string singularities arise from the region where certain of the T_i's are small, and we can handle the inversion problem in that limit.

A rather nasty complication in Eq(30) is the presence of line integrals over the anti-ghost insertions which explicitly involve the mapping function, e.g.

$$\int_0^{\pi} d\sigma_i \left(\frac{dz}{d\rho_i}\right)^2 b(z_i) = -i \int_C dz_i \left(\frac{dz}{d\rho_i}\right) b(z_i). \quad (32)$$

On the other hand these complicated expressions are just the transforms of the correct measure factors for the moduli in the original conformal frame. Those measure factors are designed to give a gauge slice independent result at least for conformally invariant external states. Thus, if we apply (30) to physical states and change variables from the Witten moduli T_i to the natural moduli for the target conformal frame (in our case the cylinder), the line integrals in (30) must become the appropriate line integrals for the new frame. This statement essentially implies an identity

$$\begin{aligned} &\frac{\partial(T_i)}{\partial(q,x_e)}\Big\langle\prod_e c(x_e)\prod_k\Big(\int\limits_{C_k} dz_k\frac{dz}{d\rho_k}b-\int\limits_{\bar{C}_k} d\bar{z}_k\frac{d\bar{z}}{d\bar{\rho}_k}\bar{b}\Big)\Big\rangle \\ &\quad=\frac{1}{q}\Big\langle\prod_e c(x_e)\prod_k\Big(\int\limits_{C'_k} dz_i\, b(z_k)-\int\limits_{\bar{C}'_k}\bar{b}(\bar{z}_k)d\bar{z}_k\Big)\Big\rangle, \end{aligned} \tag{33}$$

the right hand side of which is independent of the mapping function. One should be able to prove (33) directly, although we have so far failed in the attempt. It is an important gap to fill since it is a logical possibility that slice independence is realized in a more complicated way. After all slice independence only need apply to the integrals over moduli and for physical states; (33) is a local and state independent identity. In every test case to date, slice independence has been realized via a local identity on the integrands such as (33), but as far as I know no one has supplied an airtight proof.

As our major application of these techniques, we have used them to extract information on closed strings in Witten's open string field theory. For a discussion of these results, I refer you to our original paper[1] and to the lecture delivered by Professor Shapiro at this conference.

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